The leading order hadronic contribution to the anomalous magnetic moment of the muon

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The anomalous magnetic moment of the muon is one of the most precisely measured physical quantities. Comparing its experimental value to the theoretical prediction of the Standard Model (SM) provides a stringent test of SM, and a possible disagreement can indicate new physics. The dominant source of uncertainty in the recent theoretical determinations is the hadronic loop corrections arising from the hadronic vacuum polarization (HVP). Due to the large coupling constant of the strong interaction at low energies, the HVP contribution is only accessible by non-perturbative methods.

Here we apply lattice QCD to address these hadronic contributions non-perturbatively. Our calculations include all contributions from the u, d, s, and c quarks, directly at the physical values of their masses, in their quark-connected and quark-disconnected configurations. We find that our value for the leading order hadronic contribution to the anomalous magnetic moment of the muon, within its combined error of 2.7%, is compatible with the current experimental results.

Introduction

- Magnetic moment of an elementary particle is proportional to its spin: $\boldsymbol{\mu} = g \cdot \frac{e}{2m}\mathbf{S}$
- For $\frac{1}{2}$-spin particles in Quantum Mechanics: $g = 2$
- Due to Quantum Field Theoretical corrections:

$$a = \frac{g-2}{2} \neq 0 \quad \longleftarrow \quad \text{Anomalous Magnetic Moment}$$

- Anomalous magnetic moment of the muon in experiments:

$$a_\mu(\text{exp.}) = 11659209.1(6.3) \times 10^{-10}$$

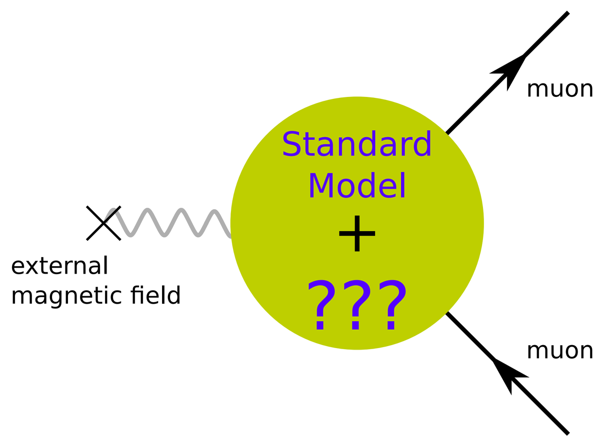
- Prediction of the Standard Model (SM) of particle physics:

$$a_\mu(\text{SM}) = 11659182.3(4.3) \times 10^{-10}$$

- When confirmed in higher accuracy, the 3.5σ tension

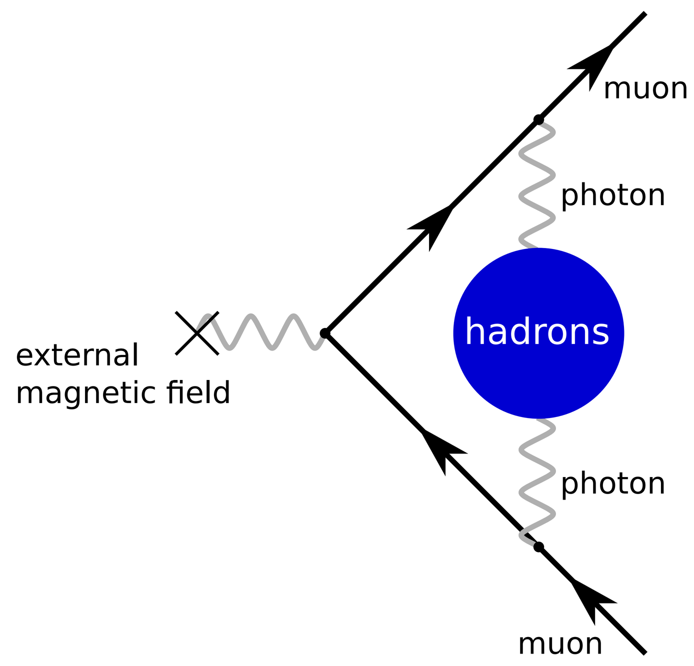
$$a_\mu(\text{exp.}) - a_\mu(\text{SM}) = 26.8(7.6) \times 10^{-10},$$

can be a signal for Beyond the Standard Model physics.



Leading order hadronic contribution

- Largest uncertainty in Standard Model prediction originates from hadronic contributions.
- Leading order comes from the hadronic vacuum polarization (HVP): the intermediate photon polarizes vacuum, such that hadronic states appear.
- HVP is described by quantum chromodynamics (QCD), the theory of the strong interaction.

