Nucleon sigma terms, corrections to Dashen's theorem and the light quark mass ratio from lattice QCD

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Abstract

We present a QCD calculation of the u, d, and s scalar quark contents of nucleons based on 47 lattice ensembles with $N_f = 2+1$ dynamical sea quarks, 5 lattice spacings down to 0.054 fm, lattice sizes up to 6 fm, and pion masses down to 120 MeV. Using the Feynman-Hellmann theorem, we obtain $f_{udN} = 0.0405(40)(35)$ and $f_{sN} = 0.113(45)(40)$, which translates into $\sigma_{\pi N} = 38(3)(3)$ MeV, $\sigma_{sN} = 105(41)(37)$ MeV for the sigma terms, where the first errors are statistical and the second errors are systematic. Using isospin relations, we also compute the individual up and down quark contents of the proton and neutron. Based on the $N_f = 2+1$ QCD simulations to which QED effects have been added in a partially quenched setup we determine the corrections to Dashen's theorem and the individual up and down quark masses. For the parameter which quantifies violations to Dashen's theorem, we obtain $\varepsilon = 0.73(2)(5)(17)$, where the third error is an estimate of the QED quenching error. For the light quark masses we obtain, $m_u = 2.27(6)(5)(4)$ MeV and $m_d = 4.67(6)(5)(4)$ MeV in the modified minimal subtraction scheme at 2 GeV and the isospin breaking ratios $m_u/m_d = 0.485(11)(8)(14)$, $R = 38.2(1.1)(0.8)(1.4)$, and $Q = 23.4(0.4)(0.3)(0.4)$. Our results exclude the $m_u = 0$ solution to the strong CP problem by more than 24 standard deviations.

Nucleon sigma terms

Nucleon sigma terms measure the scalar quark content of the nucleon

$$\sigma_{\pi N} = m_{ud}\langle N \mid \bar{u}u + \bar{d}d \mid N\rangle,$$
$$\sigma_{sN} = m_s \langle N \mid \bar{s}s \mid N\rangle.$$

They relate quark level couplings to nucleon level couplings and are important for direct dark matter detection experiments. Quark contents can be calculated via

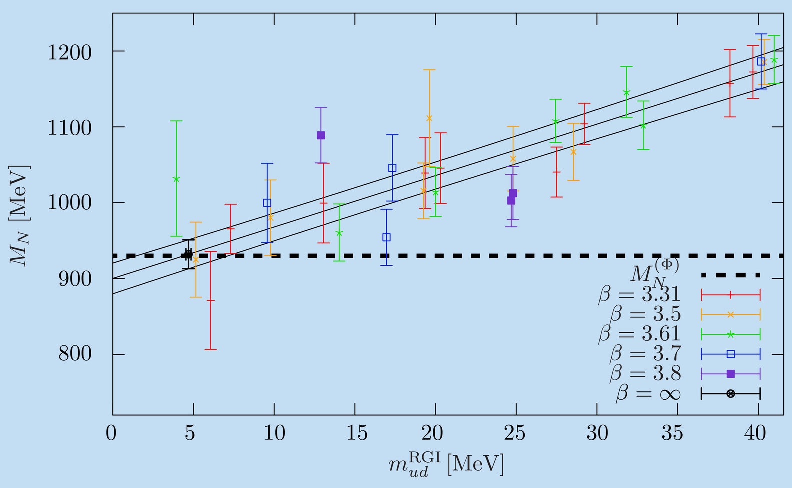
$$f^N_{ud} = \frac{\sigma_{\pi N}}{M_N} \quad , \quad f^N_s = \frac{\sigma_{sN}}{M_N}.$$

