Proton structure from lattice QCD

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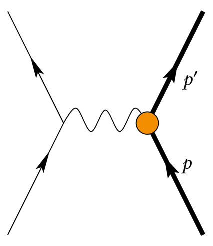
Electromagnetic form factors



The distributions of charge and magnetization in a proton are probed in elastic electron-proton scattering.

Photon-proton vertex is parameterized by two form factors,

$$\langle p'|J_\mu|p\rangle = \bar u(p')\left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}(p'-p)^\nu}{2m}F_2(Q^2)\right]u(p),\quad Q^2=-(p'-p)^2.$$

These combine to form the electric and magnetic form factors,

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m^2}F_2(Q^2) \quad \to \text{Fourier transform of charge density}$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \quad \to \text{Fourier transf. of magnetization density.}$$

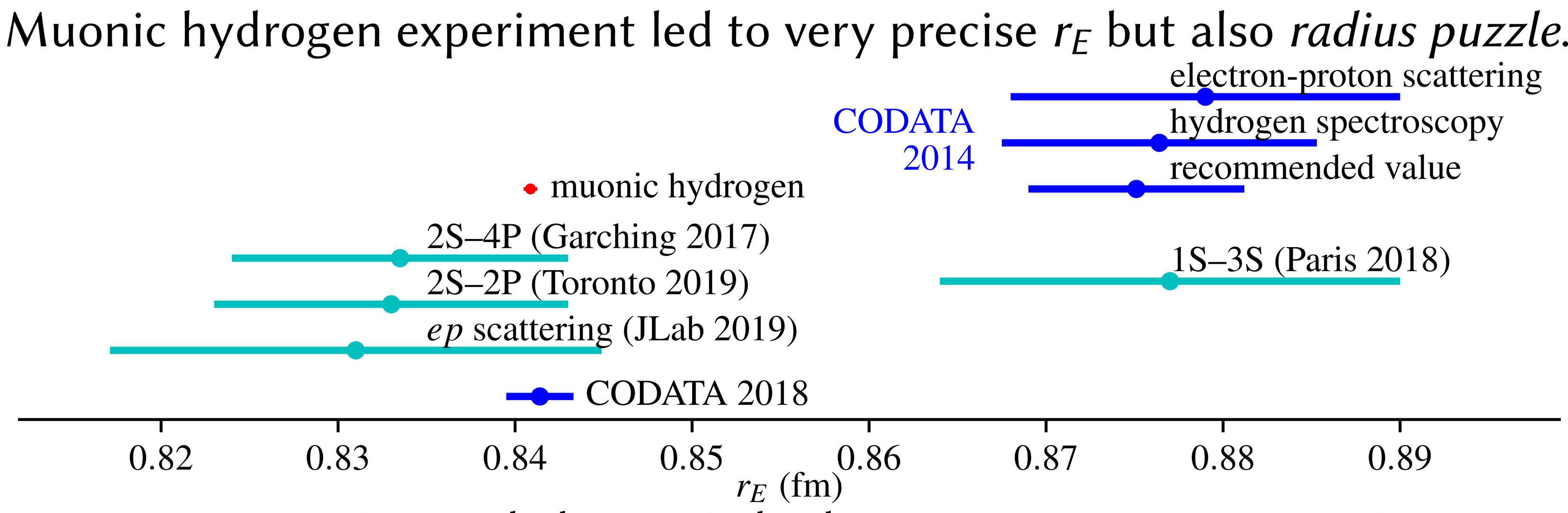
Near $Q^2=0$ they contain key properties of the proton:

electric charge $1 = G_E(0)$

rms charge radius $r_E^2 = -6\left.\frac{dG_E}{dQ^2}\right|_{Q^2=0}$

magnetic moment $\mu = G_M(0)$.

Muonic hydrogen experiment led to very precise r_E but also *radius puzzle*.



Newer *ep* scattering and electronic hydrogen spectroscopy experiments have cast doubt on precision of older measurements.

