The Spin of the Proton

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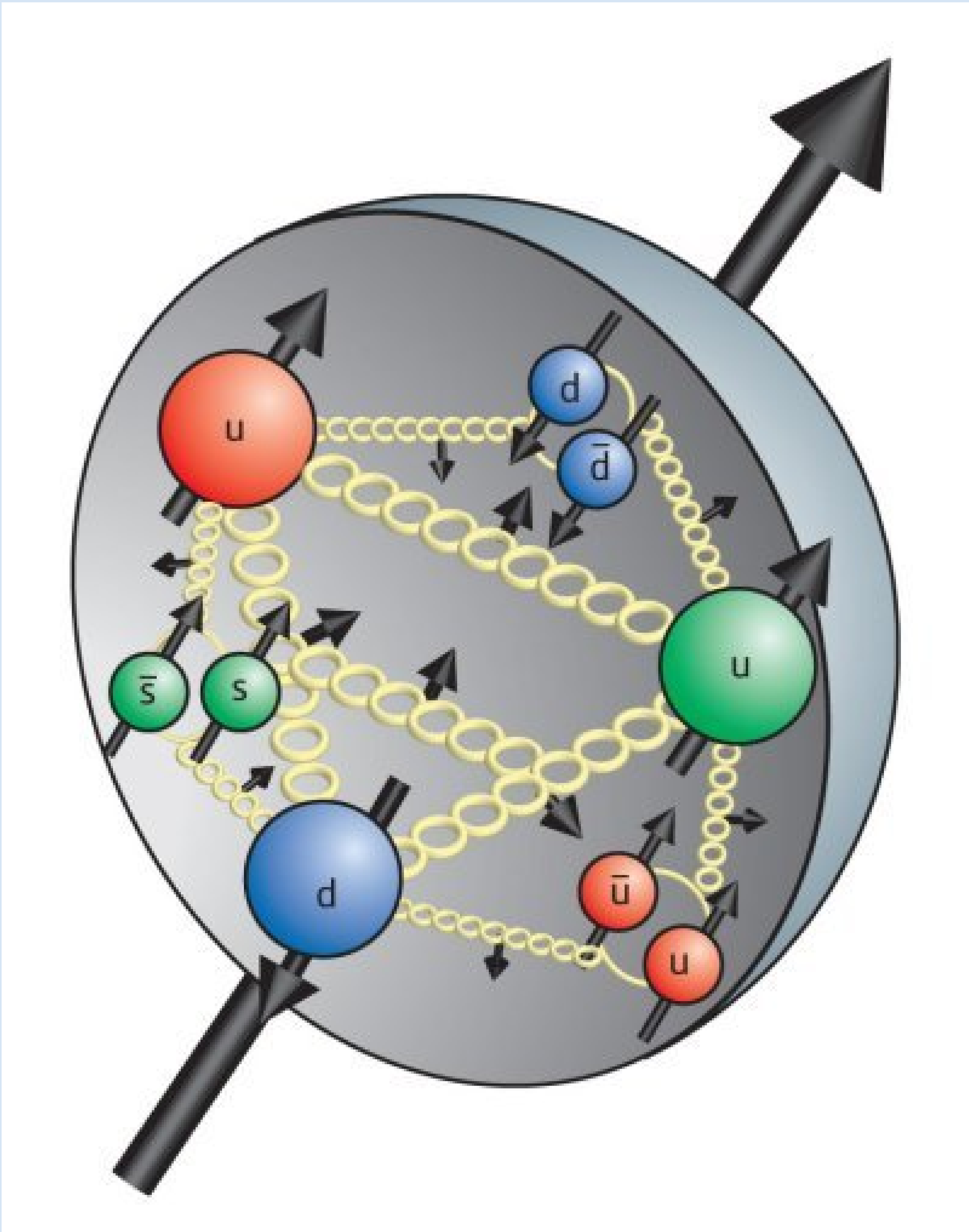
Motivation



