Non-perturbative studies of gluons and gluinos on the lattice

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$\mathcal{N}=1$ supersymmetric Yang-Mills

Why study supersymmetric gauge theories on the lattice?

- Extensions of the Standard Model of particle physics: Non-perturbative SUSY effects important to introduce SUSY breaking at low energies.
- Theoretical concepts: SUSY has led to very powerful analytical approaches to understand strong interactions, which need to be complemented and extended by numerical methods.

Specific for $\mathcal{N}=1$ supersymmetric Yang-Mills theory

- gauge sector of SUSY extension of Standard Model
- simplest model with SUSY and local gauge invariance
- Orientifold planar equivalence: SUSY Yang-Mills theory with N_c colours is equivalent to QCD with a single quark flavour, $N_f=1$ QCD, in the limit $N_c\to\infty$ with Quarks in antisymmetric repr. of SU(N_c).
- continuity to semiclassical regime

Vector supermultiplet:

- Gauge field $A^a_\mu(x)$, $a=1,\dots,N_c^2-1$, "Gluon" Gauge group SU(N_c)
- Majorana-spinor field $\lambda^a(x)$, $\overline{\lambda}=\lambda^T C$, "Gluino" adjoint representation: $\mathcal{D}_\mu\lambda^a=\partial_\mu\lambda^a+g\,f_{abc}A^b_\mu\lambda^c$
- (auxiliary field $D^a(x)$)

Lagrangean:

$$\mathcal{L}=\int d^2\theta\,\mathrm{Tr}(W^AW_A)+\mathrm{h.c.}=\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\frac{1}{2}\overline{\lambda}^a\gamma_\mu(\mathcal{D}_\mu\lambda)^a+\frac{1}{2}D^aD^a$$

- SUSY: (on-shell) $\delta A^a_\mu=-2\,\mathrm{i}\,\overline{\lambda}^a\gamma_\mu\varepsilon\,,\quad \delta\lambda^a=-\sigma_{\mu\nu}F^a_{\mu\nu}\varepsilon$
- Gluino mass term $m_{\tilde{g}}\,\overline{\lambda}^a\lambda^a$ breaks SUSY softly.
- Differences to QCD:

$$\lambda:\ 1.)\ \text{Majorana},\quad \text{"}N_f=\tfrac{1}{2}\text{"}$$
$$2.)\ \text{adjoint representation of SU}(N_c)$$

- Spontaneous breaking of chiral symmetry: U(1)$_\lambda$ (R-symmetry): $\lambda'=\mathrm{e}^{-\mathrm{i}\varphi\gamma_5}\lambda$, $\overline{\lambda}'=\overline{\lambda}\,\mathrm{e}^{-\mathrm{i}\varphi\gamma_5}$
  Anomaly: $\mathrm{U}(1)_\lambda\ \to\ Z_{2N_c}$
  Spontaneous breaking $Z_{2N_c}\ \to\ Z_2$ by Gluino condensate $<\lambda\lambda>\neq 0$
  $\leftrightarrow$ first order phase transition at $m_{\tilde{g}}=0$

Supersymmetric QCD

- additional quarks ψ and squarks Φ_i in fundamental representation
- covariant derivatives, mass terms for (ψ, Φ_i)
- Yukawa interactions and scalar potential

$$i\sqrt{2}g\bar{\lambda}^a\left(\Phi_1^\dagger T^aP_+ + \Phi_2T^aP_-\right)\psi$$
$$-\,i\sqrt{2}g\bar{\psi}\left(P_-T^a\Phi_1+P_+T^a\Phi_2^\dagger\right)\lambda^a$$
$$\frac{g^2}{2}\left(\Phi_1^\dagger T^a\Phi_1-\Phi_2^\dagger T^a\Phi_2\right)^2.$$

SUSY Yang-Mills Theory on the Lattice

SUSY breaking on the lattice:

- Local lattice theory: SUSY breaking unavoidable at any finite lattice spacing
- No general solution by Ginsparg-Wilson relation found (so far)
- Necessary to find specific solution for the model under consideration

Approach for SUSY Yang-Mills theory (Curci, Veneziano)

1. Wilson action:

$$S=-\frac{\beta}{N_c}\sum_p \mathrm{Re\,Tr}\,U_p$$
$$+\frac{1}{2}\sum_x\left\{\overline{\lambda}^a_x\lambda^a_x-K\sum_{\mu=1}^{4}\left[\overline{\lambda}^a_{x+\hat{\mu}}V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x+\overline{\lambda}^a_xV^t_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat{\mu}}\right]\right\}$$

$\beta=\frac{2N_c}{g^2}$, $K=\frac{1}{2m_0+8}$ hopping parameter, m_0 : bare gluino mass

$V_{ab,x\mu}=2\,\mathrm{Tr}\,(U^\dagger_{x\mu}T_aU_{x\mu}T_b)$, adjoint link variables

In recent studies of SU(3) SUSY Yang-Mills: one-loop $O(a)$ improvement by clover term.