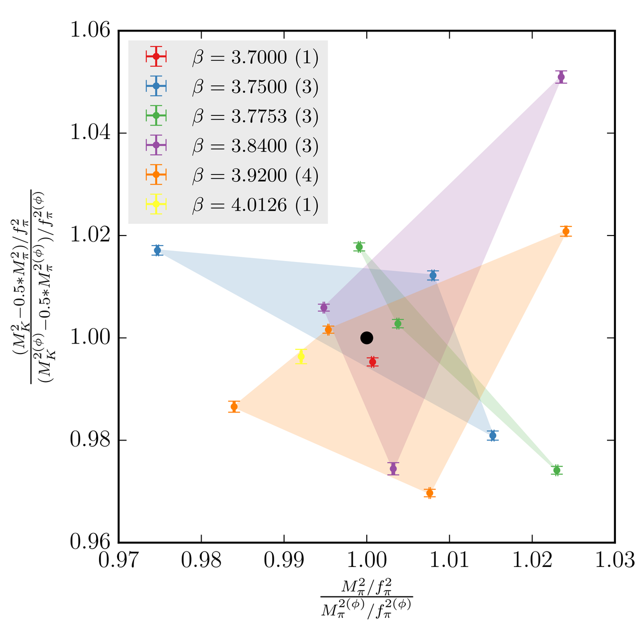


- Coupling constant of QCD is large at energy scales dominating the process $\longrightarrow$ perturbation theory fails.
- Among the currently available methods, Lattice QCD is the only, systematically improvable, non-perturbative method, which can provide purely 1st principles results.
- Lattice QCD: discretize a finite subvolume of spacetime, and solve the emerging finite, but $\mathcal{O}(10^8 - 10^{10})$ dimensional, Feynman Path Integral using Monte Carlo methods.

Simulation points

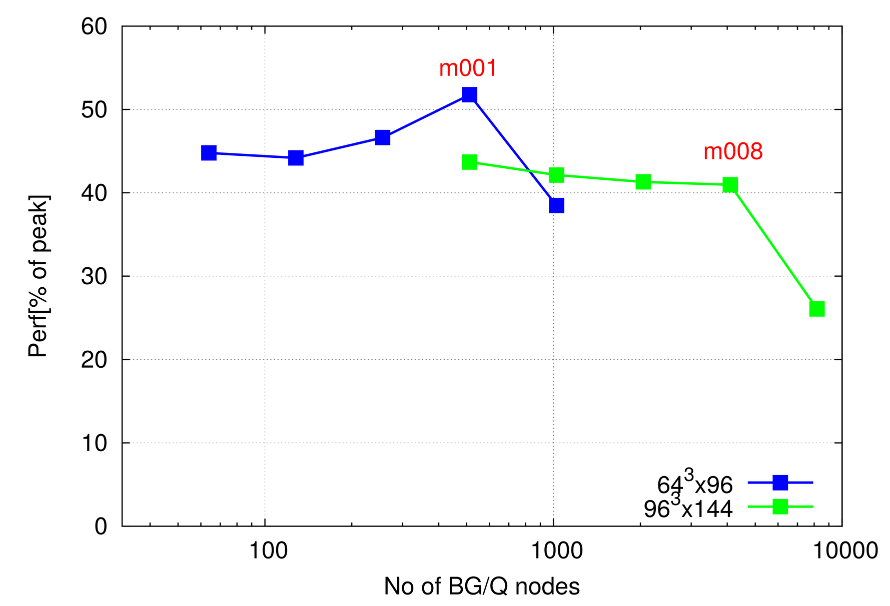
- Tree-level improved Symanzyk gauge action
- $N_f = 2+1+1$ dynamical staggered fermions
- 4 steps of stout smearing with smearing parameters $\varrho = 0.125$
- Quark masses bracketing their physical values
- Box size: $L \gtrsim 6\,\text{fm}$
- Continuum limit using 6 different lattice spacings

β	a [fm]	$T \times L$	#conf-conn	#conf-disc
3.7000	0.134	64×48	1000	1000
3.7500	0.118	96×56	1500	1500
3.7753	0.111	84×56	1500	1500
3.8400	0.095	96×64	2500	1500
3.9200	0.078	128×80	3500	1000
4.0126	0.064	144×96	450	–



