Relativistic magneto-hydrodynamics for heavy ion collisions

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Motivations

- Our aim is to study the QCD *(Quantum Chromo Dynamics)*, the theory which describes the strong interaction, i.e. the fundamental force which allows the formation of the atomic nuclei.
- Experiments are absolutely needed to validate the theory and to get hints for its extension. In our case, we are interested in the results of experiments based on Heavy Ion Collisions (like ALICE at CERN, here in Europe, or STAR at RHIC, in the USA), in which atomic nuclei are smashed together at relativistic speeds.
- All these experiments exploit very big detectors, which measure the properties of the final particles that fly into them. However, QCD governs the dynamics of the system on space-time scales many order of magnitudes smaller than the detector size, therefore we need to perform numerical simulations to connect theory with experiments, to understand whether what we observe at the very end fits well with the predictions of our models.
- In particular, we are interested in the study of the QGP *(Quark Gluon Plasma)*, a state of matter in which the constituents of protons and neutrons, i.e. quarks and gluons, are not tightly bound together anymore.
- The QGP forms at temperatures of order 10^{12} K. It is expected that a few micro-seconds after the Big Bang all the Universe was in this state.
- The QGP seems to behave like a fluid, therefore we can exploit relativistic hydrodynamics to model its evolution.
- In the last ten years it has been realized that the huge magnetic fields produced by the fast moving charges contained in the nuclei might also produce measurable effects.
- We extended the ECHO-QGP hydrodynamical code to take into account also the presence of magnetic fields, in the limit of an infinite conductivity of the fluid (ideal relativistic magneto-hydrodynamics)*[G.Inghirami et al., EPJC 76:659 (2016)]*.

