Dynamical charm effects on the QCD static potential (Gauss Project HWU17)

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We evaluate the effects of a dynamical charm quark on the QCD static potential in the continuum limit. The size of these effects is calculated through a comparison between quenched QCD and QCD with $N_f = 2$ heavy degenerate quarks of mass $M = M_c$, where M_c is the mass of a charm quark. As applications, we also determine the charm loop effects on other related observables that can be extracted from the force between two static color sources, like the strong coupling in the α_{qq}-scheme and its Renormalization Group β_{qq}-function.

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HPC-LEAP EUROPEAN JOINT DOCTORATES

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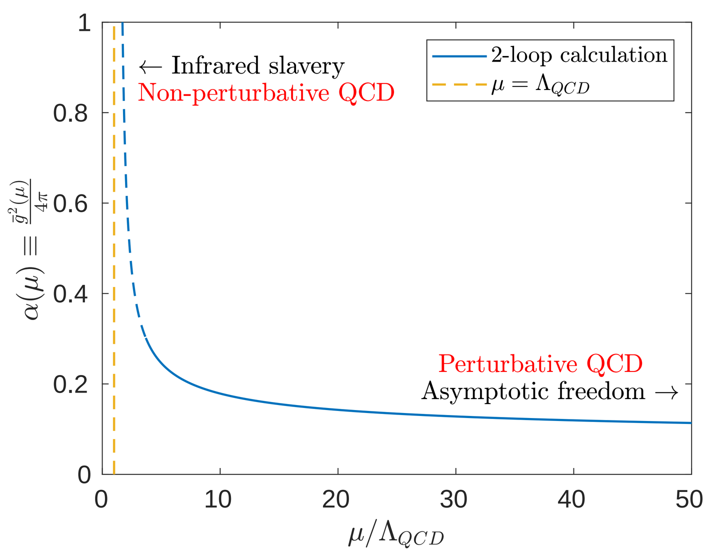
QCD: the theory of strong interactions

The forces that hold quarks together to form hadrons are called strong interactions and Quantum Chromodynamics is the theory proposed to explain their properties.

Features of Quantum Chromodynamics (QCD)

- QCD is a gauge theory based on the color group SU(3);
- quarks are described by fermionic fields ψ^f, where f denotes their flavor;
- gluons are described by vector fields A_μ;
- $\mathcal{L}_{QCD} = -\frac{1}{2}Tr[F_{\mu\nu}F^{\mu\nu}] + \sum_f \bar{\psi}^f \left[i\not{D} - m^f\right]\psi^f$,
  with $D_\mu = \partial_\mu - ig\sum_a A^a_\mu T^a$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$;
- denoting with μ the renormalization scale, the running of the coupling $\bar{g}(\mu)$ is described by the β-function:
  $\beta = \mu\frac{\partial\bar{g}(\mu)}{\partial\mu} \underset{\bar{g}\to 0}{\approx} \bar{g}^3(-b_0 - b_1\bar{g}^2 - \cdots)$;
- behavior of $\bar{g}$ at high energies:
  $\bar{g}^2 \underset{\mu\to\infty}{\approx} \frac{1}{b_0 t} - \frac{b_1\log(t)}{b_0^3 t^2} + \mathcal{O}(t^{-3}\log(t)^2)$,
  $t = \log\left(\frac{\mu^2}{\Lambda^2_{QCD}}\right)$, $\Lambda_{QCD} \approx 200$ MeV.

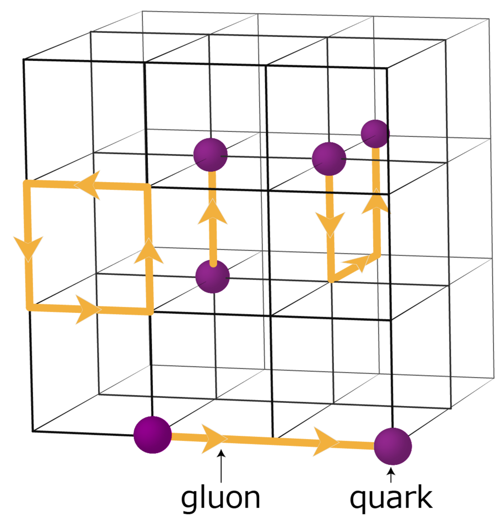


From continuum QCD to Lattice QCD

To understand the properties of hadrons and quark confinement problem we have to use non-perturbative methods. Doing that analytically is almost prohibitive and computer simulations are needed (Lattice QCD).

