More on the fate of the conformal fixed point with twelve massless fermions

presented by the Lattice Higgs collaboration:
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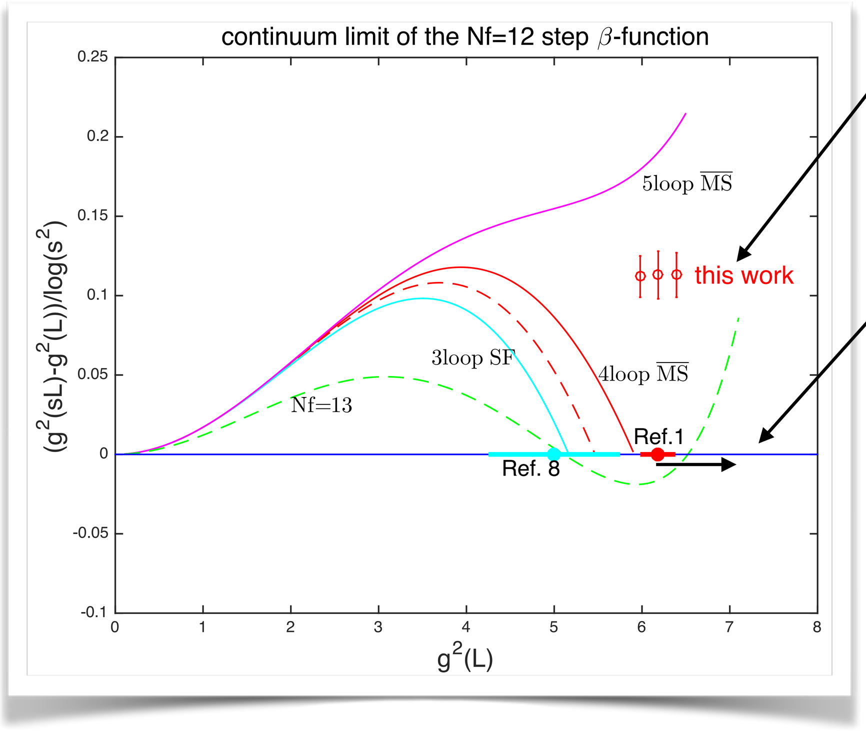
motivation of the work:

- Earlier a conformal infrared fixed point (IRFP) was reported in [1] at renormalized gauge coupling $g^2 \sim 6.2$ of the important SU(3) gauge theory with twelve massless fermions.
- In disagreement, no IRFP was seen in [2] around the $g^2 \sim 6.2$ location.
- In recent work [3] the IRFP of [1] was revived and moved by the authors to a new location in the $g^2 \sim 7$ region.
- In our new work reported here no IRFP is found in the $g^2 \sim 7$ region of the gauge coupling, in disagreement with the new results in [3]. Based on this controversy around the non-existence of the the IRFP, the (near)conformal behavior of this gauge theory remains undecided and important to resolve.

the origin of the controversy:



The plot from [2], referred as "this work" in the plot, summarizes the fate of the IRFP reported in [1]. Ref. 8 from [2] in the plot is the original publication of the IRFP by the LSD collaboration:

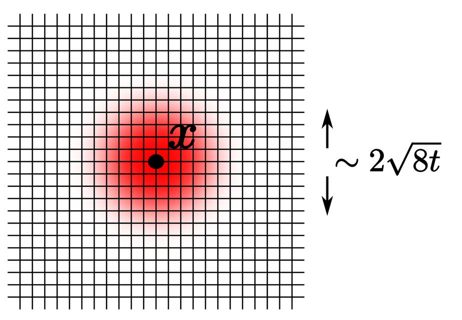
[8] T. Appelquist, G. T. Fleming, and E. T. Neil, Phys. Rev. D 79, 076010 (2009).

We are reporting new work which extends the three points of this plot into the $g^2 \sim 7$ region where the new IRFP location moved in [3].

Recent results from the 5-loop β-function calculations do not show IRFP at Nf=12 which only appears at NF=13 for the 5-loop β-function. 3-loop and 4-loop calculations indicated IRFP at Nf=12 before.

