Model development for meteorological applications

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EMCL

Low-Mach (LM) and Compressible Navier-Stokes (CNS) Model for Cyclone-Cyclone Interaction

CNS vs. LM:

- Variables:
  - **v**: Velocity field
  - ρ^*: density of fluid
  - θ_v^*: temperature of density of fluid
  - p^*: pressure within fluid
- Modelization of fluid's behavior:
  - Conservation of momentum
  - Conservation of mass
  - Conservation of energy/temperature
  - Ideal gas law
- Atmosphere: many relevant flows in LM regime

Key observation for LM:

- Split *total pressure* p in three parts:

$$p(x,t) := p_{th}(t) + p_0(x) + p^*(x,t)$$

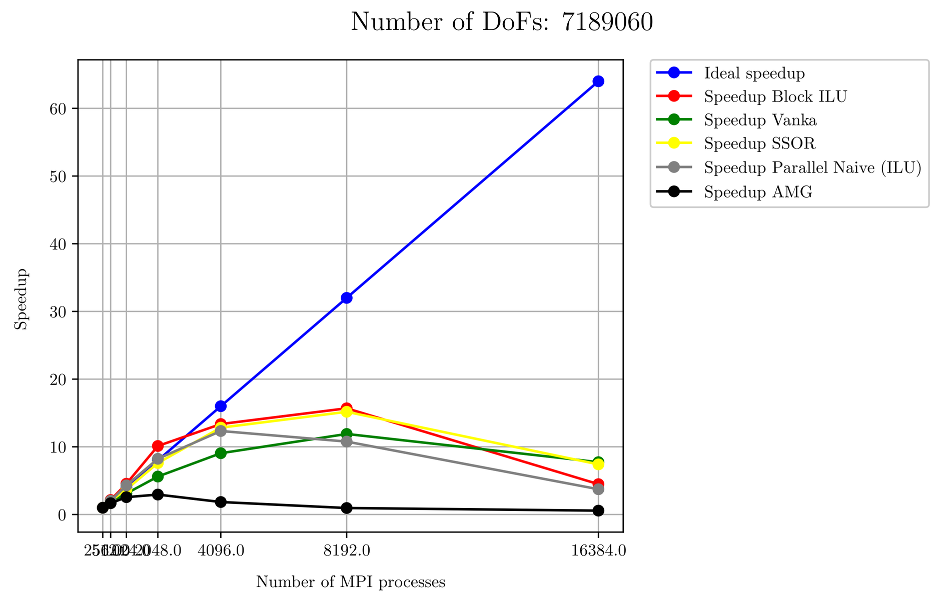
  - p_{th}: *thermodynamic pressure* (const. in space),
  - p_0: *hydrostatic pressure* (const. in time),
  - p^*: *hydrodynamic pressure*
- In LM regime:

$$|p^*(x,t)| \ll |p_{th}(t) + p_0(x)|$$

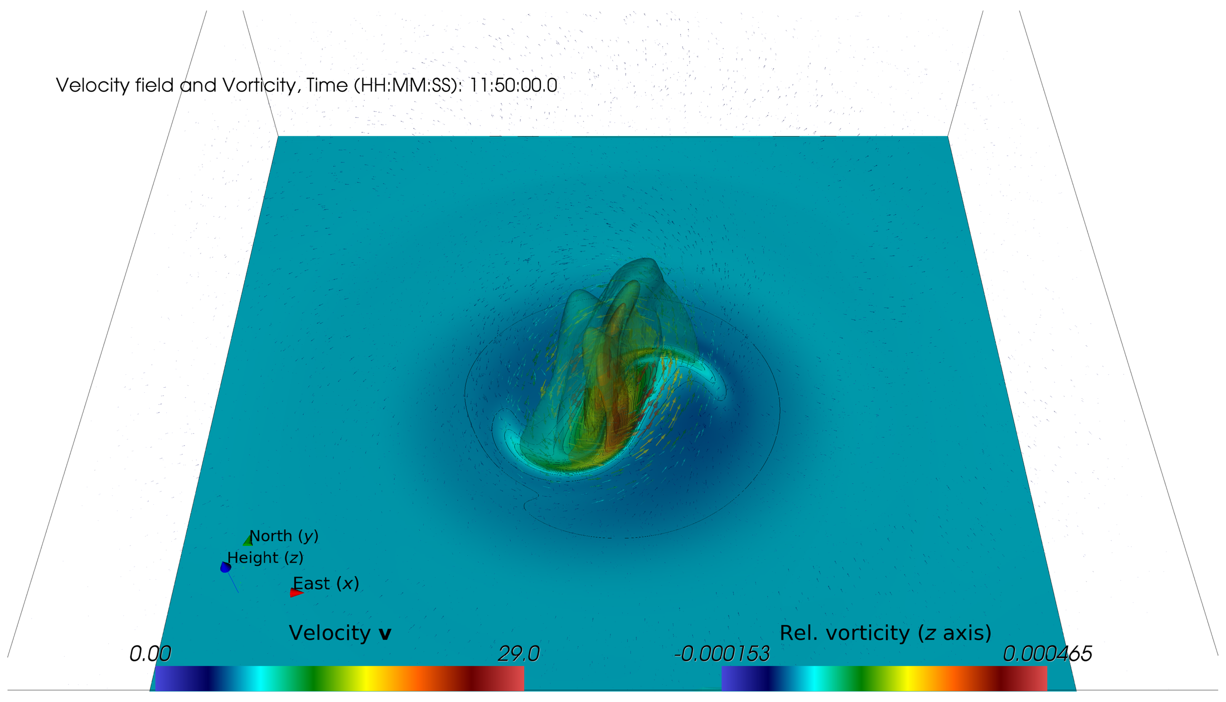
$\Rightarrow$ Neglect p^* in ideal gas law
$\Rightarrow$ Damping of acoustic modes in LM

Numerical Solver:

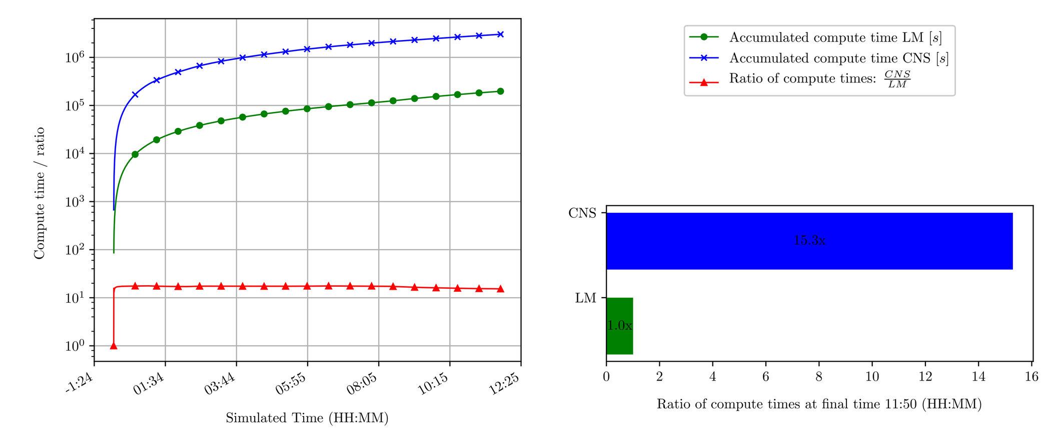
- Finite element discretization in space, finite differences in time
- Nonlinear solver: Newton's method
- Linear solver *CNS*:
  - Generalized Minimum Residual (GMRES) method
  - *Preconditioner:* BoomerAMG of *hypre* library
- Linear solver *LM*:
  - Flexible Generalized Minimum Residual (FGMRES) method
  - *Preconditioner:* nested Schur complement approach