A. Beyer *et al.*, Science **358**, 79–85 (2017), H. Fleurbaey *et al.*, Phys. Rev. Lett. **120**, 183001 (2018), N. Bezginov *et al.*, Science **365**, 1007–1012 (2019), W. Xiong *et al.*, Nature **575**, 147–150 (2019).

Finite-volume effects in form factors (preliminary)

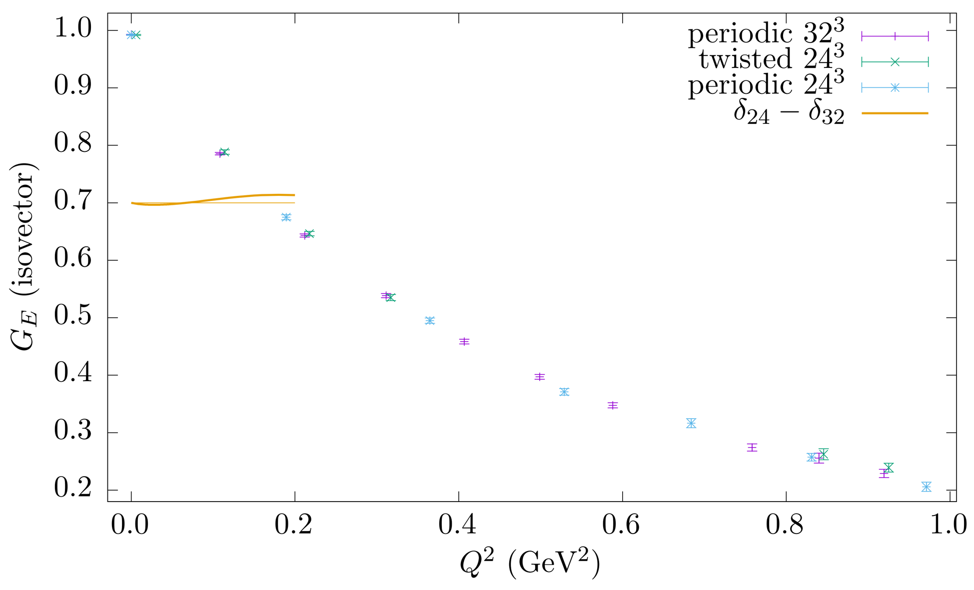
Lattice calculations are done in periodic spatial volume of size L^3.
Generically effect on nucleon observables is suppressed $\sim e^{-m_\pi L}$.
Volume also constrains Q^2: momenta $p_j = 2\pi n/L$, $n\in\mathbb{Z}$.
How to study L dependence at fixed Q^2? Use twisted boundary cond.:

$$q(\vec x) = e^{i\theta}q(\vec x + \hat j L) \implies p_j = (2\pi n+\theta)/L,\ n\in\mathbb{Z}.$$

Lattice setup: $m_\pi \approx 250$ MeV, $a = 0.116$ fm, two ensembles:

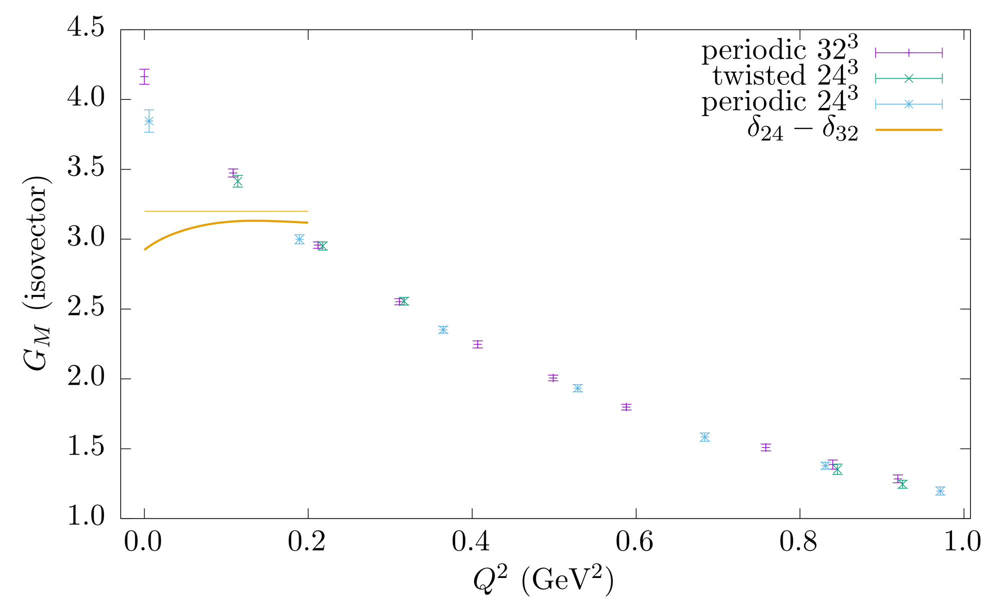
1. $32^3\times 48$, $m_\pi L = 4.8$,
2. $24^3\times 48$, $m_\pi L = 3.6$, plus twisted B.C. to match momenta.

