

# The Challenge of Small-Scale Turbulence in Planetary Boundary Layers

J. P. Mellado

*Max Planck Institute for Meteorology, Hamburg*



Canary Islands (Gran Canaria), from [www.wikipedia.org](http://www.wikipedia.org)



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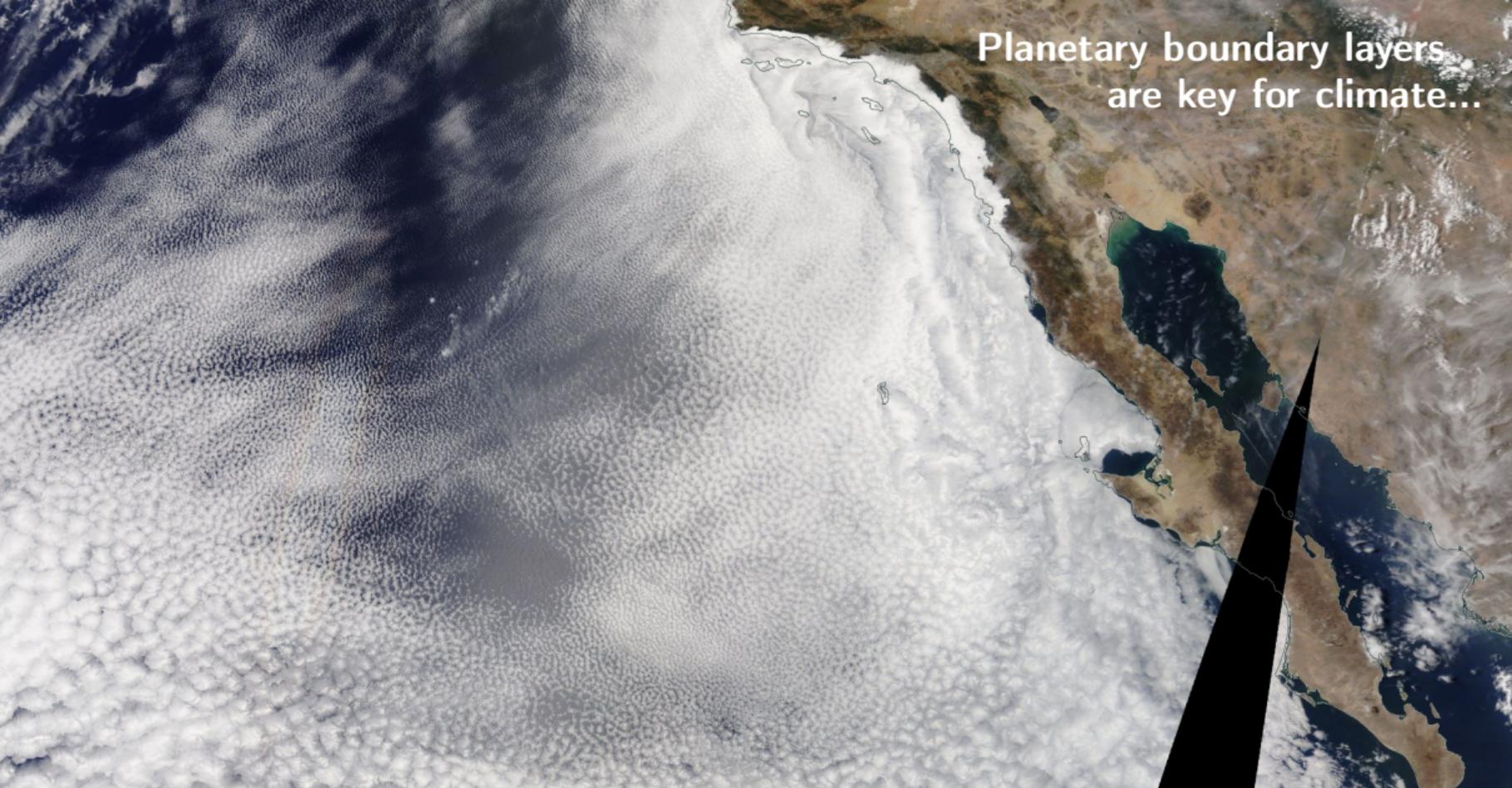
## The Planetary Boundary Layer

The planetary boundary layer is the lower layer of the atmosphere, the layer that feels directly surface effects on time scales smaller than a day.

Normally turbulent, because of wind shear or because of convection.



Planetary boundary layers  
are key for climate...



## ... planetary boundary layers are crucial in meteorology ...



...and are also a paradigm of multi-scale, complex fluid-dynamical systems.



## Understanding Meter and Submeter Scales Remains a Challenge

In important cases, small scales affect the dynamics of the whole planetary boundary layer:

1. Near the surface: Effects of density stratification in the shear production of turbulence.
2. Near the PBL top: Effects of density stratification and cloud physics in entrainment.

Understanding this coupling between the small and the large scales remains a challenge that combines fluid mechanics and atmospheric processes. This challenge motivates our work.

Our analysis is based on theory and direct numerical simulations.

Key novelty: Reynolds numbers are becoming large enough to observe Reynolds number similarity. Production runs with grid sizes  $2048^2 \times 1024$  to  $5120^2 \times 2048$ .