Lattice QCD approach

- Continuous space-time is replaced by an Euclidean (N^3N_t) lattice ($x_4 = ix_0$);
- Lattice spacing a;
- Parameters and fields are dimensionless;
- Fermions $[\psi(na)]$ lie on the sites of the lattice;
- Gluons described by links $[U_\mu(na)]$;
- Partition function
  $Z = \int D[\bar{\psi}\psi]D[U]e^{-S_F[\psi,\bar{\psi},U]-S_G[U]}$
- Extraction of observables from a lattice
  $\langle O\rangle = \frac{1}{Z}\int D[\bar{\psi}\psi]D[U]O[\psi,\bar{\psi},U]e^{-S_F[\psi,\bar{\psi},U]-S_G[U]}$
  $\approx \frac{1}{N}\sum_{j=1}^{N} O_j[\psi,\bar{\psi},U]$ (MC Methods)



Motivation and strategy

Many simulations of QCD are carried out only with light quarks (u,d,s) and this model seems to be a good approximation of the full theory at low energies. Including a dynamical charm quark in Lattice QCD simulation requires

- fine lattices to resolve the small correlation lengths associated with a charm quark;
- high precision to disentangle tiny charm loops effects on low energy observables.

We aim at estimating the charm sea effects on the QCD static potential and other related observables. In order to do that, we compare $N_f = 0$ QCD to QCD with $N_f = 2$ degenerate quarks of mass $M = M_c$, where M_c is the mass of a charm quark.

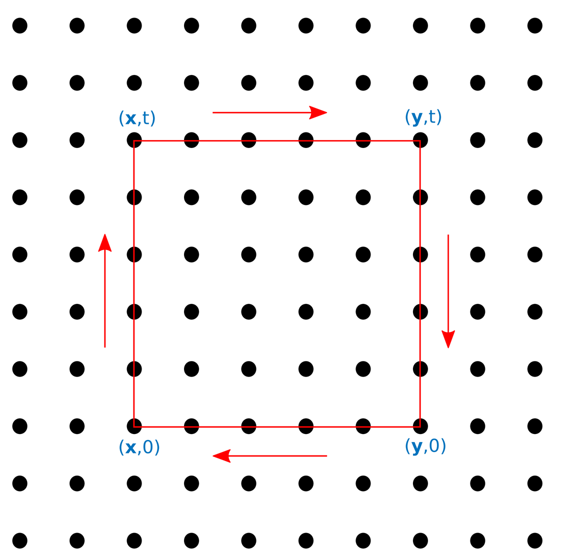
Wilson loops

The static potential between two static color sources is extracted from the expectation values of Wilson loops around a closed curve C:

$$W_C[U] = Tr[\prod_{(n,\mu)\in C} U_\mu(n)].$$

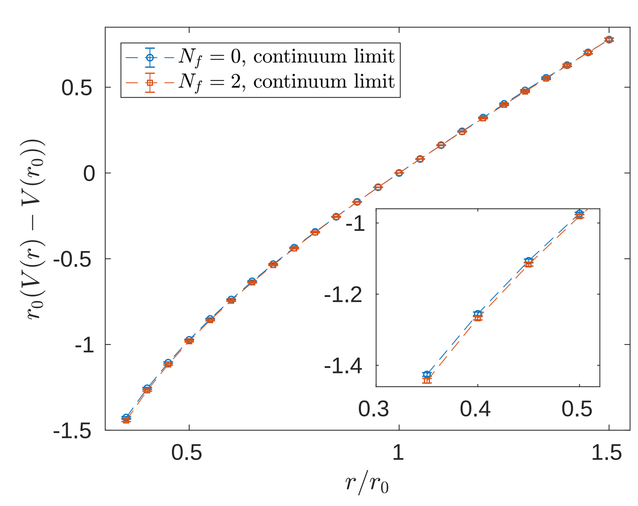
We focus on rectangular paths of lengths R and T. When $T\to\infty$, the static potential $V(r)$ at a given distance $r = Ra$ is given by:

$$aV(r) = -\lim_{T\to\infty}\frac{1}{T}\log(\langle W(R,T)\rangle)$$

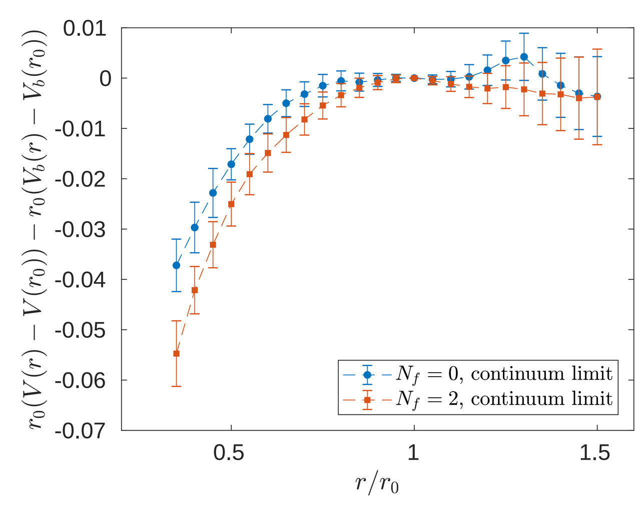


Charm sea effects on the QCD static potential

$N_f = 0$ QCD and $N_f = 2$ QCD with $M = M_c$ are simulated at four different lattice spacings such that 0.025 fm $\lesssim a \lesssim 0.075$ fm [1] using the openQCD package [2]. The static potential V is determined with the method of [3] using B. Leder's package github.com/bjoern-leder/wloop/. The reference scale r_0 [4] is determined by solving $r_0^2F(r_0) = 1.65$ with $F = V'$. We show here the results of our extrapolation to zero lattice spacing.



Since $V(r)$ can be determined only up to a constant, we study the difference $r_0(V(r) - V(r_0))$. Dynamical charm effects get visible at $r/r_0 \lesssim 0.5$.



To magnify the effects, we subtract from $V(r)$ the contribution expected from the effective bosonic string theory [5]: $V_b(r) \underset{r\to\infty}{\approx} \mu + \sigma r + \frac{\gamma}{r}$, with $\gamma = -\frac{\pi}{12}$.

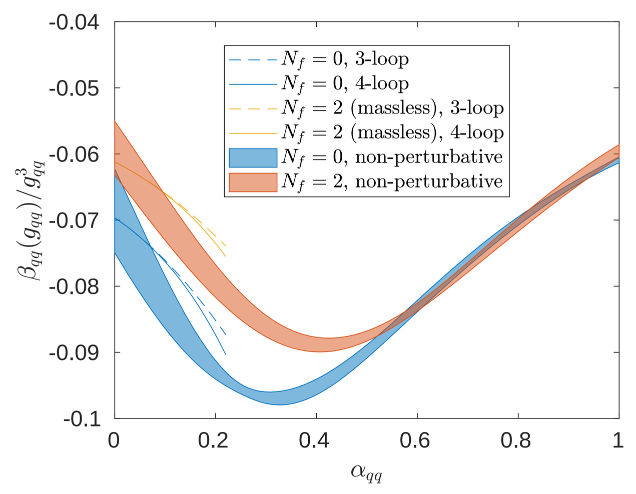
Strong coupling α_{qq} and β_{qq}-function