2. Tuning towards the chiral supersymmetric continuum limit:
   - Wilson term breaks chiral symmetry and SUSY
   - only tuning of gluino mass required to recover both symmetries in the continuum limit
   - practical implementation: extrapolation to vanishing adjoint pion mass ($m_{a-\pi}$), cross check with SUSY Ward identities

Challenging extension towards supersymmetric QCD

- Yukawa couplings and scalar potential need to be fine tuned
- order of 10 tuning parameters
- reduced tuning for chiral supersymmetric formulations (overlap fermions)

Non-perturbative Problems

- Spontaneous breaking of chiral symmetry $Z_{2N_c}\ \to\ Z_2$ $\longleftrightarrow$ Gluino condensate $<\lambda\lambda>\neq 0$
- Spectrum of bound states $\to$ Supermultiplets
- Confinement of static quarks
- Breaking of SUSY in the continuum limit?
- SUSY restauration on the lattice
- Check predictions from effective Lagrangeans (Veneziano, Yankielowicz, ...)

Spectrum of bound states

Expect colour neutral bound states of gluons and gluinos $\to$ Supermultiplets

Predictions from effective Lagrangeans[1, 2]:

chiral supermultiplet (Veneziano, Yankielowicz)

- 0^- gluinoball a - η' $\sim\ \overline{\lambda}\gamma_5\lambda$
- 0^+ gluinoball a - f_0 $\sim\ \overline{\lambda}\lambda$
- spin $\frac{1}{2}$ gluino-glueball $\sim\ \sigma_{\mu\nu}\,\mathrm{Tr}\,(F_{\mu\nu}\lambda)$

Generalization (Farrar, Gabadadze, Schwetz) additional chiral supermultiplet

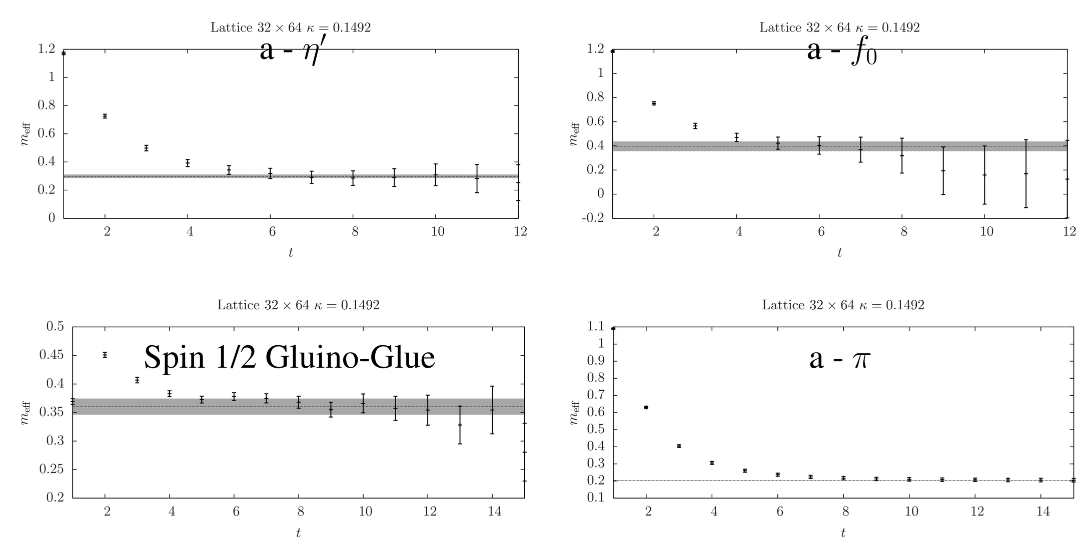
- 0^- glueball
- 0^+ glueball
- gluino-glueball

mixing of multiplets possible

Bound states on the lattice

- Glueballs: 0^+, $0^-\cong$
- Gluino-glueballs, Spin $\frac{1}{2}$ Majorana: $\chi_\alpha\simeq\frac{1}{2}F^a_{\mu\nu}(\sigma_{\mu\nu})_{\alpha\beta}\lambda^a_\beta$
- Gluino-balls: $\overline{\lambda}\gamma_5\lambda$: a - η', 0^-, $\overline{\lambda}\lambda$: a - f_0, 0^+
- Mixing of Glueballs and Gluino-balls