⇒ **HPC is providing data accurate enough for quantitative analysis.**

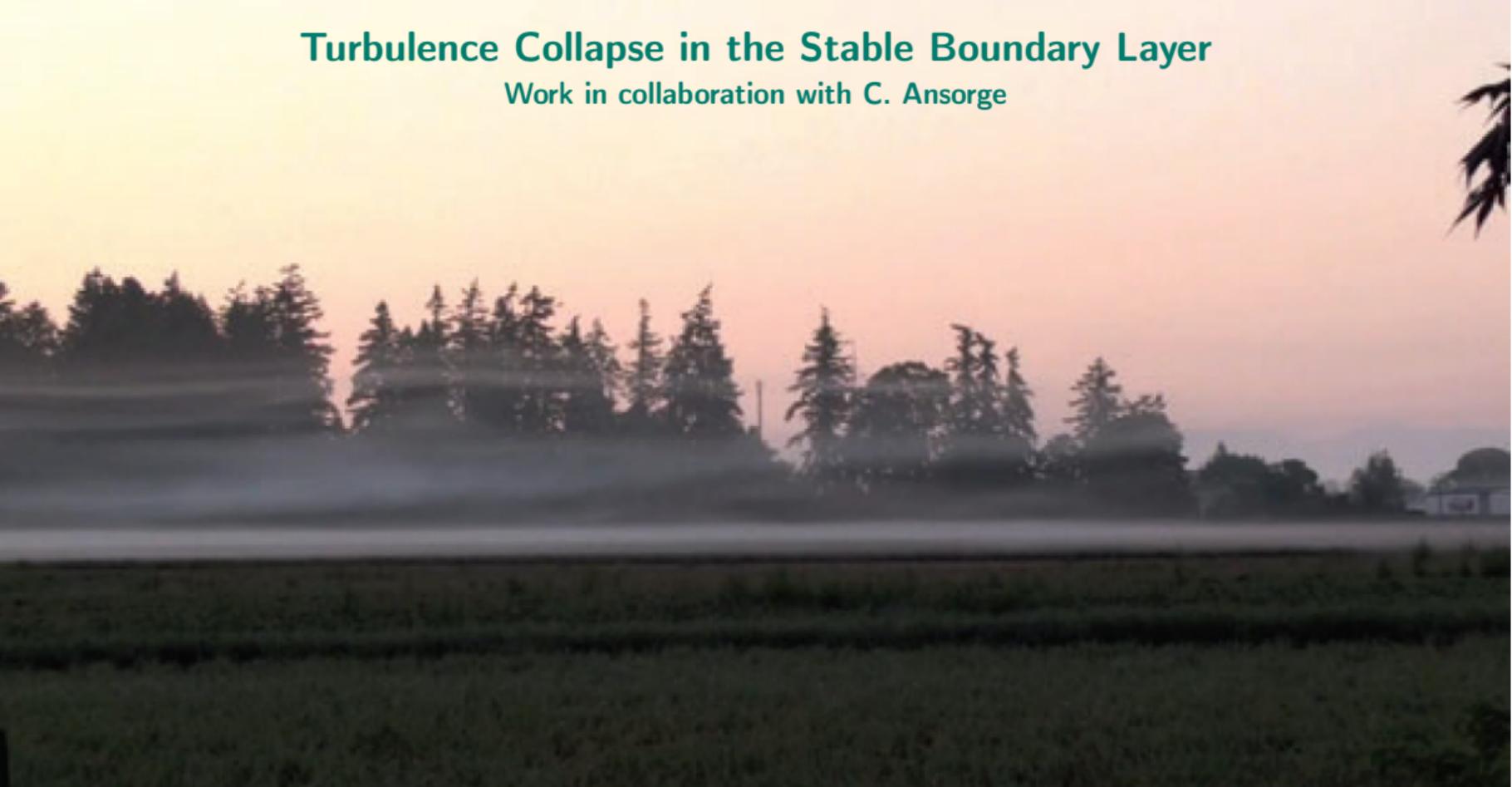
# Outline

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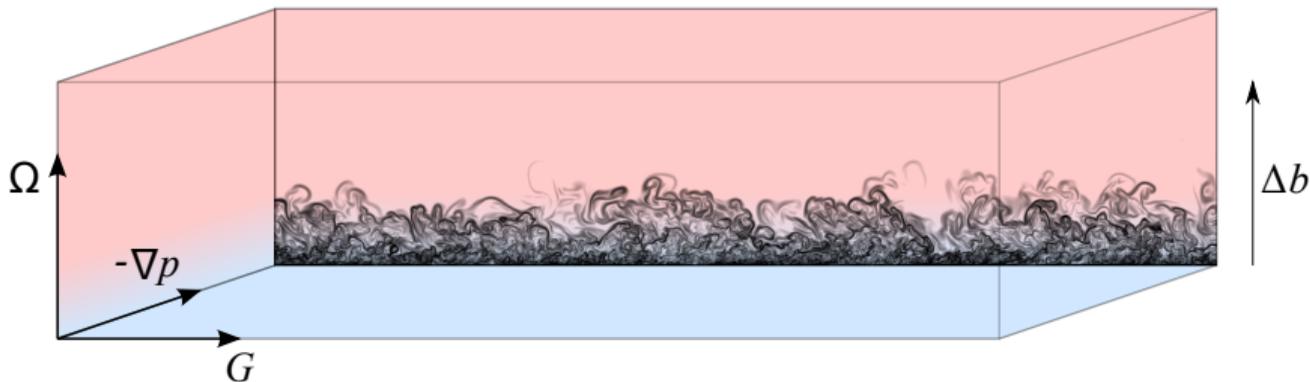
- 1. Turbulence Collapse in the Stable Boundary Layer**
2. Entrainment Effects on Moisture Statistics
3. Entrainment Reduction in Stratocumulus by Droplet Sedimentation
4. Direct Numerical Simulation of Stratocumulus-Topped Boundary Layers

# Turbulence Collapse in the Stable Boundary Layer

Work in collaboration with C. Ansorge



## Physical Model: Stably Stratified Ekman Layer



$\Omega$  angular velocity;  
 $G$  geostrophic wind velocity;  
 $\nabla p$  geostrophic pressure gradient.

$\Delta b$  buoyancy increment.

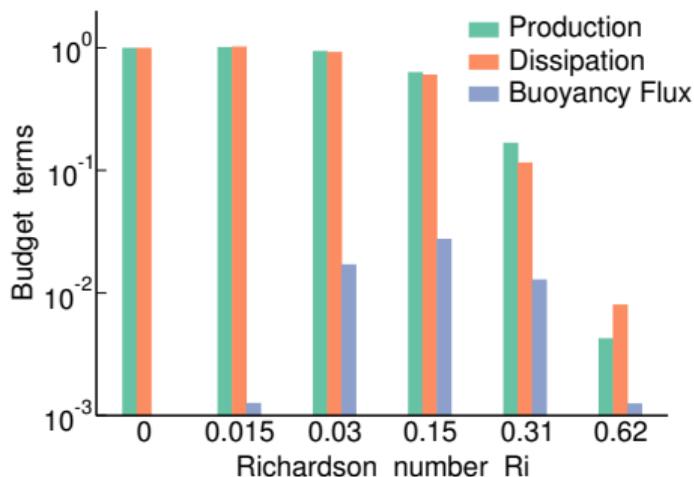
Two control parameters:

$$\text{Richardson number} = \frac{\text{buoyancy forces}}{\text{inertia forces}} \propto \Delta b$$

$$\text{Reynolds number} = \frac{\text{inertia forces}}{\text{viscous forces}} \propto \nu^{-1}$$

## Model Reproduces Regimes Observed in Nature: Weakly, Intermediately, and Strongly Stratified

$$\frac{d}{dt} \int \text{TKE} dz = \text{Prod.} - \text{Diss.} - \text{Buoy.Flux}$$