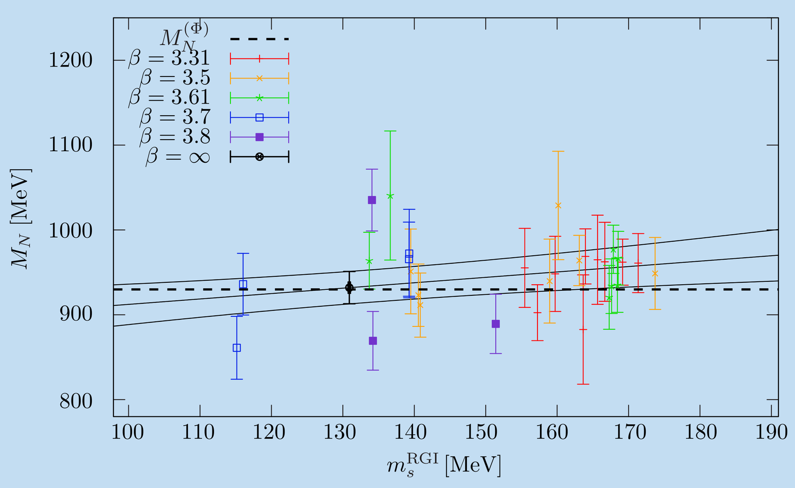
Feynman-Hellmann theorem

Nucleon sigma terms are related to quark mass derivatives of the nucleon mass via

$$\sigma_{\pi N} = m_{ud}\frac{\partial M_N}{\partial m_{ud}} \quad , \quad \sigma_{sN} = m_s \frac{\partial M_N}{\partial m_s}.$$

The derivatives are measured on $N_f = 2+1$ lattice configurations:





Lattice data is fitted with an ansatz of the form

$$M_X^{n_X} = (1+g^a_X(a))(1+g^{\rm FV}_X(M_\pi, L))M_X^{(\phi)n_X} \times$$
$$(1 + (c^{ud}_X + g^{ud,a}_X(a))\tilde{m}_{ud} + (c^s_X + g^{s,a}_X(a))\tilde{m}_s + \text{h.o.c})$$

where $X \in \{\pi, K_\chi, \Omega, N\}$, $n_X \in \{1,2\}$, $g^{a/FV}_X$ parametrize continuum and finite volume corrections and $\tilde{m}_q$ are the deviations of the quark masses from their physical values.

Individual quark contents

Individual quark contents can be calculated up to $\mathcal{O}(\delta m^2, m_{ud}\delta_m)$:

$$f^{p/n}_u = \left(\frac{r}{1+r}\right) f^N_{ud} \pm \frac{1}{2}\left(\frac{r}{1-r}\right)\frac{\Delta_{\rm QCD}M_N}{M_N}$$
$$f^{p/n}_u = \left(\frac{1}{1+r}\right) f^N_{ud} \mp \frac{1}{2}\left(\frac{1}{1-r}\right)\frac{\Delta_{\rm QCD}M_N}{M_N}$$

Here $r = m_s/m_{ud}$.

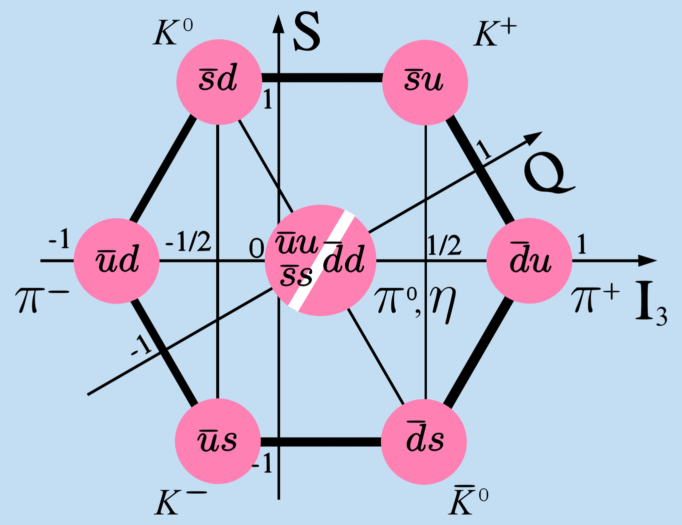
Results for sigma terms / quark contents

$$\sigma_{\pi N} = 38(3)(3)\,\text{MeV} \qquad f^p_u = 0.0139(13)(12)$$
$$\sigma_{sN} = 105(41)(37)\,\text{MeV} \qquad f^n_u = 0.0116(13)(11)$$
$$f^N_{ud} = 0.0405(40)(35) \qquad f^p_d = 0.0253(28)(24)$$
$$f_{sN} = 0.113(45)(40) \qquad f^n_d = 0.0302(28)(25)$$