Preliminary results for isovector G_E and G_M:

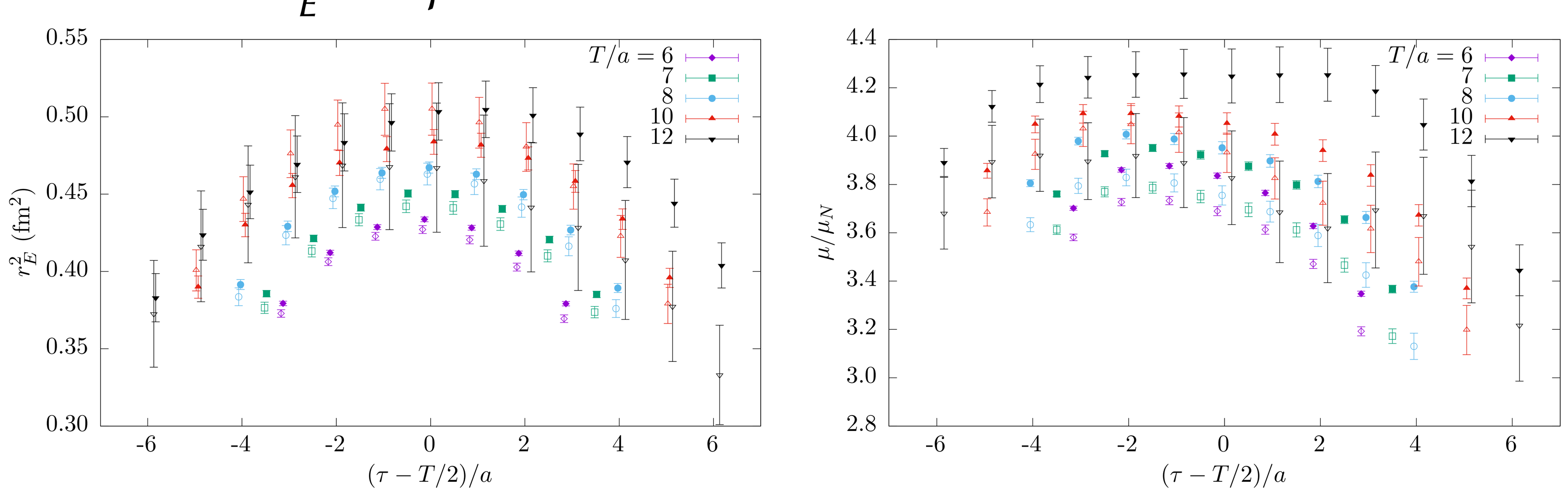


Orange curve: predicted finite-volume effect from partially quenched heavy baryon ChPT.
F.-J. Jiang and B. C. Tiburzi, Phys. Rev. D **78**, 114505 (2008)

Exact θ-derivative yields direct calculation of magnetic moment at $Q^2=0$.
N. Hasan, JG, S. Meinel *et al.*, Phys. Rev. D **97**, 034504 (2018); N. Hasan, talk at Lattice 2017

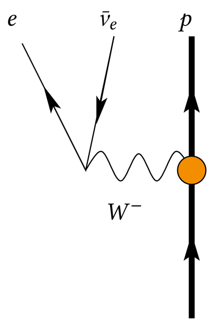


Plateaus for r_E^2 and μ from derivative method:



$\sim -2\%$ effect for r_E^2: smaller than ChPT and opposite sign.
$\sim -5\%$ effect for μ: similar to ChPT.
Excited-state contributions are significant.