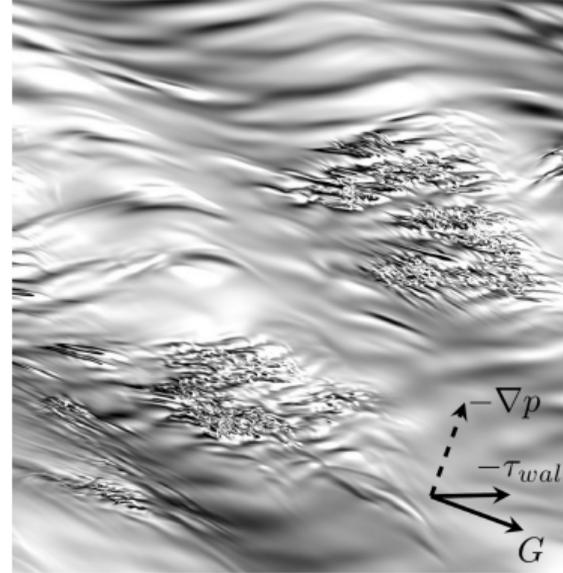
Strongly stratified regime for Richardson numbers beyond 0.1 – 0.2.

# Spatial Intermittency During Turbulence Collapse Can Occur Without External Perturbations

Neutrally Stratified



Strongly Stably Stratified



Near-surface enstrophy. Only  $1/3 \times 1/3$  of domain is shown.

...provided that large scales have enough space and time to develop.

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# Entrainment Effects on Moisture Statistics

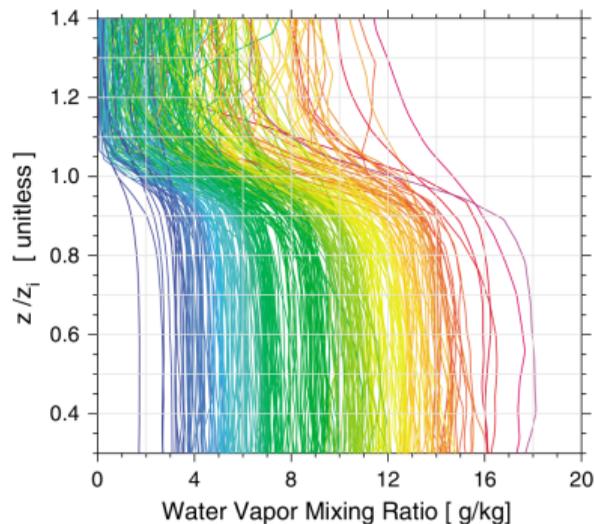
Work in collaboration with M. Puche, A. Haghshenas & C. C. van Heerwaarden



Moisture field from DNS of 1000 m deep CBL resolved to 2 m.



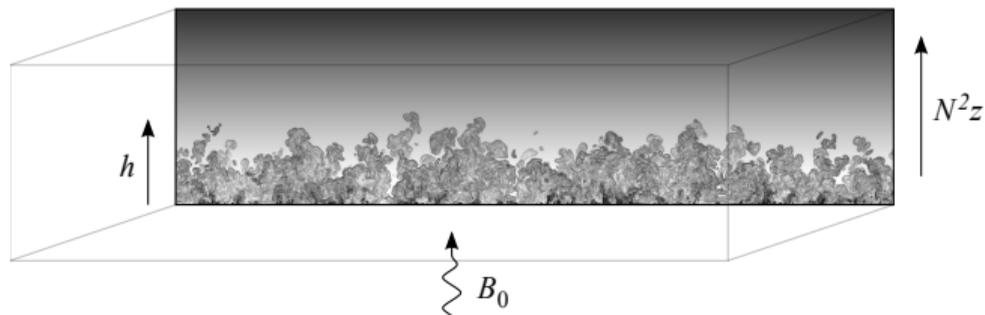
## Aim: Generalize Previous Single-Case Studies on Moisture Statistics



Specific questions:

1. Obtain dependence on environmental conditions.
2. Obtain characteristic scales in entrainment zone and surface layer.

## Physical Model: Free Convective Boundary Layer



$B_0$ : surface buoyancy flux

$N^2$ : outer buoyancy gradient

$h$ : CBL depth

System depends only on Reynolds number ( $Pr = 1$ ), and  $h/L_0$ , where

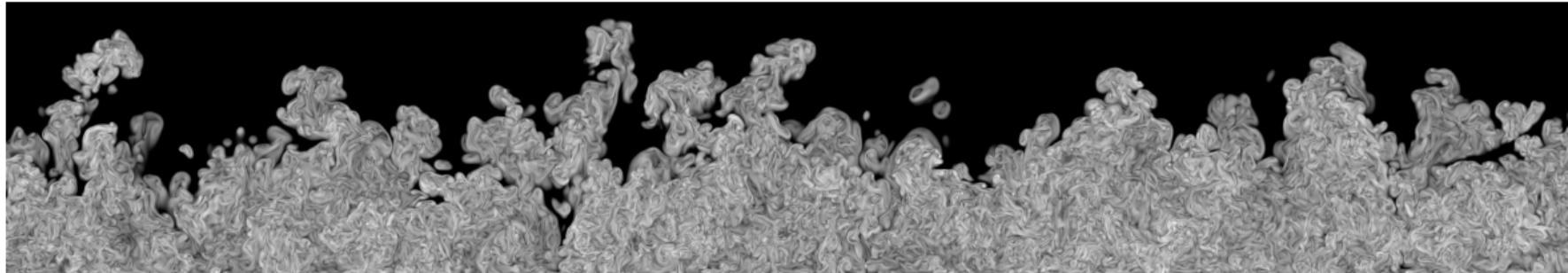
$$L_0 = (B_0/N^3)^{1/2}$$

is an Ozmidov scale. It provides a reference entrainment-zone thickness.