Dashen's theorem

Dashen's theorem states that in the $SU(3)$ flavor symmetric limit the QED contributions to pseudoscalar meson masses fulfill

$$(M^2_{\pi^\pm} - M^2_{\pi^0})_{\rm EM} = (M^2_{K^\pm} - M^2_{K^0})_{\rm EM},$$
$$(M_{\pi^0})_{\rm EM} = (M_{K^0})_{\rm EM}.$$



Corrections to Dashen's theorem can be quantified by

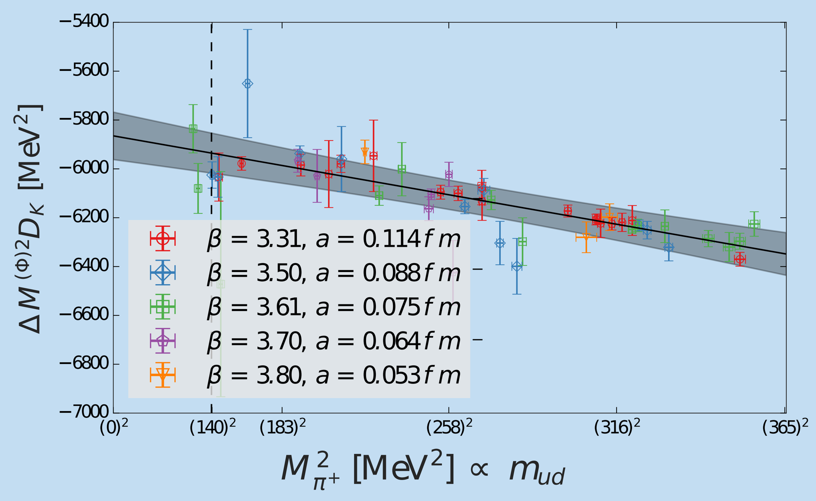
$$\varepsilon = \frac{\Delta_{\rm QED}M^2_K - \Delta_{\rm QED}M^2_\pi}{\Delta M^2_\pi}.$$

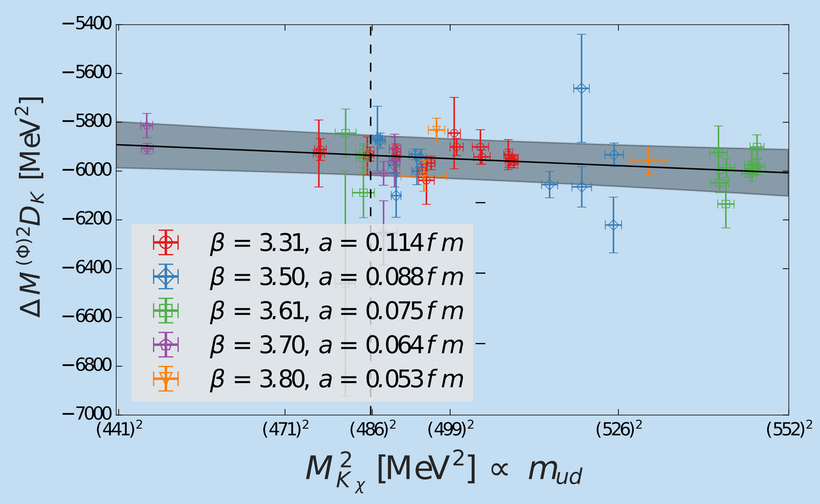
The Kaon mass splitting can be to leading order in fine structure constant α and the "connected meson splitting" $\Delta M^2 := M^2_{\bar{u}u} - M^2_{\bar{d}d}$ decomposed into