Effective masses



Simulation details

Algorithms

- TS-PHMC and RHMC algorithm

Sign Problem

Fermion action:

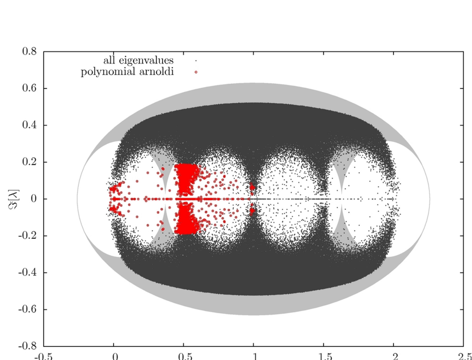
$$S_f=\frac{1}{2}\overline{\lambda}Q\lambda=\frac{1}{2}\lambda M\lambda\,,\qquad M\equiv CQ$$
$$\int[d\lambda]\,\mathrm{e}^{-S_f}=\mathrm{Pf}(M)=\pm\sqrt{\det Q}$$

Effective gauge field action

$$S_{\text{eff}}=-\frac{\beta}{N_c}\sum_p\mathrm{Re\,Tr}\,U_p-\frac{1}{2}\log\det Q[U]$$

Reweighting with sign Pf(M)

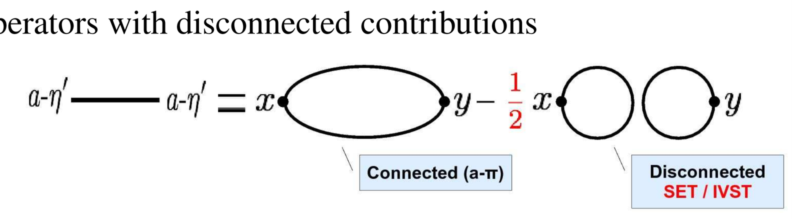
- Wilson fermions: mild sign problem, vanishes in continuum limit
- sign Pf(M): real negative eigenvalues of Q
- not relevant for the current parameter range



Challenging measurement of bound state operators

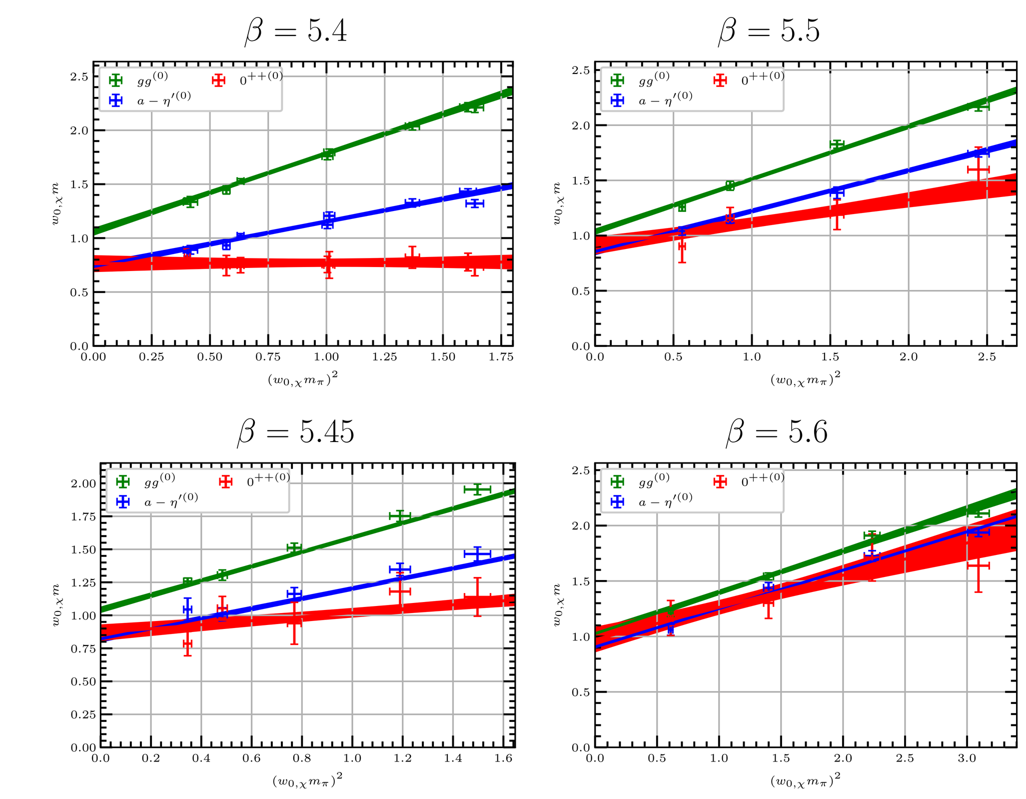
- Flavour singlet meson operators with disconnected contributions

e.g. correlators of a - η':