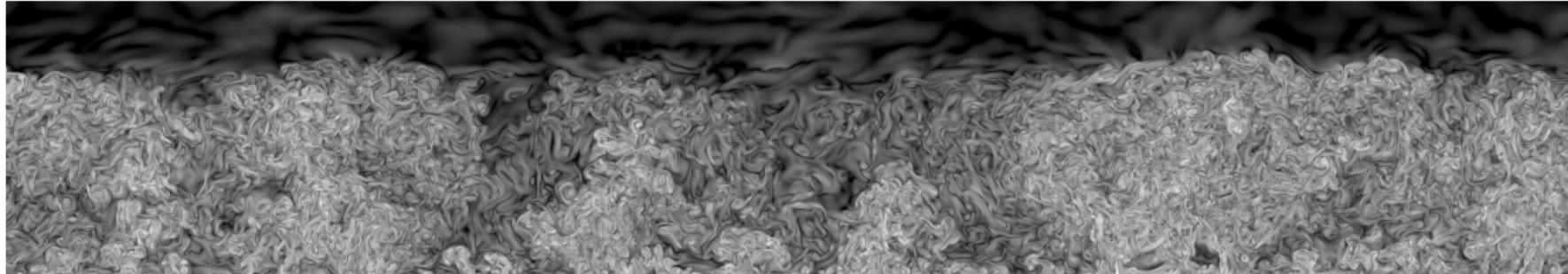
Typical atmospheric midday values:  $L_0 \simeq 20 - 200$  m ( $h/L_0 \simeq 5 - 50$ ).

## Parameter $h/L_0$ : Weak- and Strong Stratification Regimes

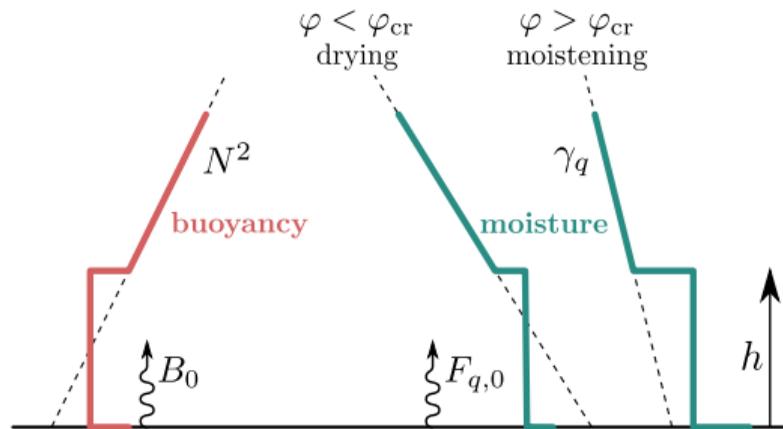
**Weak Stratification Regime:** low values of  $h/L_0$  (below 1), unsteady.



**Strong Stratification Regime:** large values of  $h/L_0$  (beyond 10–15), quasi-steady.



## Defining an Appropriate Moisture Parameter



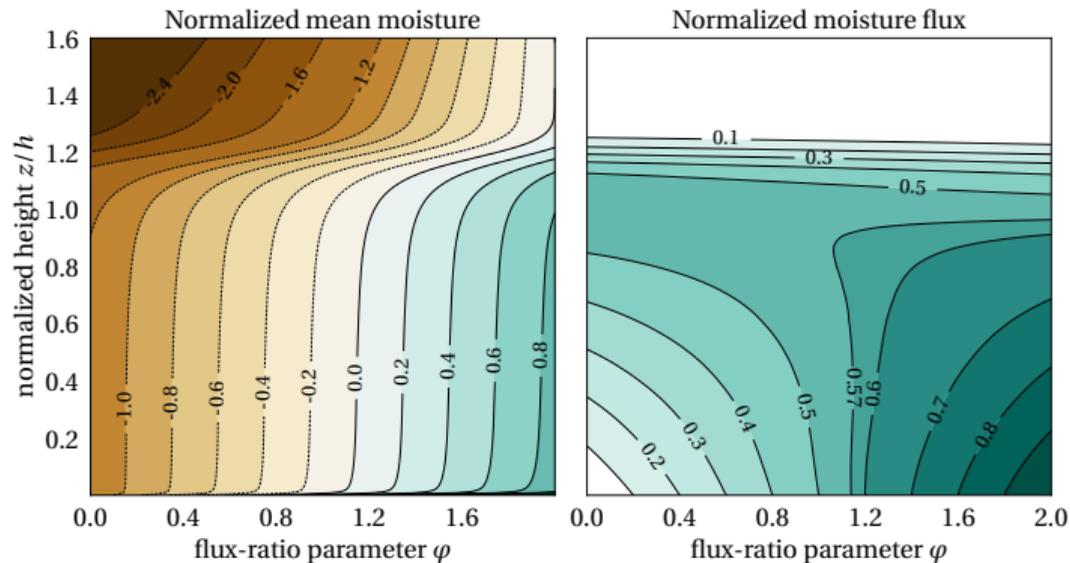
$\{B_0, F_{q,0}\}$	surface buoyancy & moisture fluxes
$\{-N^2, \gamma_q\}$	buoyancy & moisture lapse rates
$h$	CBL depth
$\varphi$	Flux-ratio parameter (surface moistening vs. entrainment drying)

Besides the Reynolds number and  $h/L_0$ , moisture introduces only one parameter. We choose:

$$\varphi = 2F_{q,0}/(F_{q,0} + F_{q,1}),$$

where  $F_{q,1} \equiv (\gamma_q L_0)(N L_0)$  is a reference entrainment flux.

## Mean Properties: Drying-to-Moistening Transition at $F_{q,0} \approx F_{q,1}$ ( $\varphi \simeq 1$ ).



Drying-to-moistening transition occurs when the surface moisture flux  $F_{q,0}$  becomes comparable with the reference entrainment flux in drying conditions  $F_{q,1} \equiv (\gamma_q L_0)(NL_0)$ .

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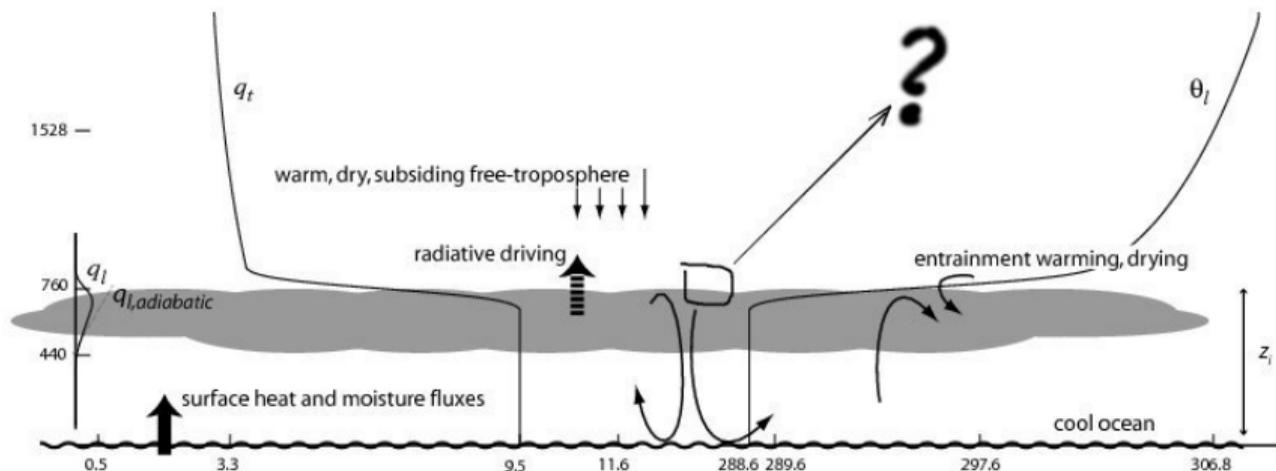
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# Stratocumulus-Topped Boundary Layers