The scale-dependent step beta function is the target of our computational strategy:

computational strategy:



The renormalized gauge coupling $\alpha(t)$ is defined at gradient flow time t of the gauge field

Lüscher (earlier work by Neuberger)

the improved Symanzik gauge action is used in the gauge field gradient flow equations with clover improved energy operator E(t) at flow time t (SSC)

$$\langle E(t)\rangle = \frac{3}{4\pi t^2}\alpha(q)\{1+k_1\alpha(q)+O(\alpha^2)\},\quad q=\frac{1}{\sqrt{8t}},\quad k_1 = 1.0978 + 0.0075\times N_f$$

t is the gradient flow time

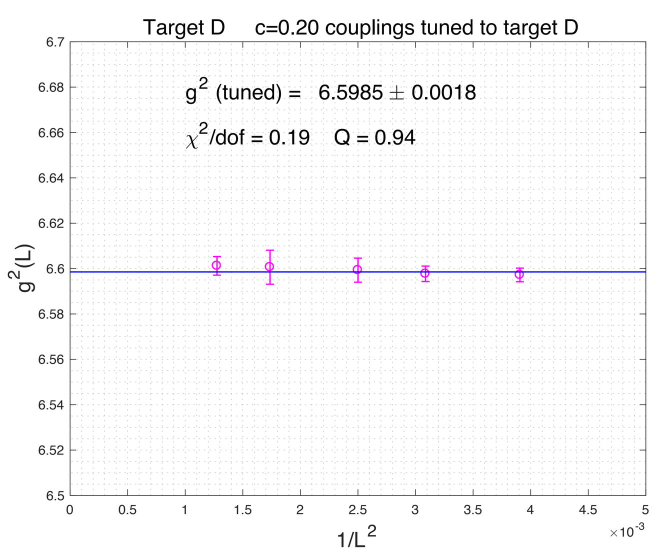
Running coupling definition (clover operator range is $\sim (8t)^{1/2}$) :

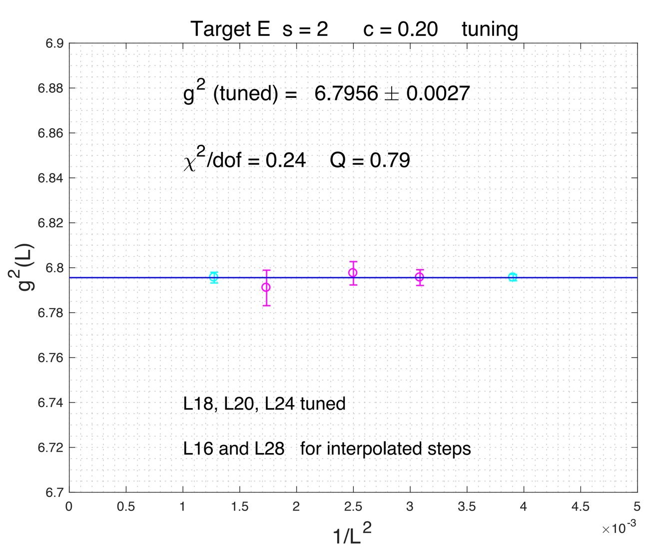
$$\text{while holding } c=(8t)^{1/2}/L \text{ fixed}\qquad \alpha_c(L) = \frac{4\pi}{3}\frac{\langle t^2E(t)\rangle}{1+\delta(c)}$$

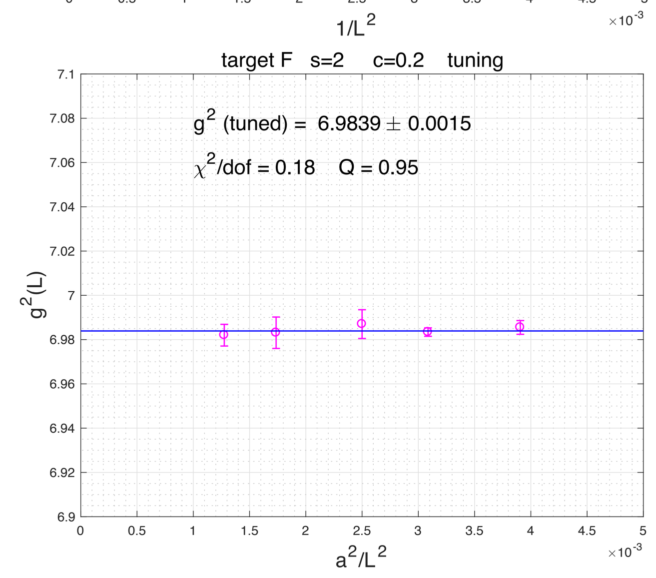
$$\delta(c) = \vartheta_3^4(e^{-1/c^2}) - 1 - \frac{c^4\pi^2}{3}$$