Neutron beta decay



In the Standard Model, neutrons decay by emitting a virtual W^- boson. The quark-level coupling to W bosons is of the form $V-A$. The baryon-level couplings are modified by QCD:

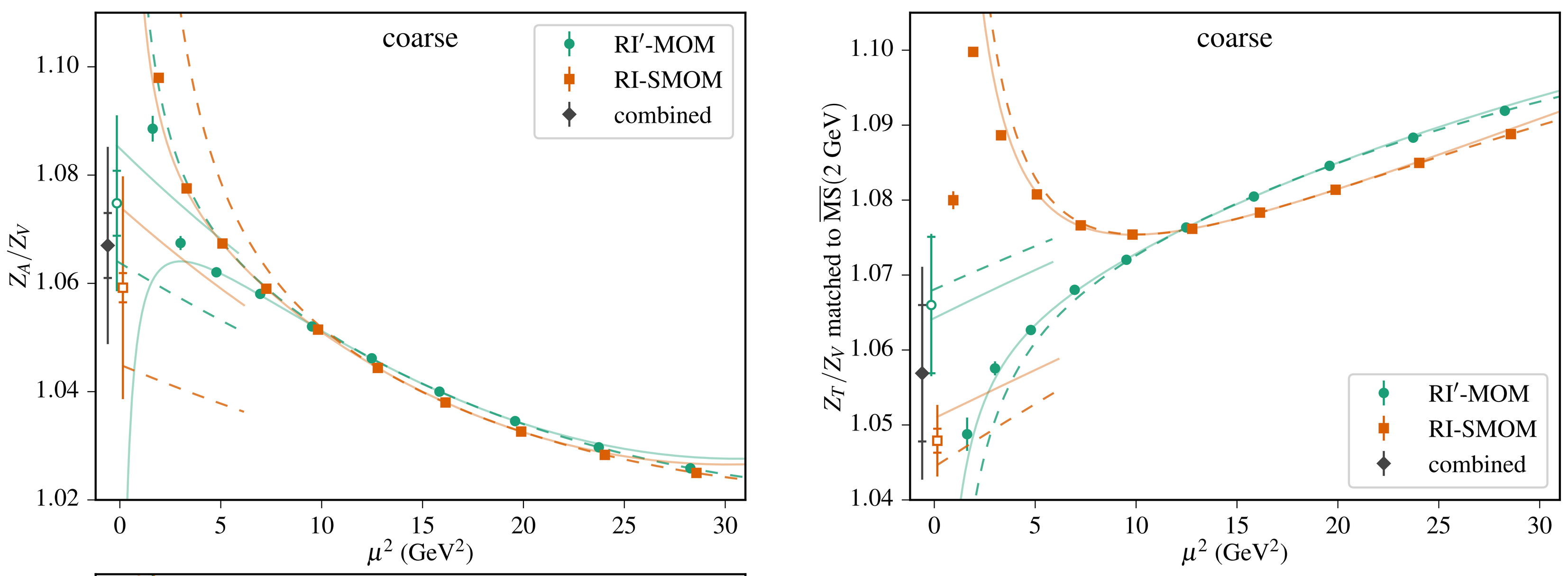
$$g_V \approx 1, \qquad g_A/g_V = 1.2732(23) \text{ PDG 2019}$$