Work in collaboration with B. Schulz, A. de Lozar, C. S. Bretherton, B. Stevens & M. C. Wyant

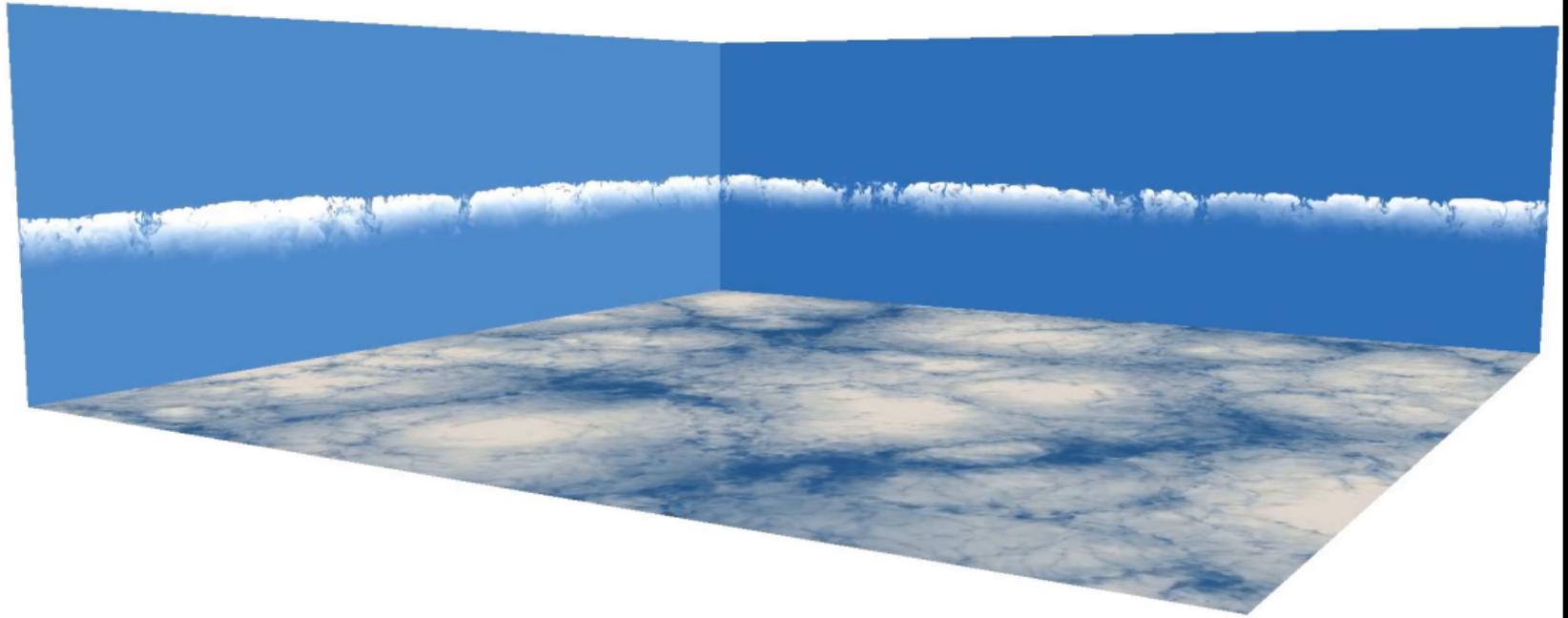


# Small-scale Processes at the Top of Stratocumulus Are Key

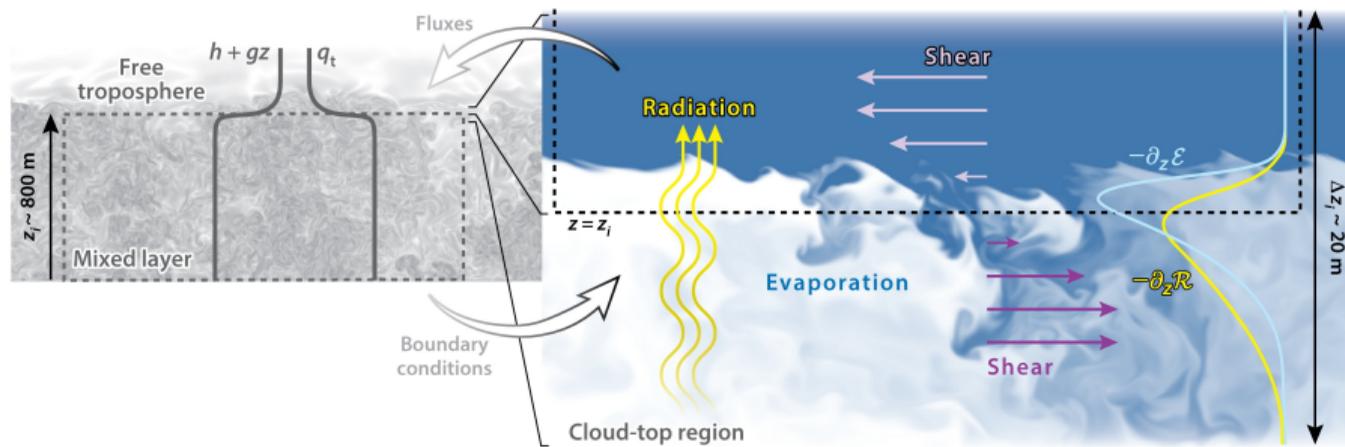


1. Longwave radiative cooling
2. Evaporative cooling
3. Turbulent entrainment across a stably stratified region
4. Droplet sedimentation (cloud microphysics)

# 1 m resolution DNS of 800 m deep stratocumulus-topped boundary layer



# Quantifying Cloud-Top Mixing: The Mean Entrainment Velocity



**Mixed-layer** analysis needs the mean entrainment velocity

$$w_e \equiv \frac{dz_i}{dt} - \langle w \rangle_{z_i} .$$

**Cloud-top** analysis of meter and submeter-scale phenomena provides it.

## Governing Equations in Eulerian Framework

Disperse and dilute multi-phase flow (liquid volume fraction  $10^{-6}$ ) with small Stokes numbers ( $< 10^{-2}$ ) and moderate settling numbers ( $\approx 0.5$ ).