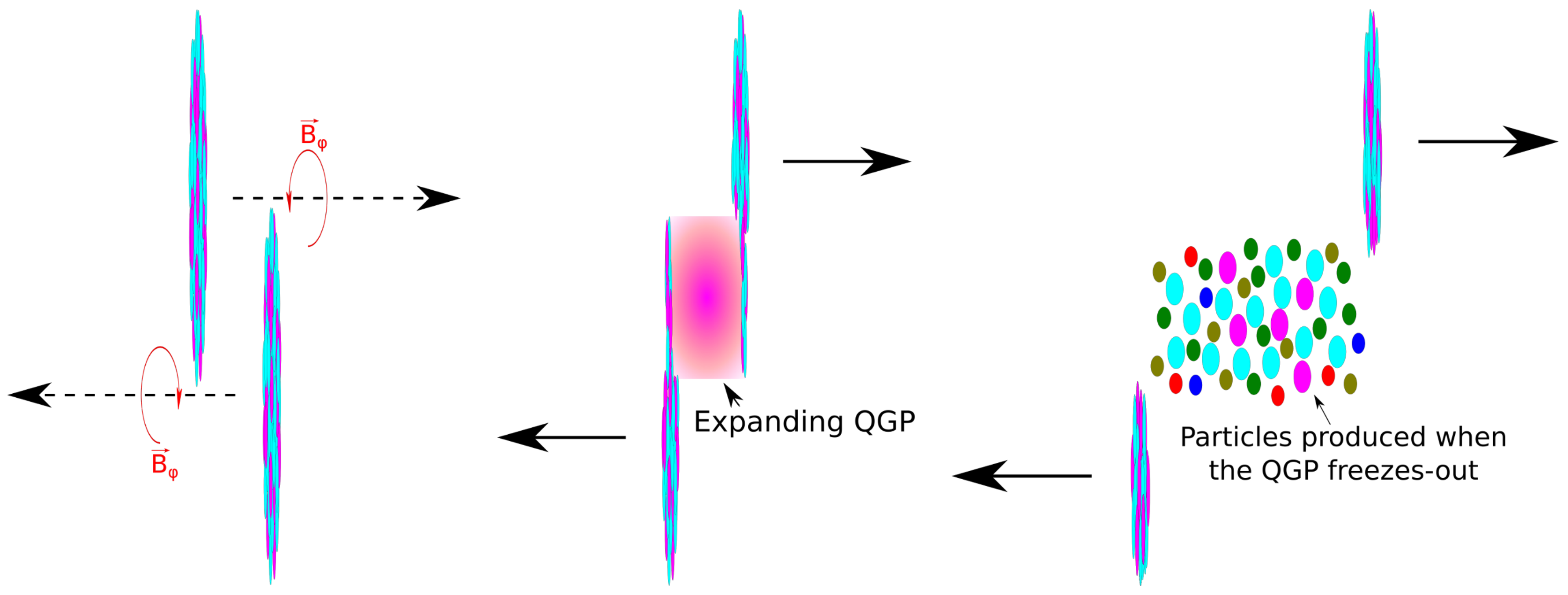


Figure 1 : **Schematic illustration of an heavy ion collision.** Two atomic nuclei traveling at more than 99.995 % of the speed of light crash together. Since they are charged, each nucleus produces a strong and approximately azimuthally symmetric magnetic field B_ϕ. Almost immediately after the collision, an hot and dense medium forms, the *Quark-Gluon Plasm*, which behaves like a fluid. We use ideal magneto-hydrodynamics to model its evolution. When the QGP cools down below a temperature of roughly $1.5 - 2 \cdot 10^{12}$ K, many particles are created. The detectors can measure the properties of the particles which are stable enough to reach them.

The equations

We solve the following system of equations:

$$\partial_0 \mathbf{U} + \partial_i \mathbf{F}^i = \mathbf{S}, \tag{1}$$

$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n \\ S_j \equiv T^0_j \\ \mathcal{E} \equiv -T^0_0 \\ B^j \end{pmatrix}, \quad \mathbf{F}^i = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n v^i \\ T^i_j \\ S^i \equiv -T^i_0 \\ v^i B^j - B^i v^j \end{pmatrix}, \quad \mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2} T^{ik} \partial_j g_{ik} \\ -\frac{1}{2} T^{ik} \partial_0 g_{ik} \\ 0 \end{pmatrix} \tag{2}$$

$$S_i = (e+p)\gamma^2 v_i + \varepsilon_{ijk} E^j B^k, \tag{3}$$

$$T_{ij} = (e+p)\gamma^2 v_i v_j + (p + u_{\text{em}}) g_{ij} - E_i E_j - B_i B_j, \tag{4}$$

$$\mathcal{E} = (e+p)\gamma^2 - p + u_{\text{em}}, \tag{5}$$

The electric field $\vec{\mathbf{E}}$ and the magnetic field $\vec{\mathbf{B}}$ are measured in the laboratory frame.
We assume that the fluid has infinite electrical conductivity: $\Rightarrow E_i = -\varepsilon_{ijk} v^j B^k$.
The system of Eqs. (1) is solved using finite difference schemes.

- reconstruction algorithm: MPE5 (others available, e.g. WENO3, CENO3, MPE3...)
- approximate Riemann solver: HLL
- time integration: RK2 (RK3 available)
- $\nabla \cdot \vec{\mathbf{B}} = 0$ enforcement: generalized Lagrange multipliers (hyperbolic divergence cleaning)
- code parallelization: MPI (OpenMP in post-processing tools)

Application: initial conditions for the energy density

This poster shows the results of a 2D+1 simulation, using optical Glauber initial conditions.