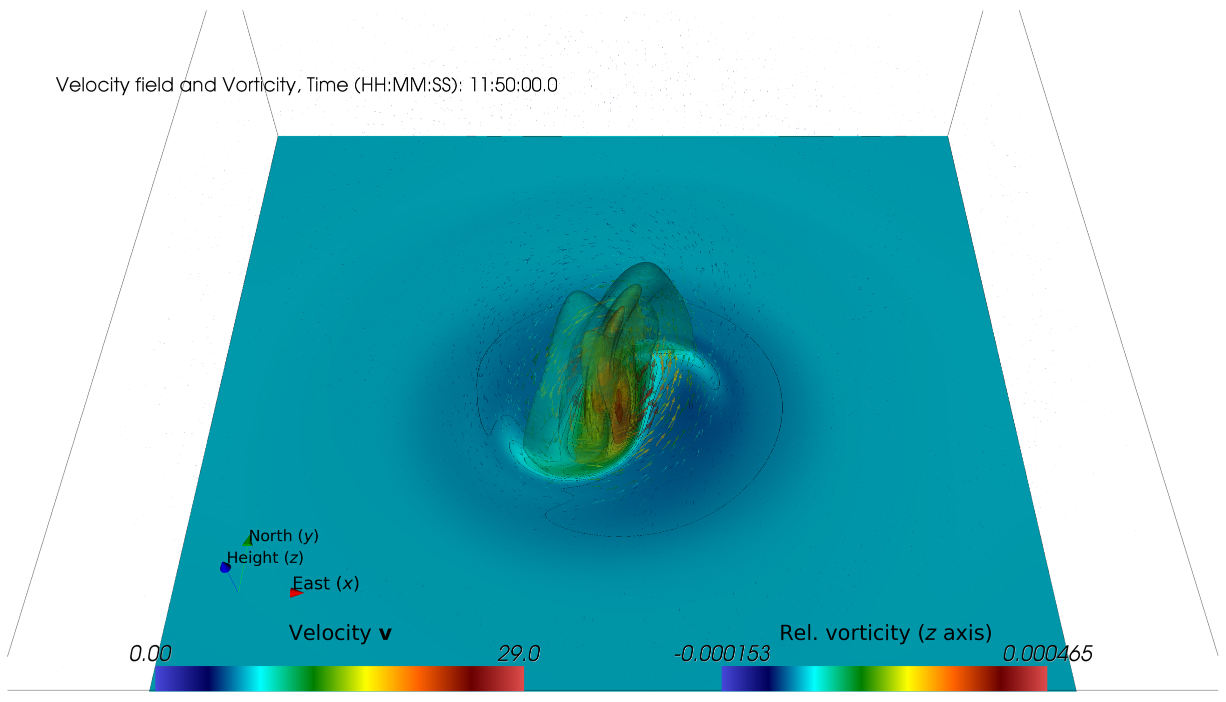
Scaling for Poisson's equation on JUQUEEN



Solution LM (bwForCluster MLS & WISO)



Runtime CNS vs. LM (bwForCluster MLS & WISO)



Solution CNS (bwForCluster MLS & WISO)

Thermal Electro-Hydrodynamical Boussinesq Equations

Boussinesq approximation, augmented by dielectrophoretic force and Gauss law

$$\partial_t u + (u\cdot\nabla)u - \nu\Delta u + \frac{1}{\rho}\nabla p = \alpha_f (\nabla\Phi)^2\nabla\theta - \alpha_g(\theta-\theta_r)\vec{g}$$
$$\nabla\cdot u = 0$$
$$\partial_t\theta + (u\cdot\nabla)\theta - \kappa\Delta\theta = 0$$
$$-\nabla\cdot(\epsilon(1+e(\theta_r-\theta))\nabla\Phi) = 0$$
$$E = \nabla\Phi$$

System matrix after FEM discretization

$$\mathcal{K} = \begin{pmatrix}\mathcal{A} & \mathcal{B}\\ \mathcal{C} & \mathcal{D}\end{pmatrix}$$