3rd Jacobi function

targeting tuned renormalized couplings at c=0.2 aspect ratio:



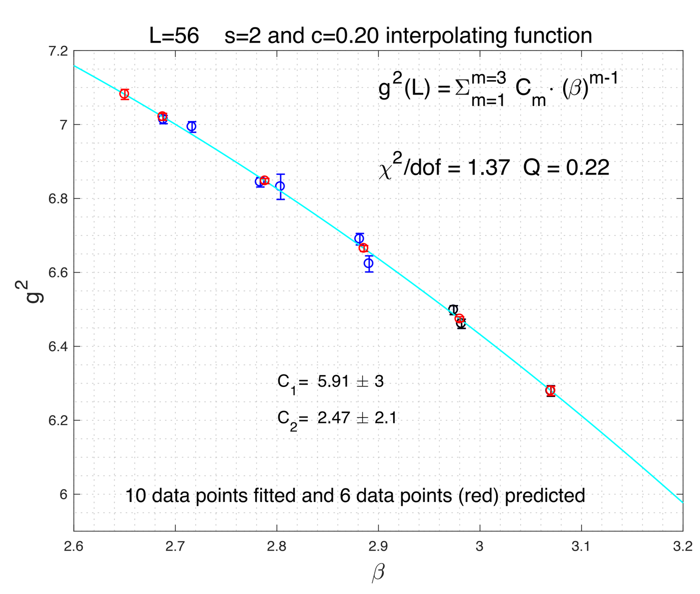


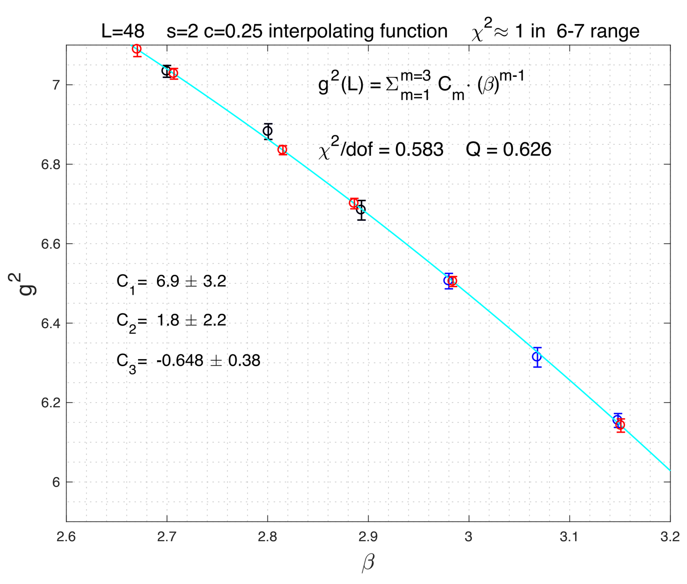


(1) Interpolation at fixed lattice volumes has been replaced by precise tuning of the lattice coupling g_0 at strategically targeted values of the renormalized gauge coupling in [2] and in the new work here. Fifteen renormalized gauge couplings were tuned at three targets D, E, F added to the previous targets A, B, C in [2] and extending the investigated range to $g^2 \sim 7$ where the new IRFP is reported in [3]. For two points interpolation was added after tuning for increased precision.