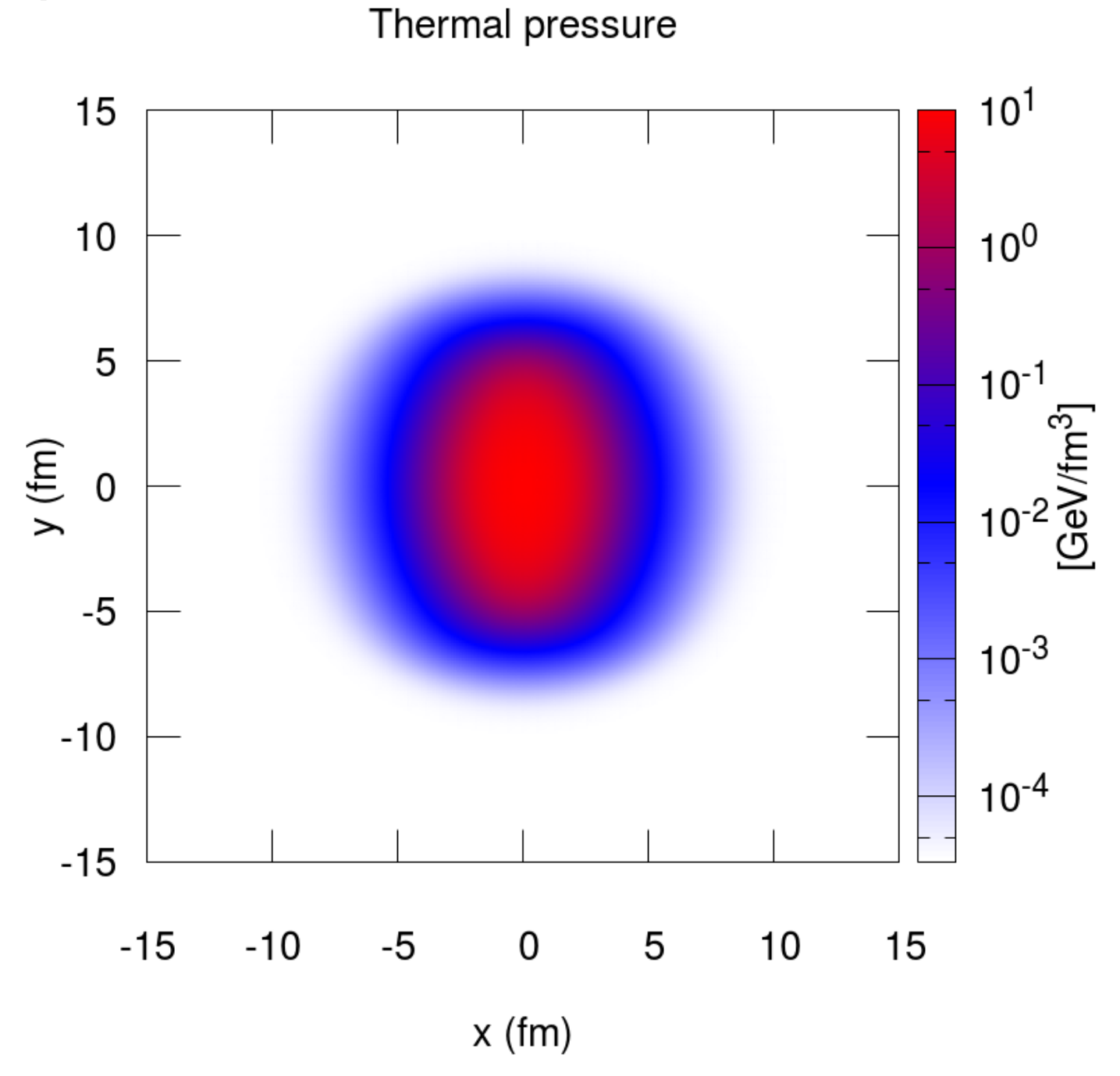


Figure 2 : Initial pressure distribution on the transverse plane.

Parameter	Value
$\sqrt{s_{NN}}$	200 GeV
b	10fm
ϵ_0	55 GeV/fm^3
τ_0	0.4 fm/c
T_{fo}	150 MeV
EOS	p = e/3

Table 1 : Summary of the key ECHO-QGP parameters chosen to set-up the initial conditions.