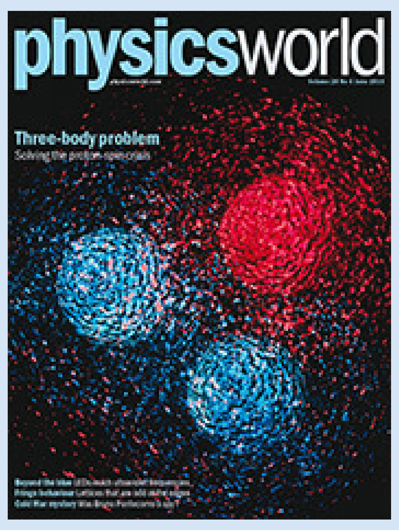
The proton consists of two valence up, u, quarks, one down, d, quark together with a 'sea' of quark anti-quark pairs, $\overline{u}-u$, $\overline{d}-d$, $\overline{s}-s$, and gluons, g. How each constituent contributes to the total spin of the proton has remained a mystery for many years. In particular the quark contribution is much smaller than expected. We discuss here our lattice QCD determination of the quark contribution, using a novel technique, based on a field theoretic application of the Feynman-Hellmann theorem.

[There are two common spin decompositions or 'schemes': Jaffe–Manohar (JM) and Ji. They both have a common quark spin term, $\Delta\Sigma/2$ but other pieces vary. In particular the JM approach has a gluon spin piece, ΔG, which can be measured in pp machines, while the Ji approach is more suitable for polarised DIS and DVCS processes (and also lattice QCD determinations).]

Proton spin $\frac{1}{2}$ can be decomposed as [Ji, [1]]

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$$

Quark orbital angular momentum (AM) L_q; Gluon AM J_g



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- Quark spin

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \qquad \begin{cases} \Delta q \propto \langle p|\widehat{A}_3|p\rangle \\ p \sim u(u^T C\gamma_5 d) \quad A_3 = \overline{q}\gamma_3\gamma_5 q \end{cases}$$

- Expectation: Quark model

$\Delta\Sigma \sim 1$ but 'Spin crisis': $\Delta\Sigma$ small $\sim 35\%$ of total spin

$\Delta s = 0$ but perhaps $\sim -10\%$

[We shall only consider Δs, $\Delta\Sigma$ evaluations here but the others can be determined.]

The Lattice approach

- Euclideanise space - time
- Discretise space - time (lattice spacing $a \to 0$)
- Path Integral $\to$ partition function which is a $V_s \times T \times d \times (n_c^2 - 1) \sim 48^3 \times 96 \times 4 \times 8$

$\sim O(500, 000, 000)$ dimensional integral

$\Rightarrow$ Monte Carlo techniques for:

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int [dU][dq d\overline{q}]\, \mathcal{O}\, e^S$$

$S = S_g + S_F$ where S_g is gluon action; S_F is fermion action given by

$$S_F = \sum_{q=u,d,s} \overline{q} D q\,, \quad D \text{ is Dirac fermion matrix}$$

Matrix elements are found from three-pt correlation functions, eg

$$\langle p(t) A_3(\tau) p(0) \rangle \propto \langle p|\widehat{A}_3|p\rangle \quad [\tfrac{1}{2}T \gg t \gg \tau \gg 0]$$

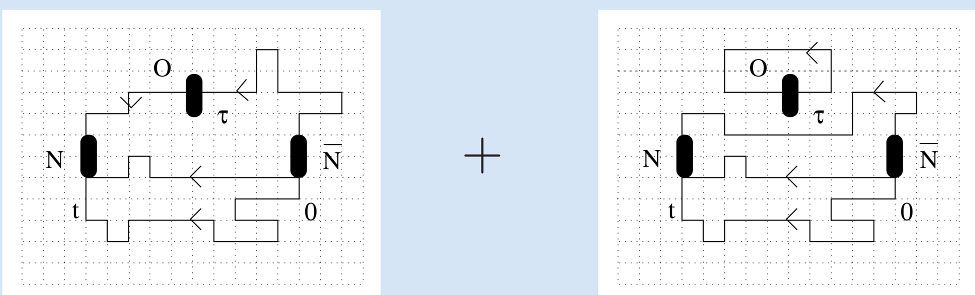
Matrix elements

D^{-1} = fermion propagator = —←—

We need to find all fermion propagator connections

$$u\,u\,d\,\overline{q}q\,\overline{u}\,\overline{u}\,\overline{d} \qquad q = u \text{ or } d \text{ or } s$$

giving quark-line connected and quark-line disconnected diagrams in a background gluon field.