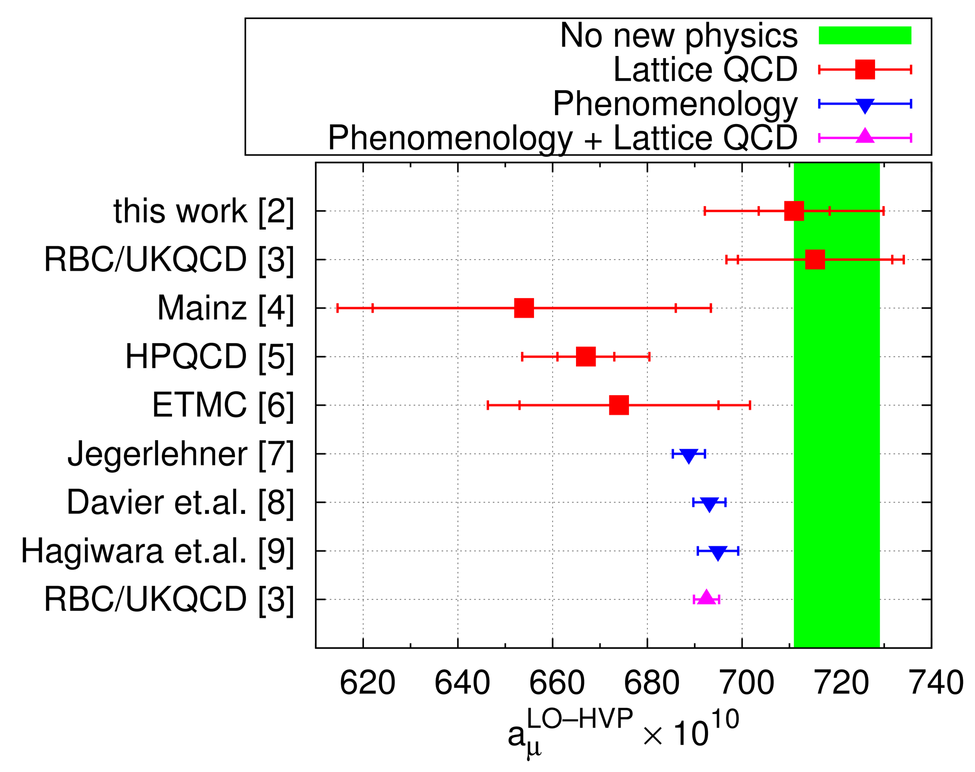
Code performance

- Vectorization of lattice sites
- Intranode parallelization using the `pthreads` library
- Internode parallelization using MPI. For the performance critical parts we use a custom software based on SPI.
- We reach about $40-50\,\%$ of the peak performance with the full mixed precision solver.
- Runs ranging from midplanes (512 nodes, 64×96) upto four rack partitions (4096 nodes, 96×144).



Results

- Our findings are compatible with the Standard Model



- Challenges that 1st principles computations have to face in the near future:
  - Increase statistics to improve signal/noise ratio.
  - Include precise estimate for finite volume corrections.
  - Include electromagnetic and strong isospin breaking effects.

Publications related to Project

[1] S. Borsanyi, Z. Fodor, T. Kawanai, S. Krieg, L. Lellouch, R. Malak, K. Miura, K. K. Szabo, C. Torrero, and B. C. Toth. "Slope and curvature of the hadronic vacuum polarization at vanishing virtuality from lattice QCD". In: *Phys. Rev.* D96.7 (2017), p. 074507.

[2] S. Borsanyi et al. "Hadronic vacuum polarization contribution to the anomalous magnetic moments of leptons from first principles". In: (2017). arXiv: 1711.04980.

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[3] T. Blum, P. A. Boyle, V. Gülpers, T. Izubuchi, L. Jin, C. Jung, A. Jüttner, C. Lehner, A. Portelli, and J. T. Tsang. "Calculation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment". In: (2018). arXiv: 1801.07224.

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[5] B. Chakraborty, C. T. H. Davies, P. G. de Oliviera, J. Koponen, G. P. Lepage, and R. S. Van de Water. "The hadronic vacuum polarization contribution to a_μ from full lattice QCD". In: *Phys. Rev.* D96.3 (2017), p. 034516.

[6] F. Burger, X. Feng, G. Hotzel, K. Jansen, M. Petschlies, and D. B. Renner. "Four-Flavour Leading-Order Hadronic Contribution To The Muon Anomalous Magnetic Moment". In: *JHEP* 02 (2014), p. 099.

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