Initial conditions for the magnetic field

We compute the initial magnetic field by approximating the two Lorentz contracted nuclei as two uniformly charged disks moving in a medium with constant, isotropic, finite electrical conductivity, which pass unaffected through each other *[K.Tuchin, PRC 88,024911 (2013)]*.

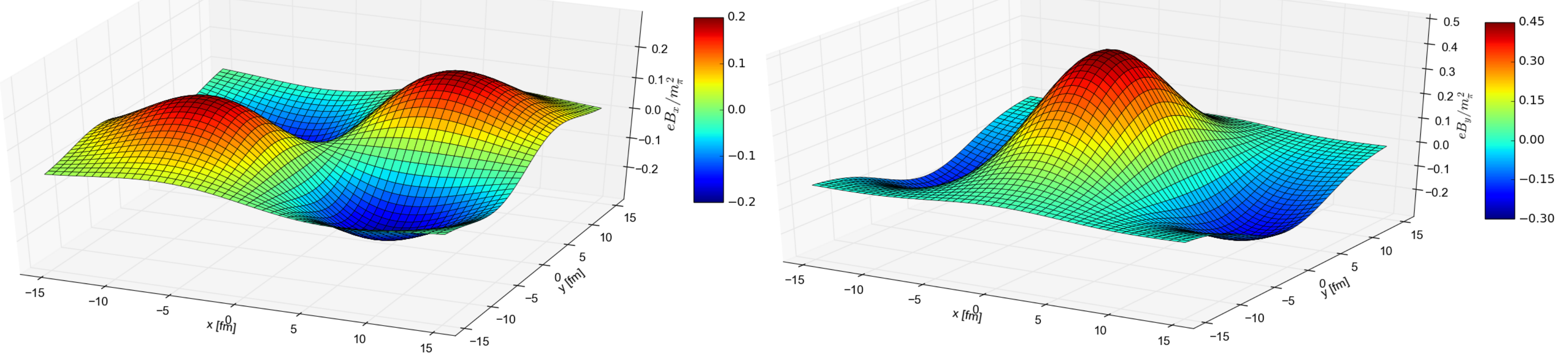
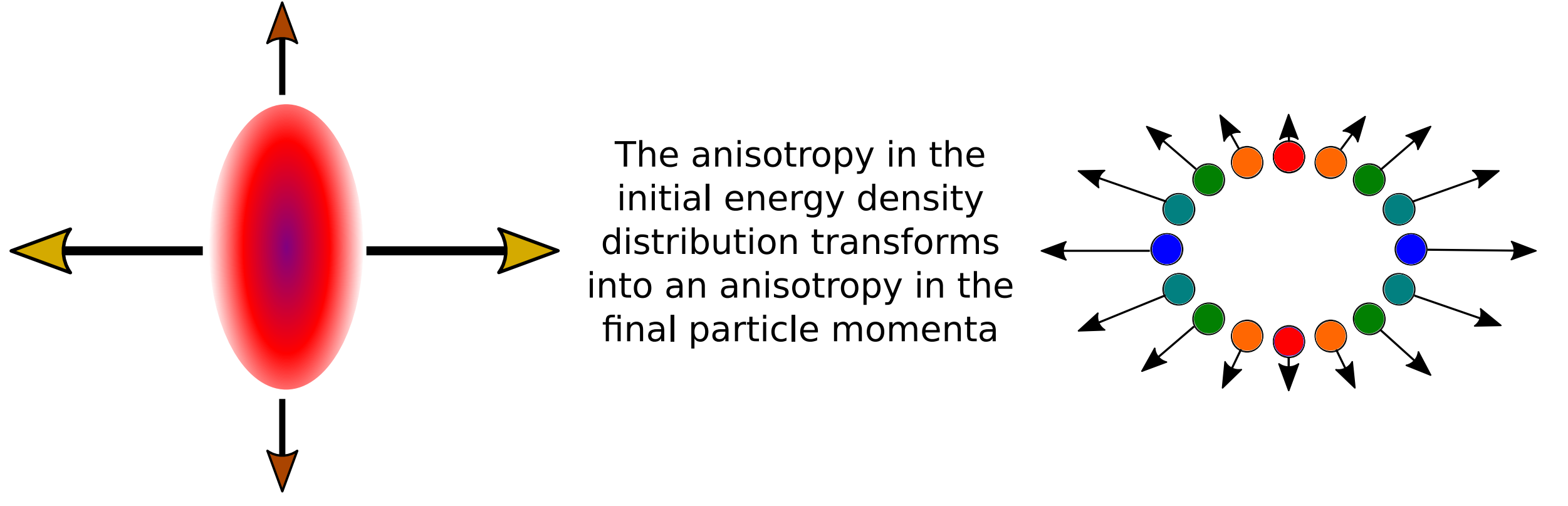