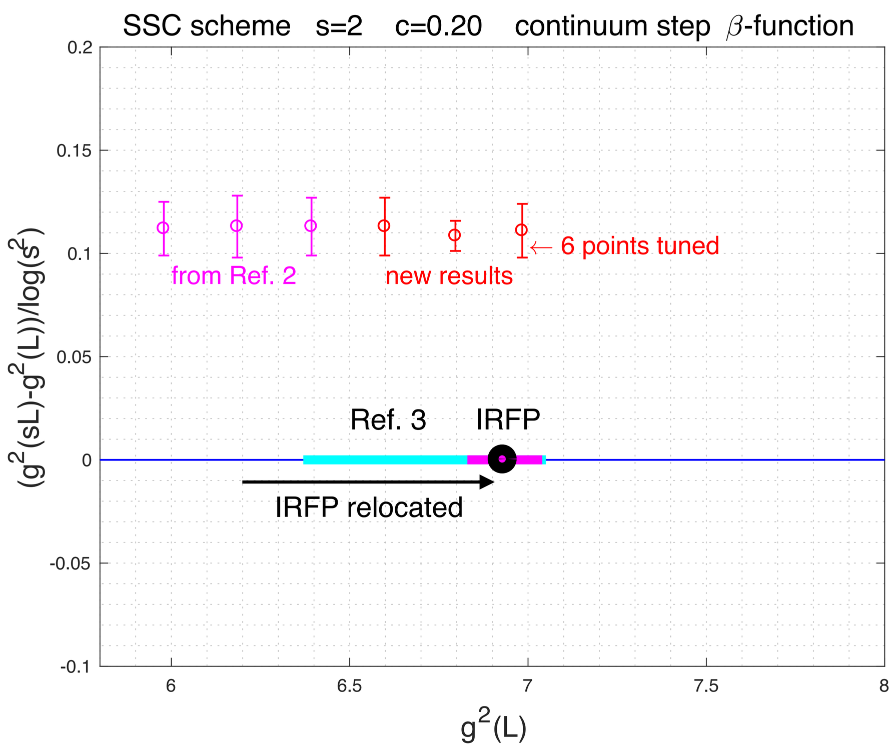
(2) Very large volumes have been deployed which was necessary to obtain definitive results around the location of the reported IRFP in [3]. The step β-function was probed for L16→L32, L18→L36, L20→L40, L24→L48, L28→L56 with tuned g^2 at the lower L of each pair.

We also investigated the interpolation based step beta functions at c=0.2 and c=0.25 for direct comparison with interpolated results in [3]. The high quality of our polynomial interpolations had excellent χ^2 values in all volumes as the representative samples demonstrate where red points are predicted and blue/black are combined data from [2] and the new tuned runs:





most important tuned new results:



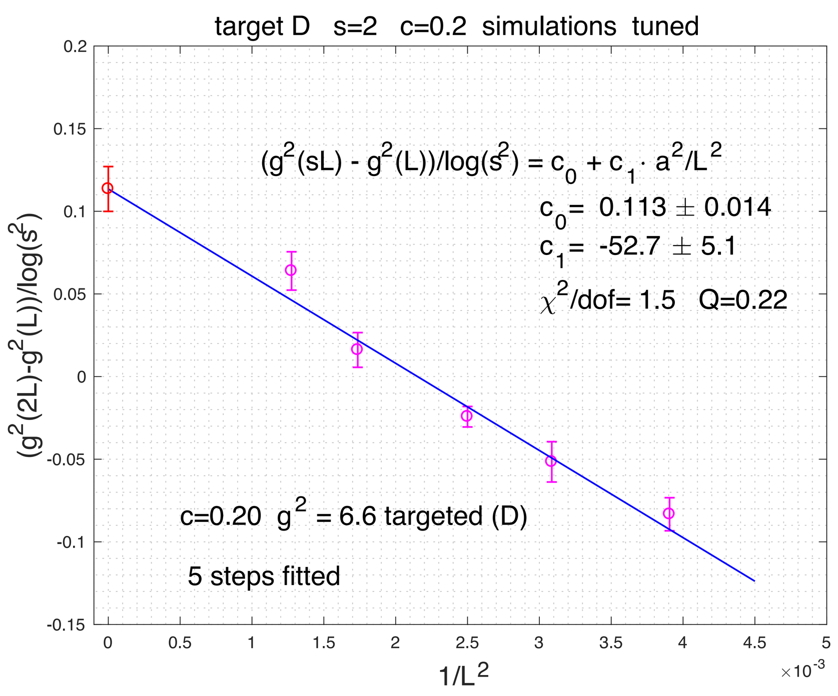
The continuum step β-function is calculated at three targeted values of the renormalized gauge coupling $g^2(L)$ where the linear size L of the finite volume is in arbitrary scale units. The physical size of the volume monotonically grows with increasing $g^2(L)$.

