# Investigation of Theories beyond the Standard Model

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# Beyond the Standard Model

## Motivation

Standard Model of elementary particle physics:

- QCD (Quantum Chromodynamics) ↔ strong interactions
- Glashow-Weinberg-Salam theory ↔ electroweak interactions

Gauge theories, gauge group SU(3)⊗SU(2)⊗U(1)
Extremely successful!
However: evidence for physics beyond the SM

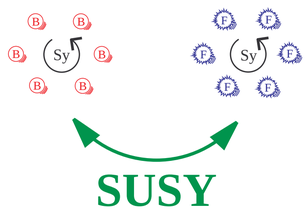
- Dark matter
- Neutrino masses
- Unnatural hierarchy of scales

Attempts: Supersymmetric models, Technicolor,
Grand Unified Theories, Supergravity, Superstring theories

# SUSY Yang-Mills Theory

## $\mathcal{N}=1$ SUSY Yang-Mills Theory

Supersymmetry (SUSY) relates bosons with fermions.




Particle multiplets: mass degenerate supermultiplets,
contain bosons and fermions.
Our investigations: $\mathcal{N}=1$ SUSY Yang-Mills Theory

- Simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended standard model

The theory:

- Gauge field $A^a_\mu(x)$, $a=1,\ldots,N_c^2-1$, "Gluon"
  Gauge group SU($N_c$)
- Majorana-spinor field $\lambda^a(x)$, $\overline{\lambda}=\lambda^T C$, "Gluino"
  adjoint representation: $\mathcal{D}_\mu\lambda^a=\partial_\mu\lambda^a+g\,f_{abc}A^b_\mu\lambda^c$

$$\mathcal{L}=-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\frac{\mathrm{i}}{2}\overline{\lambda}^a\gamma_\mu(\mathcal{D}_\mu\lambda)^a-\frac{m_{\tilde{g}}}{2}\,\bar{\lambda}^a\lambda^a$$

- Similar to QCD

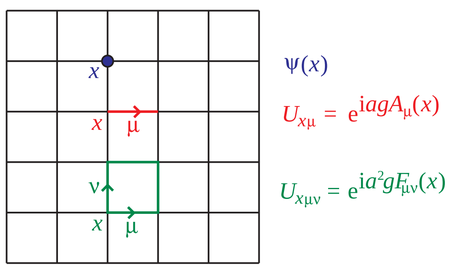
  Differences: $\lambda$: 1) Majorana, "$N_f=\frac{1}{2}$"
  2) adjoint representation of SU($N_c$)
- Gluino mass term $m_{\tilde{g}}\,\bar{\lambda}^a\lambda^a$ breaks SUSY softly.

Non-perturbative problems:

- Spectrum of bound states → Supermultiplets
- Phases of the theory
- Breaking of chiral symmetry, gluino condensate
- Confinement of static quarks
- Breaking of supersymmetry?
- and more

## SUSY on the Lattice

Numerical simulations → discretisation of space-time on a lattice
Lattice spacing $a$ ↔ momentum cut-off
Fields on the lattice:




Lattice breaks SUSY. Restoration in the continuum limit?
Curci, Veneziano: use Wilson action, search for continuum limit with SUSY

$$S=-\frac{\beta}{N_c}\sum_p \mathrm{Re}\,\mathrm{Tr}\,U_p$$

$$+\frac{1}{2}\sum_x\left\{\overline{\lambda}^a_x\lambda^a_x-\kappa\sum_{\mu=1}^4\left[\overline{\lambda}^a_{x+\hat{\mu}}V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x+\overline{\lambda}^a_x V^t_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat{\mu}}\right]\right\}$$

$\beta=\frac{2N_c}{g^2}$, $\kappa=\frac{1}{2m_0+8}$ hopping parameter, $m_0$ : bare gluino mass

$$V_{ab,x\mu}=2\,\mathrm{Tr}\,(U^\dagger_{x\mu}T_aU_{x\mu}T_b),\quad \text{adjoint link variables}$$

We study gauge groups SU(2) and SU(3).

## Light Particle Spectrum

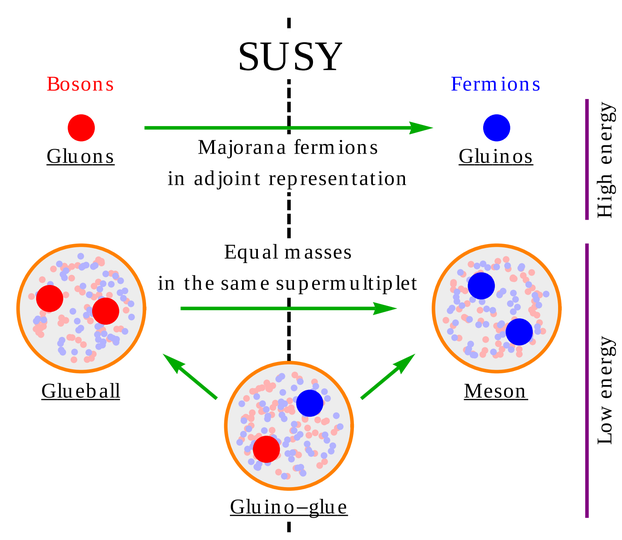
Expect colour neutral bound states of gluons and gluinos

Predictions from effective Lagrangeans:
Supermultiplet

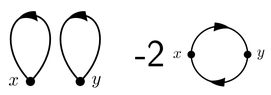
- $0^-$ meson a–$\eta'$ $\sim$ $\overline{\lambda}\gamma_5\lambda$
- $0^+$ meson a–$f_0$ $\sim$ $\overline{\lambda}\lambda$
- spin $\frac{1}{2}$ gluino-glueball $\sim$ $\sigma_{\mu\nu}\,\mathrm{Tr}\,(F_{\mu\nu}\lambda)$