Figure 3 : Initial magnetic field components, assuming a pre-hydro electrical conductivity of the medium $\sigma_{el.} = 5.8$ MeV. The unit of measurement corresponds to roughly $3 \cdot 10^{14}$ T. For a proper comparison, we recall that the magnitude of the magnetic field of the Earth on the surface is around $0.5 \cdot 10^{-4}$ T and the typical magnetic field in a Magnetic Resonance Imaging medical device is around $1 - 3$ T.

Results

The study of the azimuthal distribution of the momenta of the final particles on the transverse plane, i.e. on the plane orthogonal to the direction of the ion beam, allows to test many properties of the medium, like the Equation of State or its viscosity. The quantitative analysis is based on the Fourier decomposition of the anisotropic flow, whose second component is called the *elliptic flow* (v_2).



The anisotropy in the initial energy density distribution transforms into an anisotropy in the final particle momenta

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

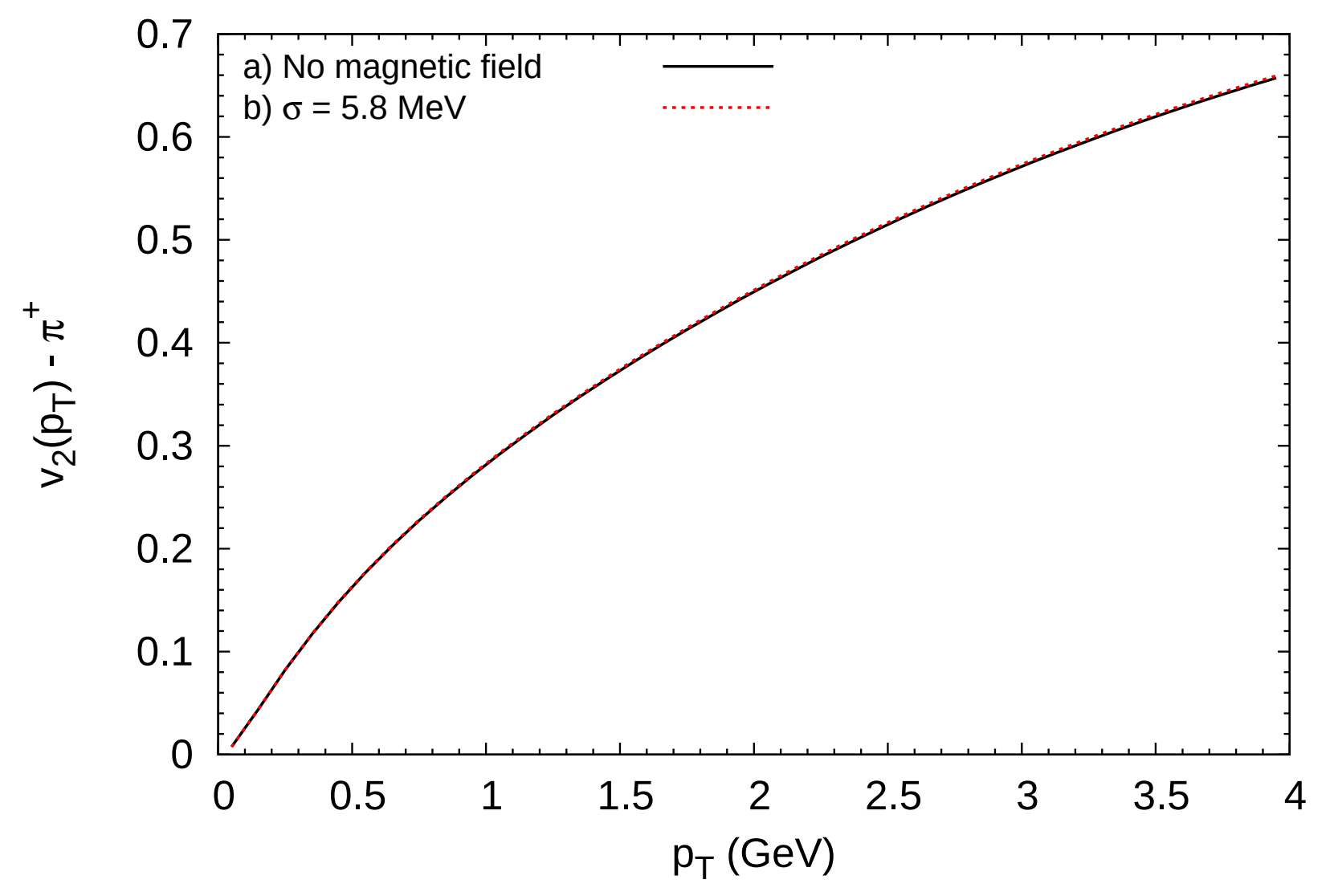


Figure 4 : Comparison of the elliptic flow (v_2) of pions with and without an initial magnetic field, with initial conditions as in table (1). At least with the initial conditions that we adopted, it seems that the magnetic field does not have measurable effects on the elliptic flow of these particles.