Anelastic approximation to Navier-Stokes equations plus:

$$\begin{aligned} \text{enthalpy} \quad \rho_{\text{ref}} D_t h &= \nabla \cdot [\rho \kappa_h \nabla h - \rho \mathbf{j}_\mu (h_\ell - h)] - \nabla \cdot (\rho \mathbf{j}_r) , \\ \text{total water} \quad \rho_{\text{ref}} D_t q_t &= \nabla \cdot [\rho \kappa_v \nabla q_t - \rho \mathbf{j}_\mu (1 - q_t)] , \\ \text{liquid water} \quad \rho_{\text{ref}} D_t q_\ell &= \nabla \cdot [\rho \kappa_v \nabla q_\ell - \rho \mathbf{j}_\mu (1 - q_\ell)] + (\partial_t \rho q_\ell)_{\text{con}} . \end{aligned}$$

Cloud processes to be modeled:

1. Radiative flux  $\rho \mathbf{j}_r$ .
2. Rate of phase change  $(\partial_t \rho q_\ell)_{\text{con}}$ : Latent heat effects.
3. Transport flux  $\rho \mathbf{j}_\mu$ : Droplet sedimentation.

## Cloud-Top Integral Analysis Provides Expression for $w_e$

Evolution equation for the buoyancy  $b$  (normalized density anomaly) can be derived from the linearized equations of state:

$$D_t b = \nabla \cdot [\kappa_h \nabla b - \mathbf{j}_\mu (b_\ell - b)] - \beta_h \nabla \cdot \mathbf{j}_r + \beta_{q_\ell} (\partial_t q_\ell)_{\text{con}} .$$

( $\beta_i$  are thermodynamic partial derivatives.)

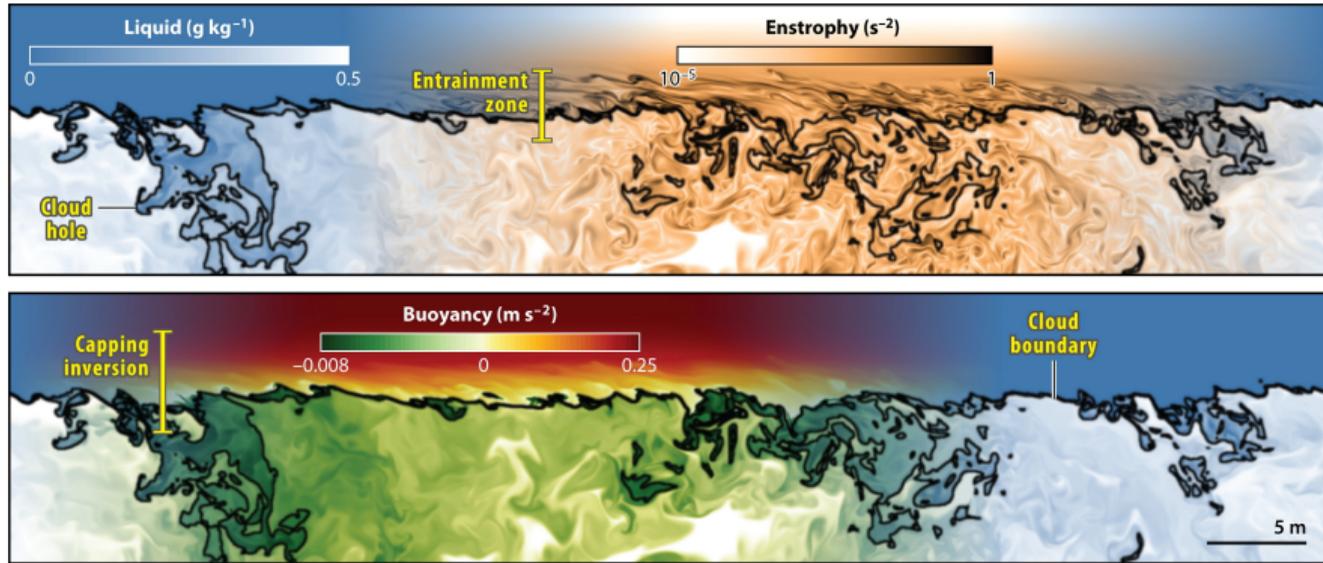
Integral analysis from inversion height  $z_i$  upwards yields analytical expressions to calculate  $w_e$ :

$$w_e (\Delta b)_{z_i} \simeq -\langle w' b' \rangle_{z_i} + \beta_h (\Delta j_r)_{z_i} - \beta_{q_\ell} \int_{z_i}^{z_\infty} (\partial_t q_\ell)_{\text{con}} dz - g \langle |j_\mu| \rangle_{z_i}$$
$$\Rightarrow w_e = (w_e)_{\text{mix}} + (w_e)_{\text{rad}} + (w_e)_{\text{eva}} + (w_e)_{\text{sed}}$$

( $g$  is the magnitude of the gravity acceleration.)

# Resolving Meter and Submeter Scales at the Cloud Top

Cloud Boundary + Turbulence Interface + Capping Inversion



Example of the need for this resolution: Reduction of  $w_e$  by droplet sedimentation.

# Model for Gravitational Settling of Droplets: Assumed Droplet-Size Distribution

Since the transport flux is

$$\rho \mathbf{j}_\mu = \rho q_\ell [\overline{d^5} / (\overline{d^3} \overline{d_0^2})] \mathbf{u}_{s,0} = \rho_0 q_{\ell,0} (n/n_0) (\overline{d^5} / \overline{d_0^5}) \mathbf{u}_{s,0} ,$$

we need a model for the 5.-order moment of the droplet-size distribution, and then either the 3.-order moment or the cloud-droplet number density.