Additional Supermultiplet

- $0^-$ glueball, $0^+$ glueball, gluino-glueball

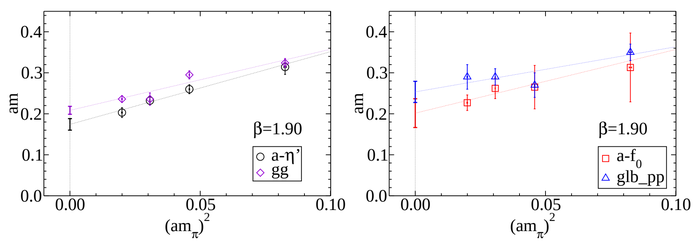



Masses are obtained from corresponding correlation functions.
Correlators of mesons have disconnected pieces, which are numerically demanding and require sophisticated techniques.

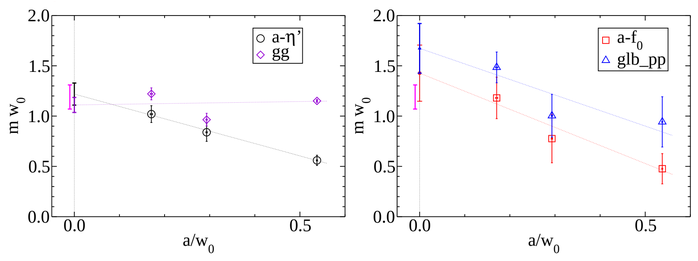



Spectrum for SU(2):
Lattices $16^3\cdot 32$, $24^3\cdot 48$, $32^3\cdot 64$,
$a=0.087, 0.054, 0.036$ fm, in QCD units

Extrapolations to $m_{\tilde{g}}=0$




Extrapolations to the continuum




| a–$\eta'$ | a–$f_0$ | $\tilde{g}g$ | glueball $0^{++}$ |
|---|---|---|---|
| 1.06(10) | 1.25(24) | 0.97(6) | 1.46(22) |

Bound state masses in units of GeV (QCD units)

Results are consistent with the formation of degenerate supermultiplets.

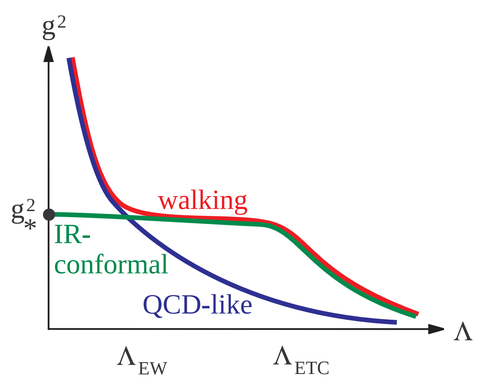
Current investigations:

- excited states
- gauge group SU(3)

# Technicolor Candidates

## Infrared Conformal Models

Higgs mass is not protected against large radiative corrections.
→ Technicolor models: Higgs = fermion bound state
Phenomenological constraints → find theories with a (nearby) infrared fixed point.



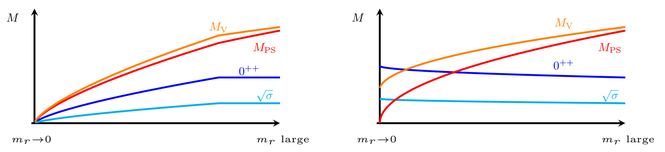

## Adjoint Fermions

Scenario of a theory depends strongly on the gauge group and the number $N_f$ of fermion flavours.

lower $N_f$ ≤ conformal window ≤ upper $N_f$

Fermions in the adjoint representation ↔ smaller $N_f$

Scaling of masses:




infrared conformal QCD-like

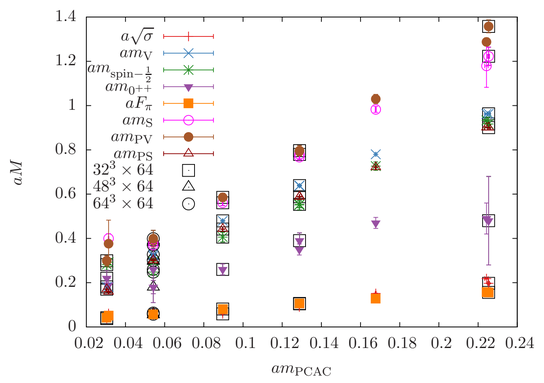
$M\propto m_r^\alpha$

with mass anomalous dimension $\alpha$

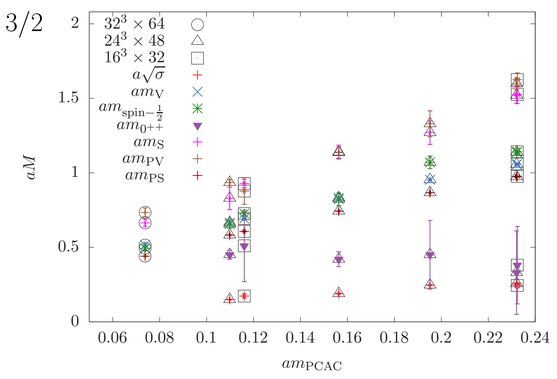
Our investigations: SU(2) gauge theory, $N_f=2,\ 3/2,\ 1$ and $1/2$
(half-integer $N_f$ = Majorana fermions)

Examples:

$N_f=2$




$N_f=3/2$




Results:

from mass spectrum and eigenvalue spectrum of the Wilson-Dirac operator

$N_f=2,\ 3/2,\ 1$: infrared conformal,
numerical values for the mass anomalous dimension $\alpha$

$N_f=1/2$ ≃ SUSY Yang-Mills: QCD-like

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