Temperature-potential matrix

$$\mathcal{A} = \begin{pmatrix}A_\theta & 0\\ * & A_\Phi\end{pmatrix}$$

Incompressible N-S matrix

$$\mathcal{D} = \begin{pmatrix}A_v + N & B^T\\ -B & 0\end{pmatrix}$$

Optimal preconditioner

$$\mathcal{P} = \begin{pmatrix}\mathcal{A} & \mathcal{B}\\ 0 & \mathcal{S}\end{pmatrix}$$

with spectrum

$$\sigma(\mathcal{P}^{-1}\mathcal{K}) = \{1\}$$

Computable preconditioner

$$\hat{\mathcal{P}} = \begin{pmatrix}\mathcal{A} & \mathcal{B}\\ 0 & \hat{\mathcal{S}}\end{pmatrix}$$

Exact SC

$$\mathcal{S} = \mathcal{D} - \mathcal{C}\mathcal{A}^{-1}\mathcal{B} = \begin{pmatrix}A_v + N + Q(\mathcal{A}) & B^T\\ -B & 0\end{pmatrix}$$

Approximate SC

$$\hat{\mathcal{S}} = \begin{pmatrix}A_v & B^T\\ -B & 0\end{pmatrix}$$

$\rightarrow$ Instationary Stokes matrix

Involved sub solvers

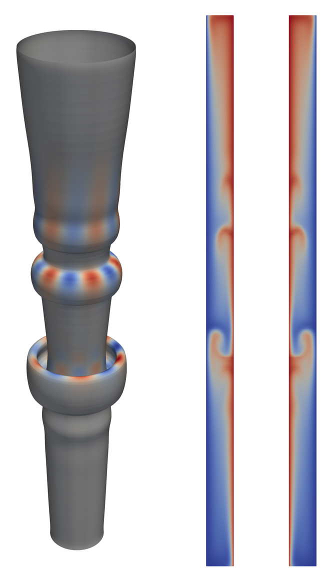
- Pressure convection diffusion preconditioner for Stokes system
- Algebraic Multigrid for elliptic sub matrices

Parameters

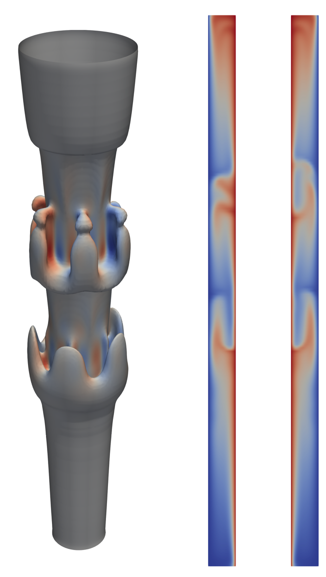
ν	$5\cdot10^{-6}$
κ	$7.74\cdot10^{-8}$
ρ	$9.23\cdot10^{2}$
e	$1.07\cdot10^{-3}$
α_f	$1.4\cdot10^{-17}$
α_g	$1.08\cdot10^{-3}$
$\delta\theta$	5 K
V	10 kV
H	100 mm
r_i	5 mm
r_o	10 mm
$DOFs$	$2.6\cdot10^{7}$

Figures: isosurface $\{\theta = 25°C\}$ with axial vorticity, θ on cutplane

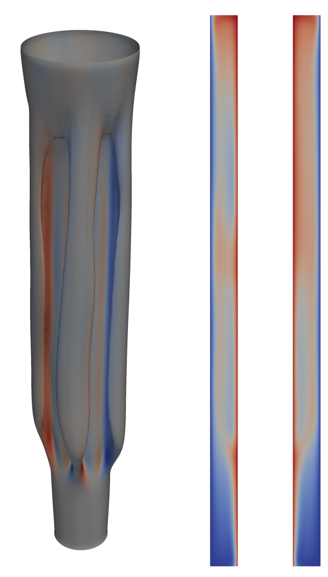
Axial symmetric phase (25s)



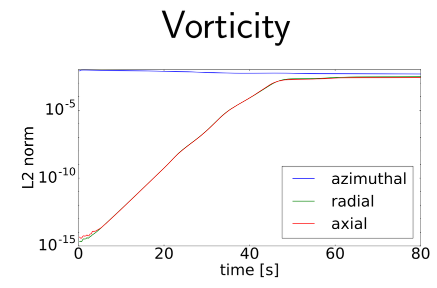
Transition phase (47.5s)