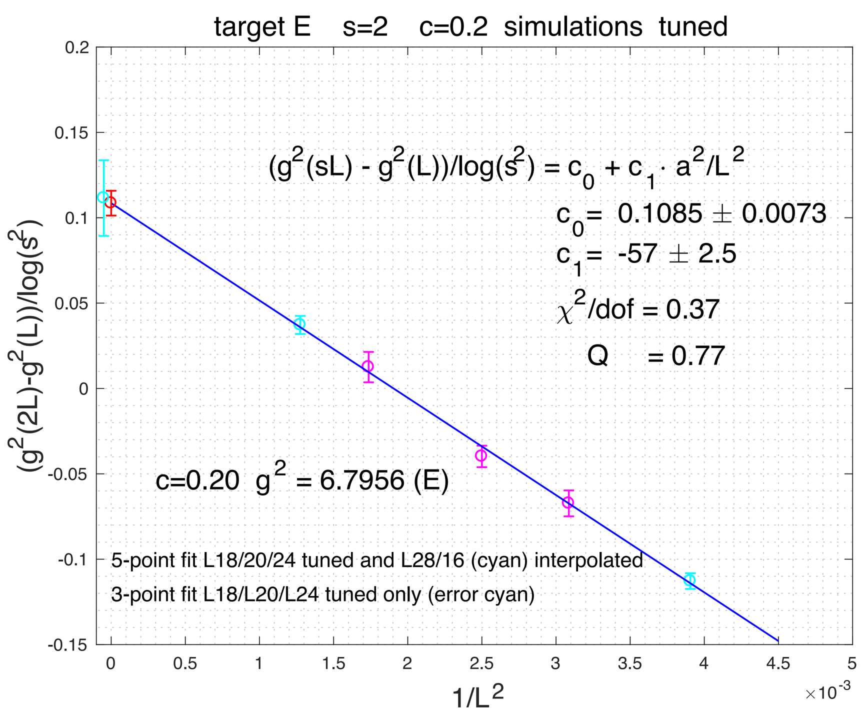
The three red points are new results for targets D, E, F. The three magenta points are targets A, B, C from [2] in disagreement with the earlier location of the IRFP in [1]. All 6 points of the step β-function were determined from precisely tuned and targeted renormalized gauge couplings $g^2(L)$ eliminating systematic errors from interpolation.

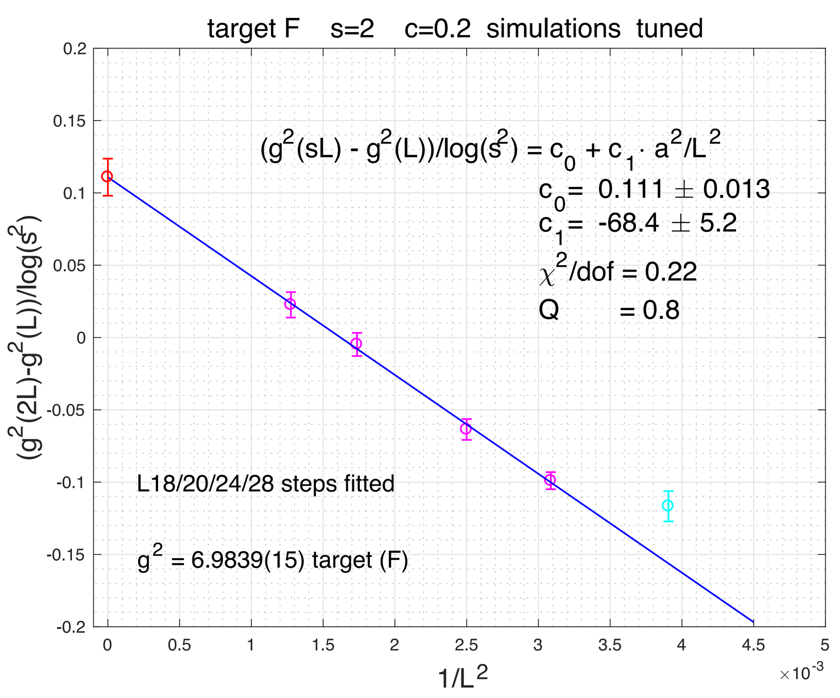
The relocated IRFP from [3] is shown with statistical error band (magenta) and what is described in [3] as systematic error band (cyan).