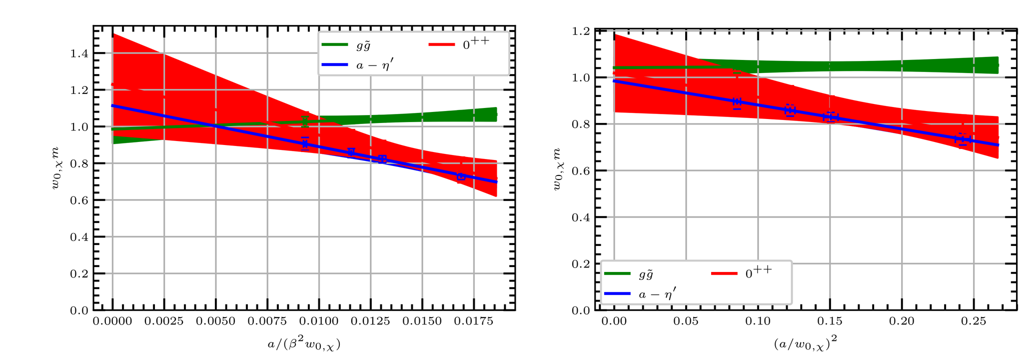


- Gluino-glue: variational methods (optimized ground state overlap)
- Glueballs, mixed with Gluino-balls: variational methods in large operator basis
- Baryon operators: spectacle contributions

Extrapolation to the supersymmetric continuum limit of SU(3) SUSY Yang-Mills theory



Continuum limit

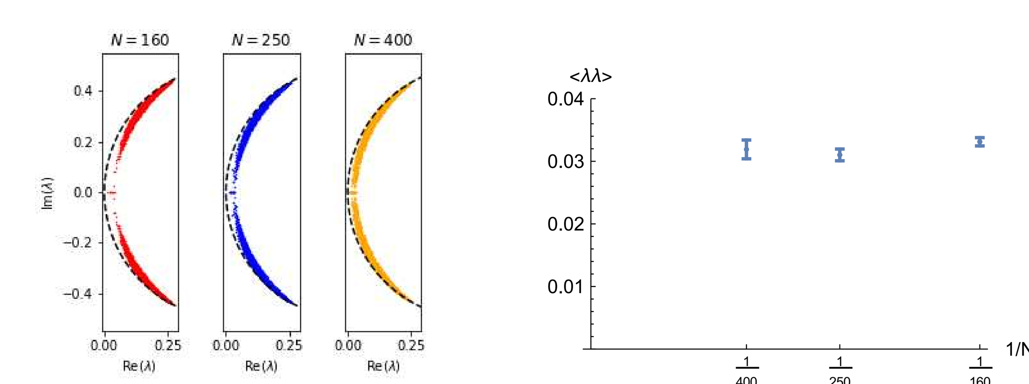


parameter ranges: $0.2<am_{a-\pi}<0.7$, lattice spacing $0.053\,\mathrm{fm}<a<0.082\,\mathrm{fm}$, lattice sizes $12^3\times 24$ to $24^3\times 48$

Fit	$w_0m_{g\tilde{g}}$	$w_0m_{0^{++}}$	$w_0m_{a-\eta'}$
linear fit	0.917(91)	1.15(30)	1.05(10)
quadratic fit	0.991(55)	0.97(18)	0.950(63)
SU(2) SYM	0.93(6)	1.3(2)	0.98(6)

(in units of the gradient flow scale w_0)

SUSY Yang-Mills with overlap fermions



- overlap fermions implement chirals symmetry on the lattice
- eigenvalue spectrum on circle
- RHMC + Overlap: stable polynomial approximation of sign-function to order N:

$$D_{\text{ov}}=\frac{1}{2}+\frac{1}{2}\gamma_5\mathrm{sign}(\gamma_5D_W)$$

Summary

- finalized analysis of SU(2) SUSY Yang-Mills particle spectrum [3]
- investigated phase transitions of SU(2) SUSY Yang-Mills [4, 5]
- most recently: final continuum extrapolations of the bound state spectrum of SU(3) SUSY Yang-Mills (gauge group of QCD) [6, 7]

SUSY breaking under control

- formation of bound state multiplets verified by numerical investigations
- SUSY restoration verified by Ward identities [8]

Outlook

- Overlap formulation as alternative, in particular for investigations of chiral symmetry breaking
- extensions to SQCD under investigation: mixed representations (fund.+adjoint), scalar fields, tuning

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