The major technical problem is the evaluation of the quark-line disconnected terms – these are short distance quantities and suffer numerically from large fluctuations.

While all $\Delta q = \Delta q^{con} + \Delta q^{dis}$ have quark line disconnected pieces this is particularly obvious for Δs, which only has a disconnected piece.

Feynman–Hellmann (FH)

Replace, eg

$$S \to S(\lambda) = S + \lambda \sum_{q,x} \overline{q}(x)\gamma_3\gamma_5 q(x)$$

This choice determines $\Delta\Sigma$. Then (FH)

$$\left.\frac{\partial E_p(\lambda)}{\partial\lambda}\right| \propto \left\langle p \left| \widehat{A}_3 \right| p \right\rangle \propto \Delta\Sigma$$

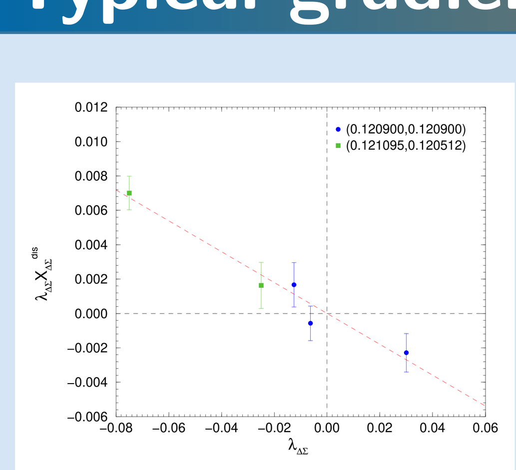
seen by $\partial/\partial\lambda$ of two-pt correlation function $\langle p(t)p(0)\rangle_\lambda = A_p(\lambda)\exp(-E_p(\lambda)t)$.

Constraints on the action means that the energy can develop an imaginary part, $E \to E + \phi$.