The existence of the recently relocated IRFP from [3] is inconsistent with our new results. The statistical evidence represented by the error bars of the independent data points is overwhelming. The error on target E is from the 5-point fit. With unlikely effect on the conclusions, no additional systematics is provided.

continuum extrapolation of the step β-function is shown for the new targets D, E, F:







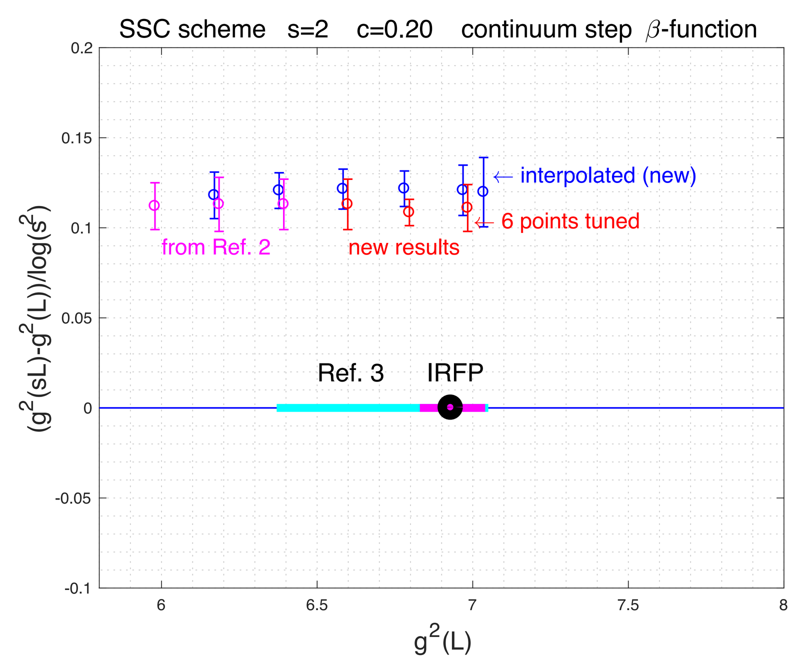
Two key ingredients in our work overcome the most important limitations on the results reported in [3]. They were also applied to our recent work in [2] which was a response to the reported IRFP in [1]:

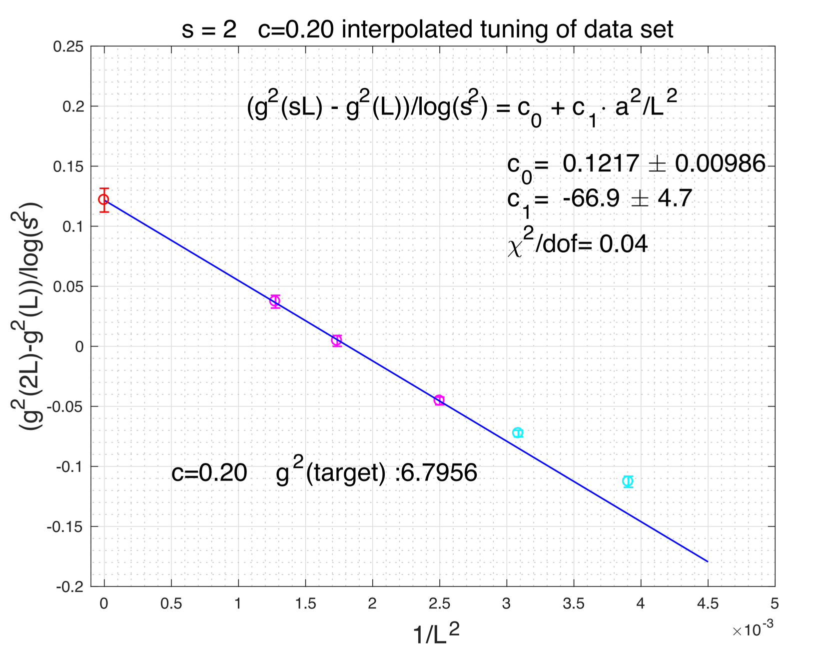
(1) Targeting selected values of the renormalized gauge coupling, interpolation at fixed lattice volumes is replaced by precise tuning of the lattice coupling g_0

(2) Very large volumes have been deployed which was necessary to obtain decisive results on the (non)existence of the reported IRFP in [3].