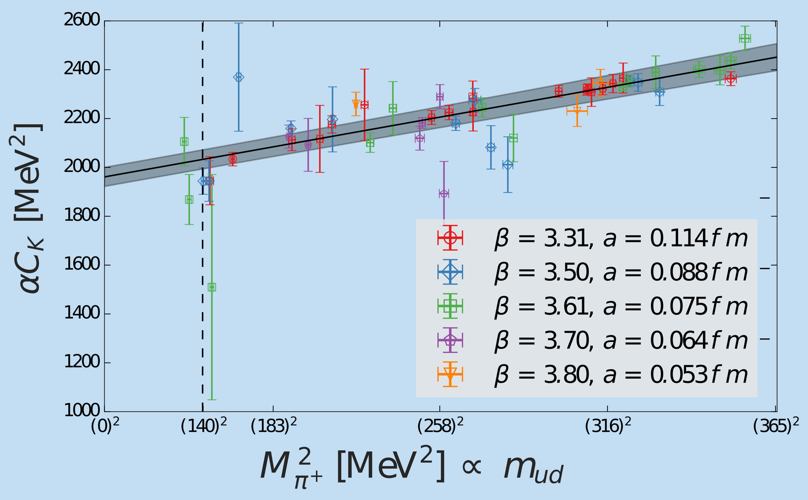
$$\Delta M^2_K = C_K\alpha + D_K\Delta M^2.$$

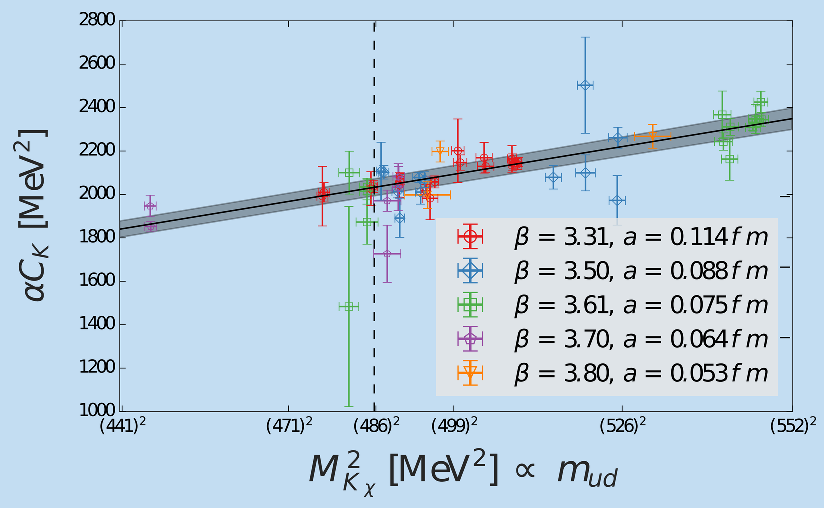
Determination of C_K and D_K

C_K and D_K are functions of the light and strange quark masses. These are represented by M^2_π and $M^2_{K_\chi}$ and C_K and D_K are fitted:









Datapoints are from $N_f = 2+1$ dynamical configurations with isospin effects added via quenched QED and partial quenched non degenerate light quarks.

Result from corr. to Dashen's theorem

In the following results the first error is statistical, the second is systematic and the last error is an estimate of the QED quenching uncertainty:

$$m_u = 2.27(6)(5)(4)\,\text{MeV} \;,\; m_d = 4.67(6)(5)(4)\,\text{MeV} \;,\; m_u/m_d = 0.485(11)(8)(14)$$
$$\varepsilon = 0.73(2)(5)(17) \;,\; R = 38.2(1.1)(0.8)(1.4) \;,\; Q = 23.4(0.4)(0.3)(0.4)$$

Refernces

S. Durr *et al.*, Phys. Rev. Lett. **116** (2016) no.17, 172001 [arXiv:1510.08013 [hep-lat]].

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