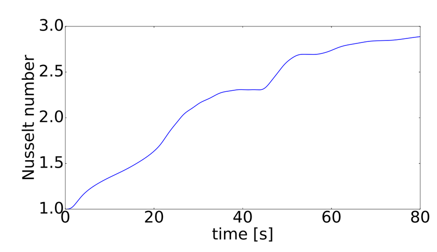
Stationary phase (80s)



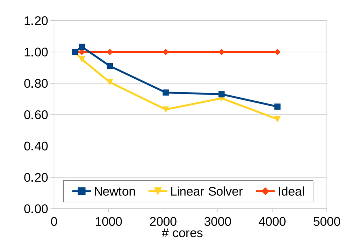
Vorticity



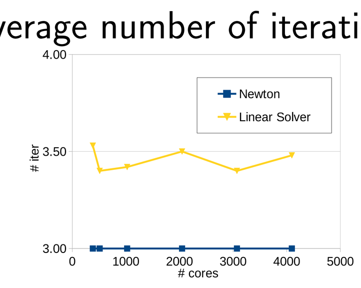
Heat transfer



Parallel efficiency



Average number of iterations



Uncertainty Quantification for Blood Pump Simulation

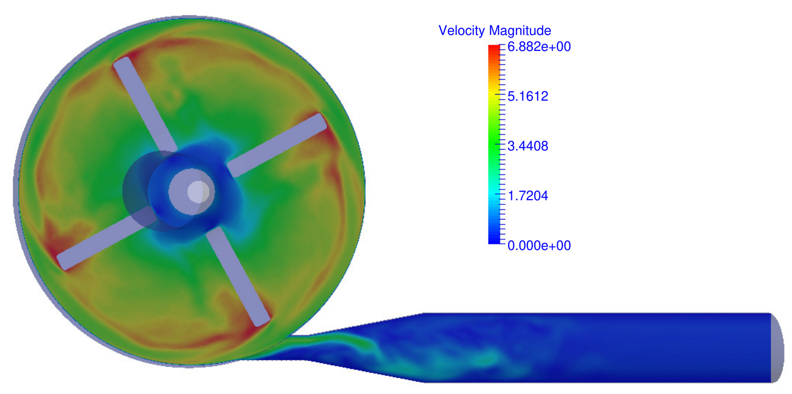
Three sources of uncertainty are considered:

- inflow boundary condition $g(x)$.
- angular speed ω.
- dynamic viscosity μ.

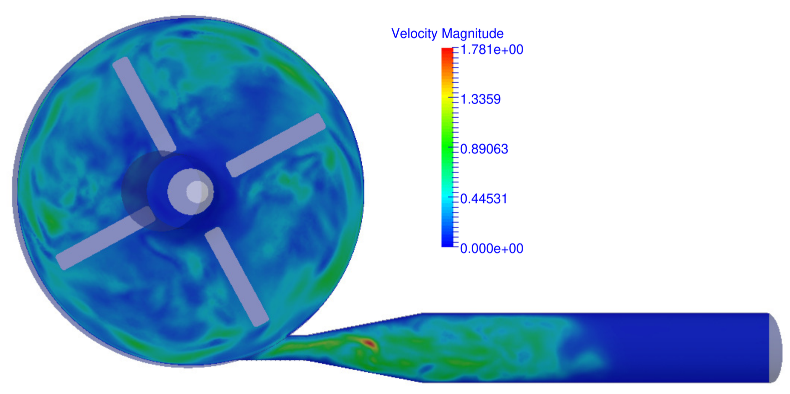
Three random variables are modeled as Uniform distribution:
$\xi_i \sim U(-1,1), i = 1,2,3.$

$$g(x) := g_0(x) + \sigma_1 g_0(x)\xi_1\ ,$$
$$\omega := \omega_0 + \sigma_2\omega_0\xi_2\ ,$$
$$\mu := \mu_0 + \sigma_3\mu_0\xi_3\ .$$