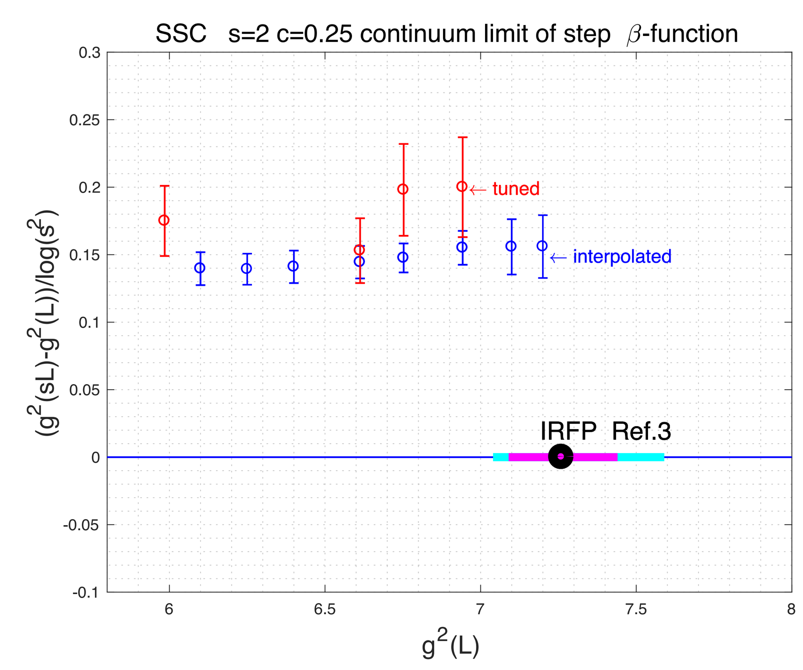
tuned results compared with interpolations:

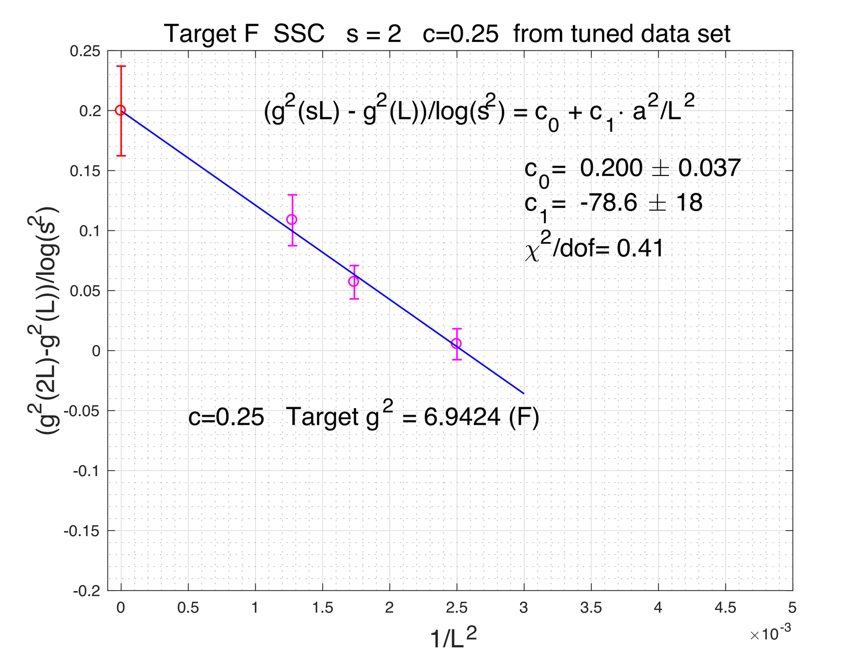
the consistency of tuned and interpolated results is shown at c=0.2 :





the consistency of tuned and interpolated results is shown at c=0.25 :





References

[1] A. Cheng, A. Hasenfratz, Y. Liu, G. Petropoulos, and D. Schaich, J. High Energy Phys. 05 (2014) 137.

[2] Z. Fodor, K. Holland, J. Kuti, D. Nogradi, and C.H. Wong, Phys. Rev. D 94, 091501(R) (2016).

[3] A. Hasenfratz and D. Schaich, arXiv:1610.10004 (2016).