Strategy for FH application

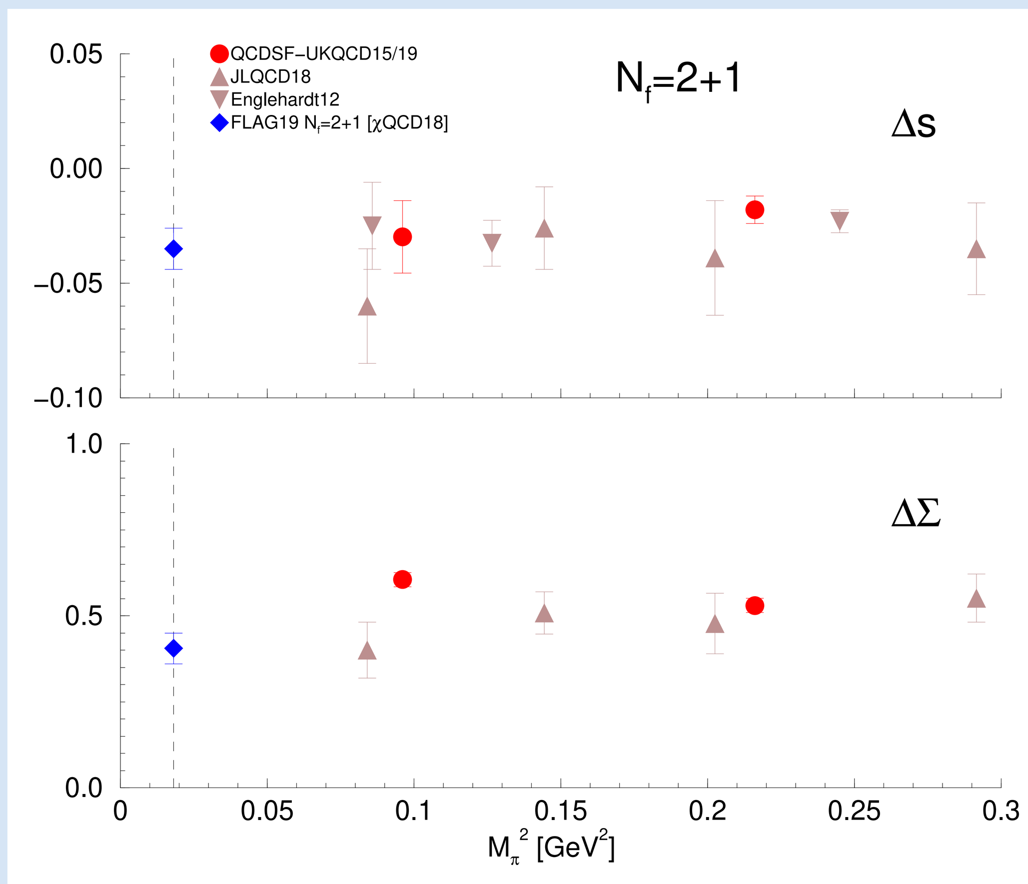
Modify matrix before quark propagator inversion	Modify field weighting during configuration generation
$D'^{\,-1} = [D + \lambda O]^{-1}$	$\det D'\, e^{-S_g} = \det[D + \lambda O]\, e^{-S_g}$
Inserts connected contributions on every line:	Access disconnected contributions:
$\left.\frac{\partial}{\partial\lambda} D'^{\,-1}\right\|_{\lambda=0} = D^{-1} O D^{-1}$	$\left.\frac{\partial}{\partial\lambda} \det D'\right\|_{\lambda=0} = \mathrm{tr}(D^{-1}O)\, \det D$
Gives connected insertation in LH plot	Gives disconnected insertion on RH plot
Easy to implement	Need to generate new gauge ensembles

Typical gradient example, [3]



- Quark-line disconnected
- Flavour symmetric point, $M_\pi \sim 465$ MeV and 310 MeV for 500 configs on a $32^3 \times 64$ lattice with $a \approx 0.074$ fm
- Measure gradient, gives $\Delta\Sigma^{dis}$ (can be related to Δs)
- Renormalise to the $\overline{\mathrm{MS}}$ scheme at a scale of 2 GeV [Use RI-MOM and FH]

Results



- (Various) $N_f = 2 + 1$ dynamical fermion results
- Red circles are our results, [2, 3]; triangles are comparison results; the FLAG19 lattice review result [4] is also shown

Conclusions

- Result for $\Delta\Sigma$ slightly larger than present experimental result
- Further simulations at additional quark masses to extrapolate matrix element using $SU(3)$ flavour breaking expansion, [3]
- Further experiments [5] planned to measure all components of spin decomposition at the (proposed) Electron–Ion-Collider (EIC) and LHC

References

[1] X. Ji, Phys. Rev. Lett. **78** (1997) 610, [arXiv:hep-ph/9603249].

[2] A. J. Chambers *et al.* [CSSM-QCDSF-UKQCD], Phys. Rev. **D 92** (2015) 114517, [arXiv:1508.06856 [hep-lat]].

[3] R. Horsley *et al.* [QCDSF-UKQCD Collaboration], PoS LATTICE **2018** (2018) 119, [arXiv:1901.04792 [hep-lat]].

[4] S. Aoki *et al.* [Flavour Lattice Averaging Group], arXiv:1902.08191 [hep-lat].

[5] D. Boer, DIS2019, arXiv:1907.09344 [hep-ph]; M. G. Echevarria *et al.*, SPIN2018, arXiv:1903.03379 [hep-ph].