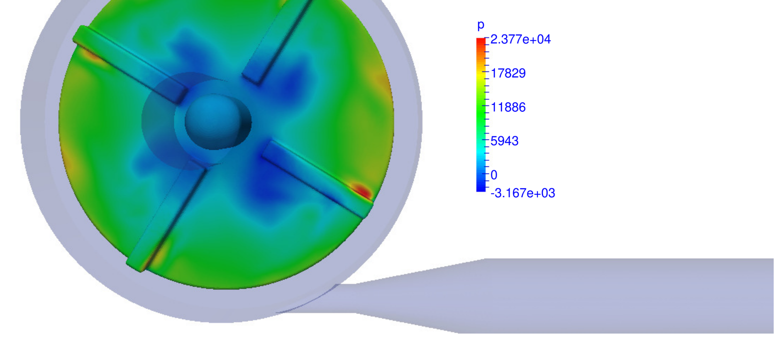
$$\Big(\frac{u_{h,k}}{\partial t}, v_h\Big) + \sum_{i=0}^{P}\sum_{j=0}^{P}((\hat{u}_{h,i} - u^r_{h,i})\cdot\nabla u_{h,j}, v_h)\, C_{ijk} + \sum_{i=0}^{P}\sum_{j=0}^{P}\frac{\mu_i}{\rho}(\nabla u_{h,j}, \nabla v_h) C_{ijk} - \frac{1}{\rho}(p_{h,k}, \nabla\cdot v_h) - (f_{h,k}, v_h)$$
$$+ (\tau_M[\frac{\partial u_{h,k}}{\partial t} + \sum_{i=0}^{P}\sum_{j=0}^{P}(u_{h,i} - u^r_{h,i})\cdot\nabla u_{h,j}\, C_{ijk} - \sum_{i=0}^{P}\sum_{j=0}^{P}\frac{\mu_i}{\rho}\Delta u_{h,j}\, C_{ijk} + \frac{1}{\rho}p_{h,k} - f_{h,k}]$$
$$, (\hat{u}_{h,0} - u^r_{h,0})\cdot\nabla v_h) + (\tau_C\nabla\cdot u_{h,k}, \nabla\cdot v_h) = 0\ ,$$
$$(\frac{1}{\rho}\nabla u_{h,k}, q_h) + (\tau_M[\frac{\partial u_{h,k}}{\partial t} + \sum_{i=0}^{P}\sum_{j=0}^{P}(u_{h,i} - u^r_{h,i})\cdot\nabla u_{h,j}\, C_{ijk} - \sum_{i=0}^{P}\sum_{j=0}^{P}\frac{\mu_i}{\rho}\Delta u_{h,j}\, C_{ijk} + \frac{1}{\rho}\nabla p_{h,k} - f_{h,k}], \nabla q_h) = 0\ ,$$
$$\forall\ v_h,\ q_h$$



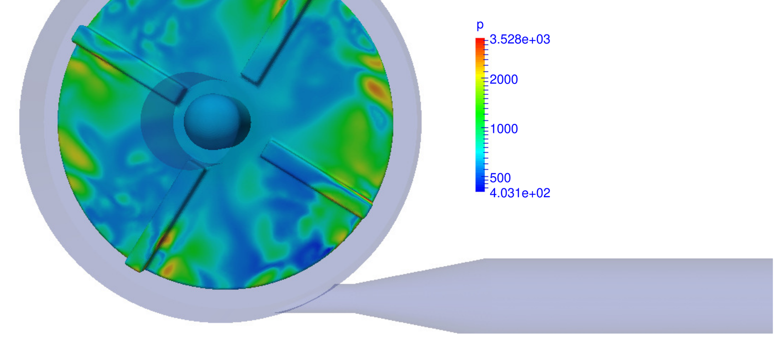
Mean value of velocity



Standard deviation of velocity



Mean value of pressure



Standard deviation of pressure

Acknowledgements

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