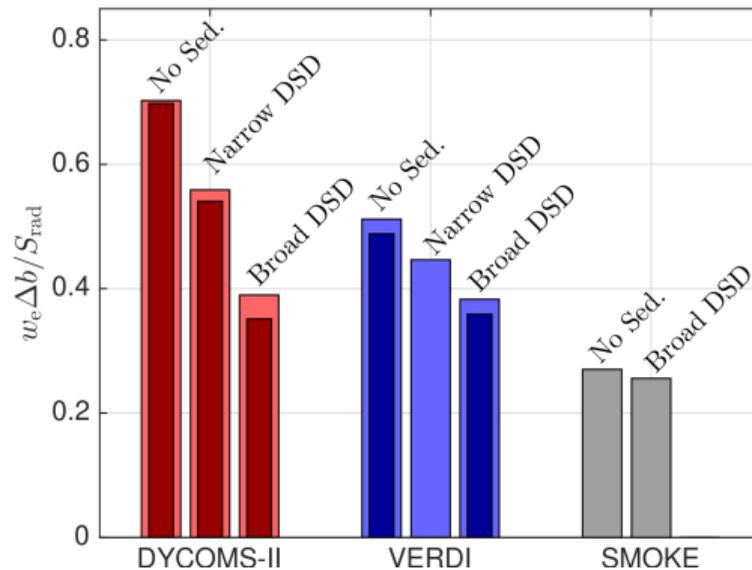
Following previous work, we assume a log-normal distribution with a constant number density, which leads to

$$\overline{d^5} / (\overline{d^3} \overline{d_0^2}) = \exp[5\sigma^2] (q_\ell / q_{\ell,0})^{2/3} .$$

We consider a narrow distribution ( $\sigma_g \simeq 1.0$ ) and a broad one ( $\sigma_g \simeq 1.5$ ), where  $\sigma_c = \exp(\sigma)$ .

**What is the effect of small-scale turbulence?**

# Droplet Sedimentation Can Reduce Entrainment Significantly



1. It depends on the meteorological conditions.
2. It depends on droplet-size distribution.
3. **Almost 50% reduction of  $w_e$ , 2–3 times larger than previously reported.**

## Two Contributions from Droplet Sedimentation to Entrainment Velocity

Integral analysis yields analytical expressions to calculate  $w_e$ :

$$w_e = (w_e)_{\text{mix}} + (w_e)_{\text{rad}} + (w_e)_{\text{eva}} + (w_e)_{\text{sed}} .$$

Two contributions:

1. Direct contribution: Increase of mean buoyancy for  $z > z_i$  by removal of droplets translates into a negative  $(w_e)_{\text{sed}}$

$$(w_e)_{\text{sed}} = -g \langle |j_\mu| \rangle / (\Delta b)_{z_i} \propto q_l \overline{d^5} / \overline{d^3} \propto n \overline{d^5}$$

Responsible for almost 30% of the reduction.

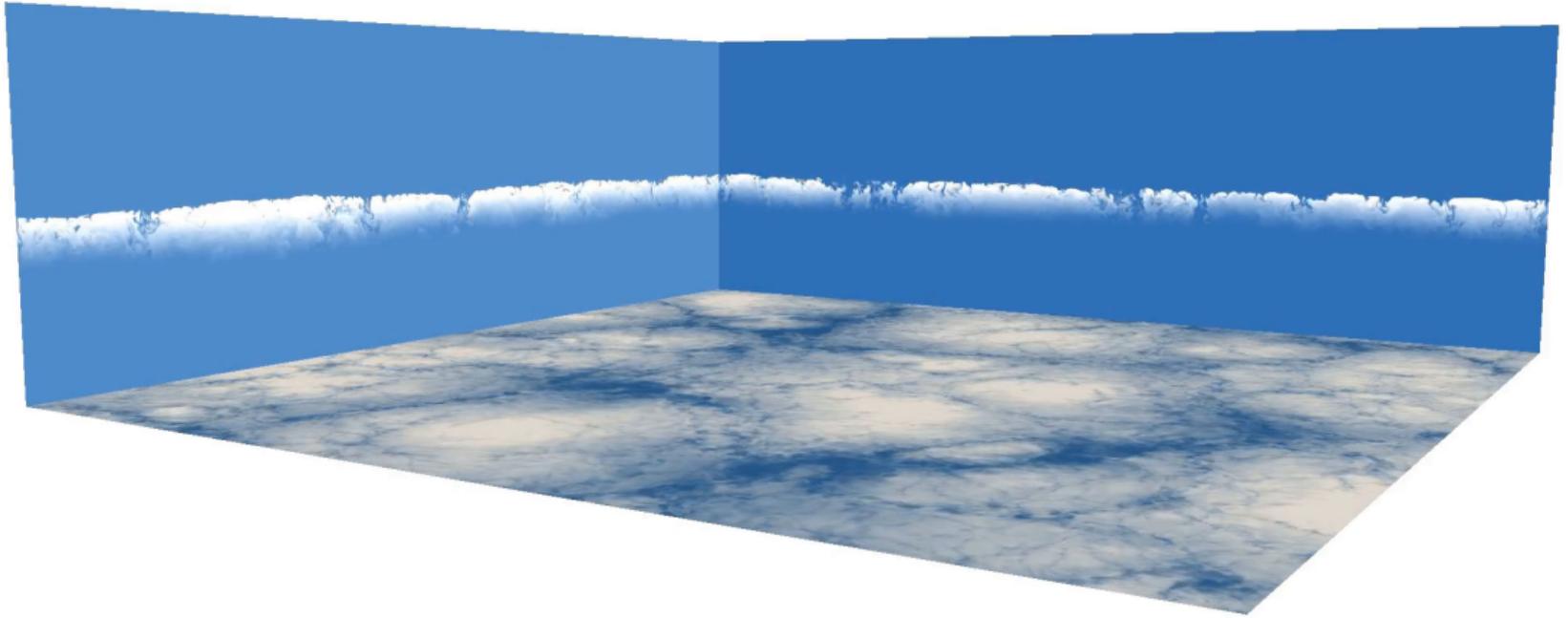
2. Indirect contribution: Changes in  $(w_e)_{\text{mix}} + (w_e)_{\text{rad}} + (w_e)_{\text{eva}}$ . In particular, reduction of cloud-top cooling because of removal of droplets.