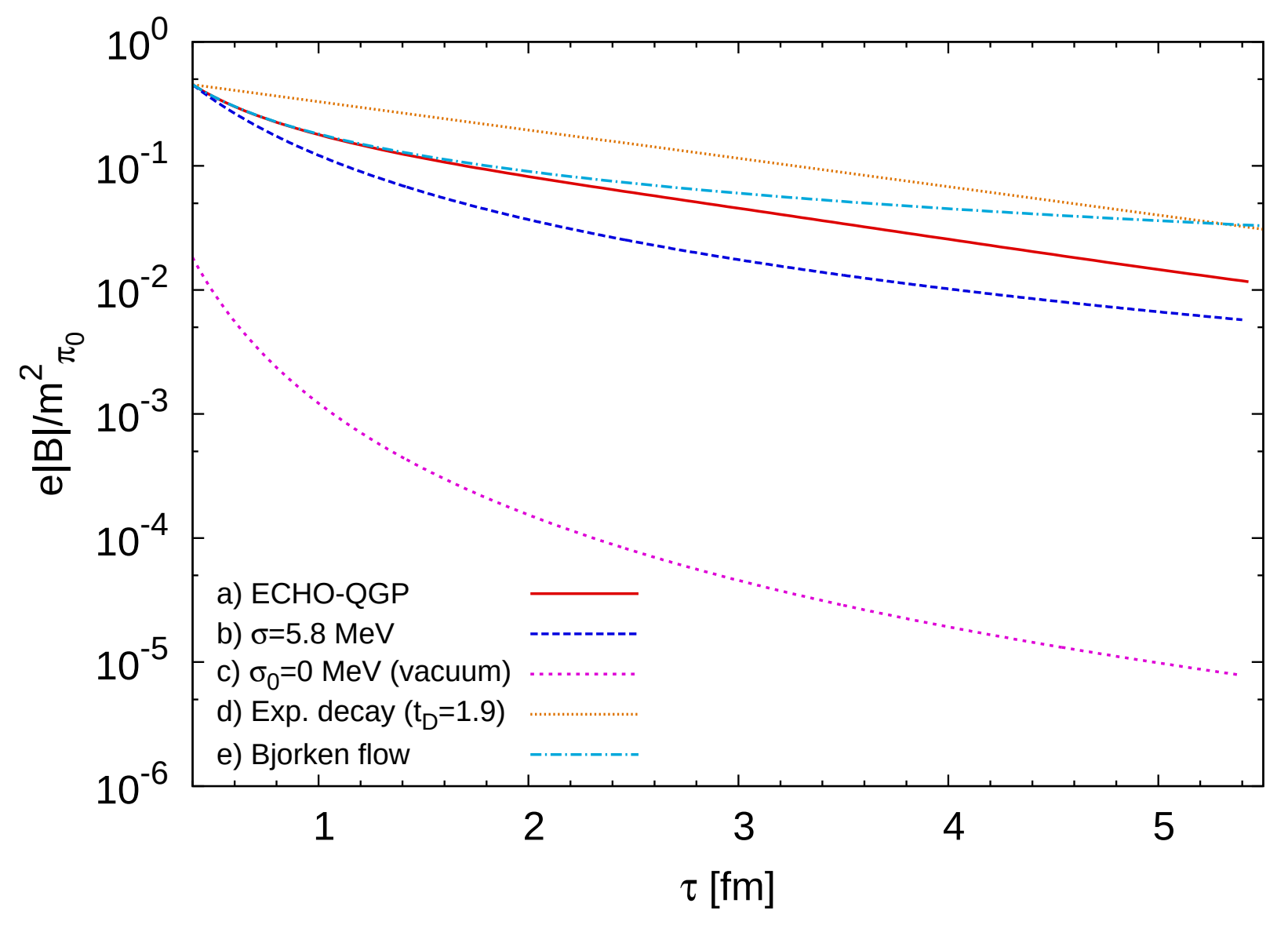


Figure 5 : Time evolution of the magnetic field in the center of the computational grid, computed with different models. ECHO-QGP initial conditions are in table (1). The results provided by ECHO-QGP might allow more precise studies of other effects which depend on the magnitude of the magnetic field.