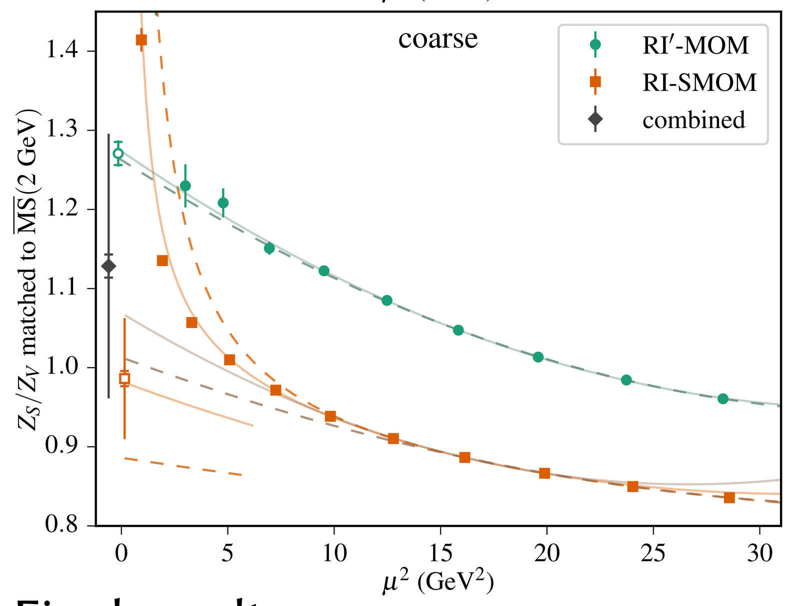
Precision β-decay experiments may be sensitive to beyond-the-Standard-Model physics by detecting scalar or tensor couplings.
Need to compute the corresponding “charges”, g_S and g_T on the lattice.

Lattice setup: $m_\pi = m_\pi^{\text{phys}}$, two ensembles: N. Hasan, JG *et al.*, Phys. Rev. D **99**, 114505 (2019)

1. $a = 0.116$ fm, 48^4, $m_\pi L = 3.9$,
2. $a = 0.093$ fm, 64^4, $m_\pi L = 4.0$.

Need to compute renormalization factors $Z_{A,S,T}$.





Two different intermediate renormalization schemes used.

Reasonable agreement for $Z_{A,T}$.

Large difference for Z_S $\to$ large uncertainty for g_S.

Final results:

$$g_A = 1.265(49),\quad g_T = 0.972(41),\quad g_S = 0.927(303).$$

Large discrepancy between two renormalization schemes for scalar needs further study. Could affect results by other collaborations!

