**HWB07 - Multidimensional hydrodynamic simulations of stars**

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### Modelling Stars

- **Time-scale problem (values for the Sun):**
  - Nuclear time scale \( \tau_{\text{nuc}} = 10^{11} \) years.
  - Free-fall time scale \( \tau_f = 27 \) min.
  - \( \tau_f \gg \tau_{\text{nuc}} \) stellar evolution models are assumed to be spherically symmetric and hydrostatic to make the computation feasible.
  - Multidimensional phenomena (turbulent convection, rotation, magnetic fields) are added through simplistic prescriptions.
  - Hydrostatic structure \( \nabla P = \rho \ddot{g} \)
  - Hydrodynamics \( \rho \ddot{u} + \nabla \cdot (\rho \dot{u} \otimes \dot{u}) + \nabla P = \rho \dot{g} \)

### Seven-League Hydro (SLH) Code

- Compressible hydrodynamics in 1D, 2D, and 3D.
- Explicit and implicit time integration.
- Special solvers for all Mach numbers (e.g. AUSM⁺-up).
- Hybrid (MPI, OpenMP) parallelization.
- Arbitrary curvilinear meshes using a logically rectangular computational mesh.
- Radiation in the diffusion limit.
- General equation of state (fully-ionized ideal gas, radiation, partial degeneracy, Coulomb corrections).
- General nuclear reaction network.
- Self-gravity solvers.

### Scaling Tests

- Full machine tests on JUQUEEN during the 2016 Extreme Scaling Workshop. The runs computed the Taylor-Green vortex on a 2688² grid with 16 MPI tasks per node and 4 threads per task.

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### Case I: Shell Helium Burning

- **Goals of the study:**
  - Measure the mixing rate across the upper boundary of a He-burning shell to improve 1D mixing prescriptions for stellar evolution.
  - Investigate the impact of using specialized low-Mach flux functions.
  - Model of a 25 \( M_\odot \) star after core helium burning: He is burned into C & O in a convective shell.
  - Transport of fresh He through the upper convective boundary sustains nuclear burning at the bottom.
  - **Numerical challenges:**
    - Low-Mach number flows in a strongly stratified medium.
    - Mass transport through narrow convective boundary layers.

**Above:** Volume rendering of the radial component of velocity in a 3D SLH simulation of a convective He-burning shell (the high velocity layer) with a stably-stratified layer on top of it.

### Case II: Shell Silicon Burning

- **Goals of the study:**
  - Improve our understanding of convective shell interactions in core-collapse supernova progenitors in terms of their dynamics and of the convective-reactive nucleosynthesis involved.
  - Create improved initial conditions for supernova explosion models by simulating late stages of stellar evolution in 2D and 3D.
  - SLH simulations based on a 1D model of a 25 \( M_\odot \) star \( \sim \) 2 min before the ultimate core collapse.
  - **Multi-physics problem:**
    - 2D and 3D fluid dynamics at typical Mach numbers in the range \( 10^{-2} \) to \( 10^{-1} \).
    - Strong self-gravity in the monopole approximation.
    - Equation of state of a partially-degenerate mixture of ideal gas with radiation, including Coulomb corrections.
    - Complex network of nuclear reactions with 21 species advected with the flow.
    - Energy losses via plasma neutrino emission.

**Above:** Distribution of the mass fraction of \( ^{28}\text{Si} \) during a strong mixing event in a 2D SLH run performed on a \( 384 \times 768 \) polar grid. The simulation includes part of the Fe core, a Si-burning shell, an O-burning shell and part of a C-burning shell in a volume comparable to that of the Earth (dimensions on the axes in cm).

**Above:** Mach number distribution in a convective He-burning shell computed on a \( 810 \times 540 \times 540 \) spherical wedge grid with energy generation rate boosted by a factor of \( 3 \times 10^{12} \). The two simulations have the same initial setup and they only differ in the solver used: the AUSM⁺ solver not specialized for low-Mach flows (left panel) versus the low-Mach solver AUSM⁺-up (right panel). The turbulent cascade extends to significantly smaller scales with the low Mach solver on the same grid.