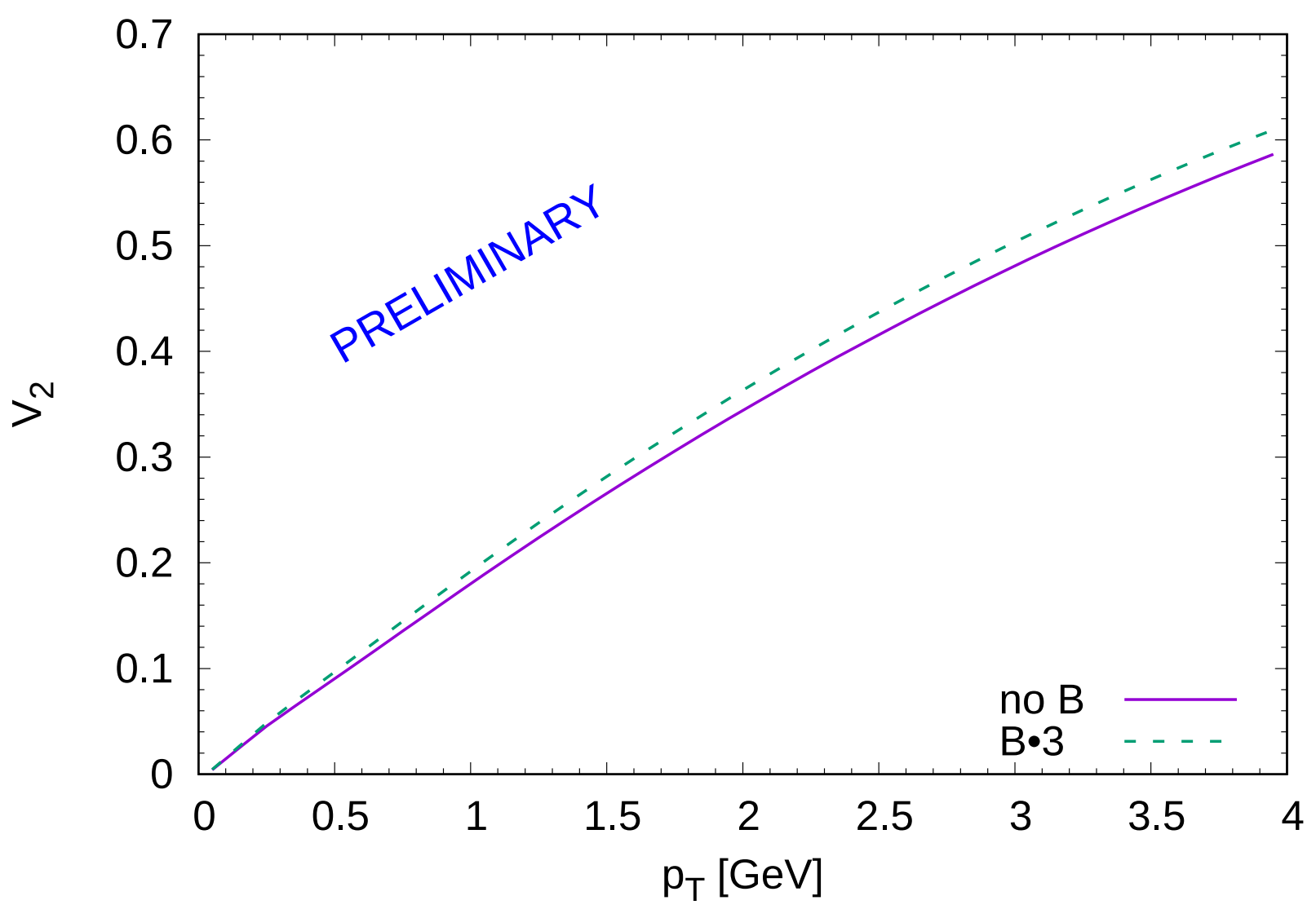


Figure 6 : Elliptic flow (v_2) of pions in peripheral (b=12fm) Pb-Pb collisions at LHC energies ($\sqrt{s_{NN}} = 2.76$ TeV), with and without magnetic field. Given the many uncertainties affecting the computation of the magnetic field at the beginning of the simulations, in our initial investigations we might have underestimated its magnitude. With an initial magnetic field three times larger, we can appreciate a mild enhancement of v_2.

Conclusions and outlooks

- Magnetic fields might have an impact on some observables studied in heavy ion collisions
- We extended the ECHO-QGP code to evolve magnetic fields coupled with the fluid
- The first 2D+1 simulations did not show relevant effects on v_2 due to magnetic fields
- The initial conditions for the magnetic fields are affected by large uncertainties
- More extensive 3D+1 simulations with a broader set of initial conditions are in progress
- Recent preliminary results suggest a possible enhancement of v_2 in peripheral collisions
- The flows are not the only observables and other phenomena might be investigated
- We are working on including resistive effects and the dynamical evolution of axial charges
- Event by event simulations and multi-particle correlations might bring new insights
- Detailed explorations of the whole parameter space are computationally expensive
- Our researches rely on large computational resources like those provided by the J. von Neumann Institute for Computing in the Forschungszentrum Jülich

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Reference article:
G.Inghirami et al.,
EPJC 76:659 (2016)
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https://arxiv.org/abs/1609.03042