Reducing excited-state effects

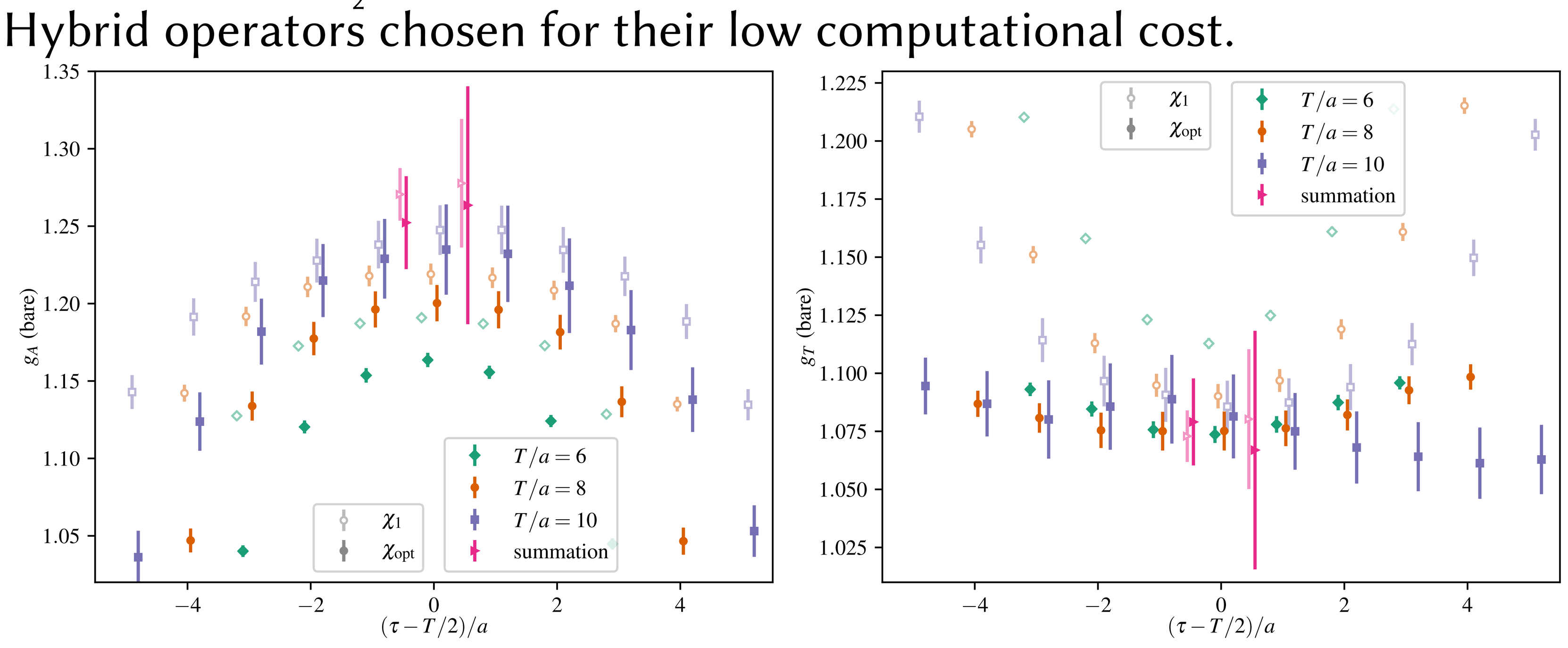
Challenge in lattice calculations: can't exactly create proton.
Must use Euclidean time evolution to suppress excited states $\sim e^{-Et}$.
Can the standard creation operator $\chi_1^\dagger$ be improved?
JG *et al.*, Phys. Rev. D **100**, 074510 (2019)

$\chi_1 \sim (uud)_{\frac12}$ (standard)
$\chi_2 \sim (uud)_{\frac12}g$ (hybrid)
$\chi_3 \sim (uud)_{\frac32}g$ (hybrid)

Use variational setup: optimize

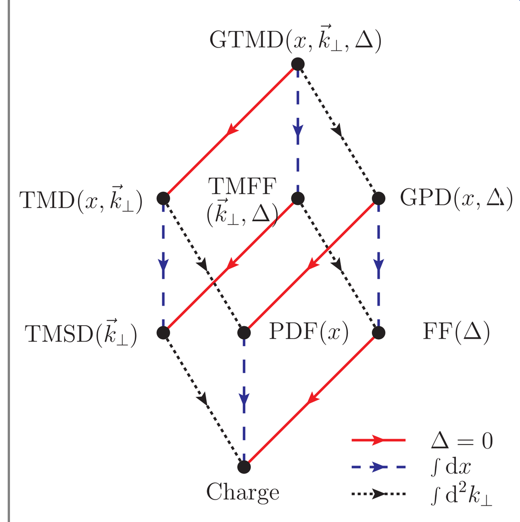
$$\chi_{\text{opt}} = c_1\chi_1 + c_2\chi_2 + c_3\chi_3$$

Hybrid operators chosen for their low computational cost.



Optimized operator χ_{opt} has significantly reduced excited-state effects in g_T but increased effects in g_A. No universal improvement.
Need to systematically target lowest-lying excitations!

Generalized TMDs (ongoing calculations)



C. Lorcé, B. Pasquini, and M. Vanderhaeghen, JHEP **2011** (05), 041

Distributions of quarks in the proton that are multidimensional: longitudinal momentum, transverse momentum, and transverse position.

Focus on quark orbital angular momentum and spin-orbit correlation. Use momentum derivative method and chiral fermions.
M. Engelhardt, Phys. Rev. D **95**, 094505 (2017); M. Engelhardt *et al.*, PoS **Lattice 2018**, 115, **SPIN 2018**, 047 [1901.00843]