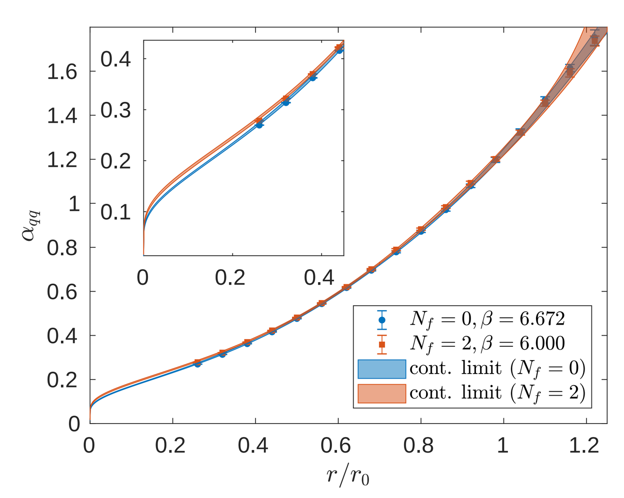
From the force between a static $q\bar{q}$ pair, one can define the strong coupling in the α_{qq}-scheme and its Renormalization Group β_{qq}-function:

$$\alpha_{qq}(r) \equiv \frac{g^2_{qq}}{4\pi} \equiv \frac{1}{C_F}r^2F(r), \qquad \beta_{qq} \equiv -r\frac{\partial g_{qq}}{\partial r}.$$

Strategy to extract the β_{qq}-function

- Lattice regularization of $F(r)$ [4,6]:
  $F(r_I) = \frac{V(Ra) - V(Ra-a)}{a} = \frac{C_F g^2}{4\pi r_I^2} + \mathcal{O}(g^4a^2)$.
- Study of the step scaling function σ [7] :
  $\sigma(f,u) = g^2_{qq}(f\times r)|_{g^2_{qq}(r)=u}$ (we fix $f = 2$)
  $\Rightarrow \log(f) = -\int_{\sqrt{u}}^{\sqrt{\sigma(f,u)}}\frac{dx}{\beta_{qq}(x)}$.
- Parametrize β_{qq} and cutoff effects [8]:
  $\beta_{qq} = -\frac{g^3_{qq}}{P(g^2_{qq})}$, $P(g^2_{qq}) = p_0 + p_1 g^2_{qq} + \cdots$;
  $\log(f) \to \log(f) + h(f, u, a/r_0)$
  $h(f,u,a/r_0) = \left(\sum_{i=0}^{n_\rho-1}\rho_i(f)u^i\right)\times\frac{a^2}{r_0^2}$.





Charm sea effects on α_{qq} and β_{qq} get visible at high energies.

<table>
<tr><th>Theory</th><th colspan="5">Best-fit parameters</th><th>$\frac{\chi^2}{N_{dof}}$</th></tr>
<tr><td rowspan="4">$N_f = 0$</td><td>p_0</td><td>p_1</td><td>p_2</td><td>p_3</td><td>p_4</td><td rowspan="4">$\frac{6.41}{12}$</td></tr>
<tr><td>14.6(1.3)</td><td>−2.47(78)</td><td>0.44(15)</td><td>−0.025(11)</td><td>0.00050(26)</td></tr>
<tr><td>ρ_0</td><td>ρ_1</td><td></td><td></td><td></td></tr>
<tr><td>1.58(81)</td><td>−0.12(11)</td><td></td><td></td><td></td></tr>
<tr><td rowspan="4">$N_f = 2$
($M = M_c$)</td><td>p_0</td><td>p_1</td><td>p_2</td><td>p_3</td><td></td><td rowspan="4">$\frac{1.00}{8}$</td></tr>
<tr><td>16.9(1.2)</td><td>−2.40(45)</td><td>0.294(56)</td><td>−0.0083(20)</td><td></td></tr>
<tr><td>ρ_0</td><td>ρ_1</td><td>ρ_2</td><td>ρ_3</td><td></td></tr>
<tr><td>84.3(18.1)</td><td>−42.1(9.2)</td><td>7.1(1.6)</td><td>−0.399(96)</td><td></td></tr>
</table>

Conclusions

- dynamical charm effects on $V(r)$, on the strong coupling α_{qq} and its β_{qq}-function get visible at $r/r_0 \lesssim 0.5$ and significant with increasing energy.

Outlook

- estimate string breaking distance in $N_f = 2$ QCD from the study of the static-light meson spectrum.

References

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[8] M. Dalla Brida et al., arXiv:1607.0642