## Outline

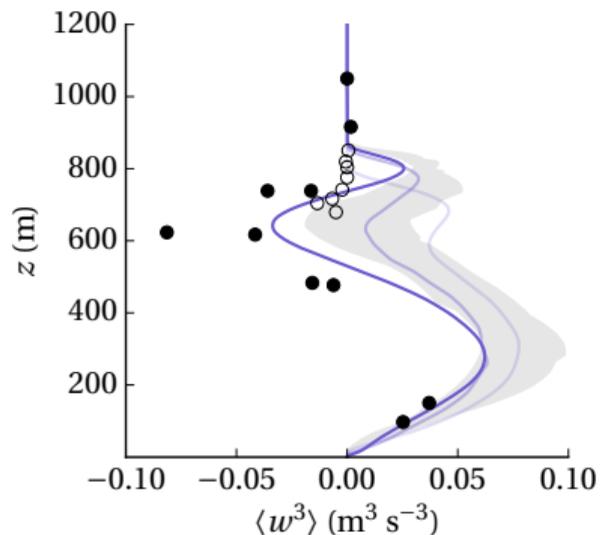
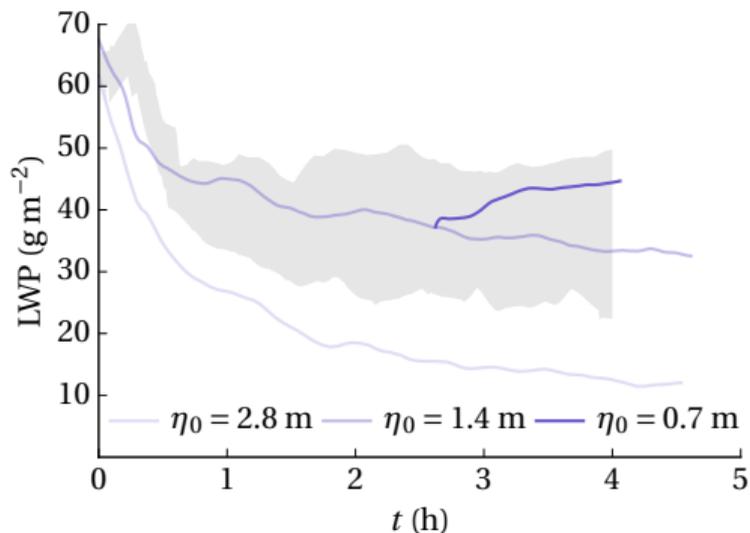
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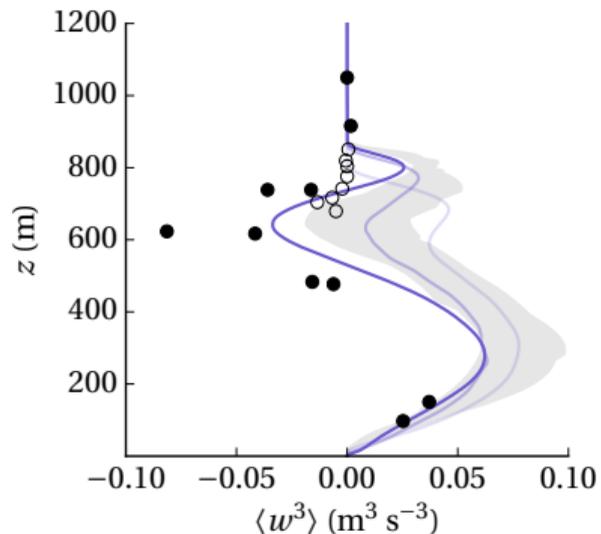
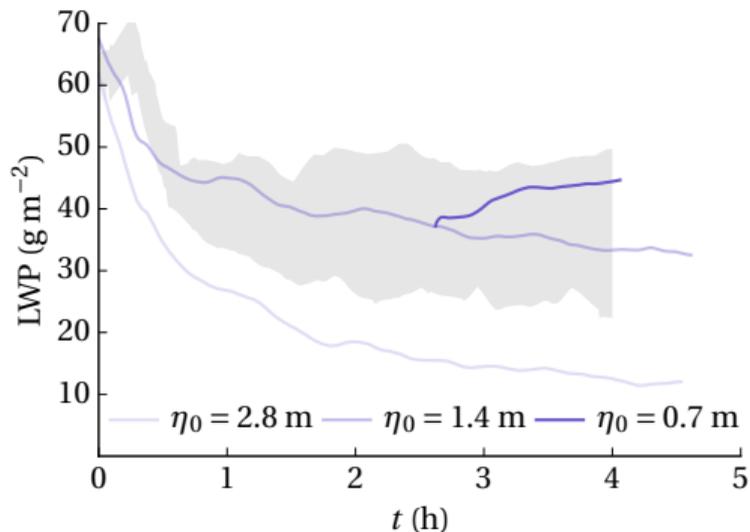


## Approaching Reynolds Number Similarity



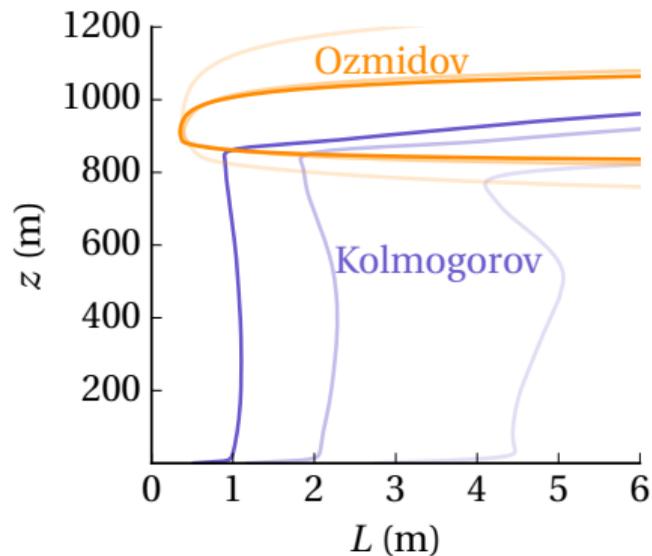
1. A Kolmogorov scale of  $\simeq 1.4$  m reproduces the central distribution of LES models.
2. A Kolmogorov scale of  $\simeq 0.7$  m reproduces more than 70% of measured LWP, about 90% of skewness of vertical velocity.

## Approaching Reynolds Number Similarity



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2. A Kolmogorov scale of  $\simeq 0.7$  m reproduces more than 70% of measured LWP, about 90% of skewness of vertical velocity. **Why?**

## Resolving the Ozmidov Scale in the Cloud-Top Region



$Re_0$	$\eta_0$	$(L_{Oz}/\eta)_{ct}$
2000	2.8 m	1.5
5000	1.4 m	2.7
12500	0.7 m	3.7

We start to represent motions smaller than the Ozmidov scale, which is the lower bound of length scales strongly influenced by stable stratification.

## Summary & Conclusions

### Meter and submeter scales can be key for the dynamics of planetary boundary layers



Near the surface:

Turbulence collapse in stable boundary layers can occur intermittently in space without external perturbations.



Near the boundary-layer top:

Droplet sedimentation can reduce substantially cloud-top mixing: better characterization of droplet-size distribution needed.

**Laboratory experiments, field measurements, and numerical simulations are reaching the accuracy necessary to solve long-standing problems**