Scale setting for QCD with four dynamical quarks (Gauss Project HWU35)

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We generated various lattice QCD ensembles with four dynamical quarks, set the scale and studied the charmonium spectrum in the continuum limit.
This poster is based on [1].

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QCD: the theory of strong interactions

The forces that hold quarks together to form hadrons are called strong interactions and Quantum Chromodynamics is the theory proposed to explain their properties.

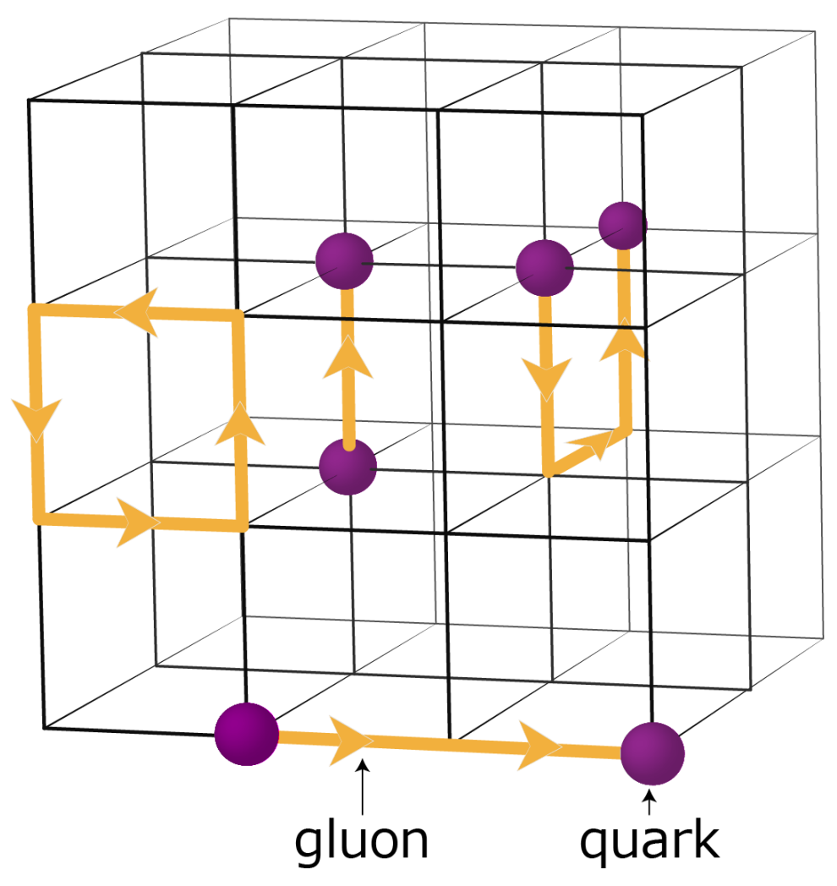
Features of Quantum Chromodynamics (QCD)

- QCD is a gauge theory based on the color group SU(3)
- At high energies (particle accelerators), strong interactions are weak
  ⇒ perturbation theory works well
- At low energies the QCD coupling is strong
  ⇒ Many non-perturbative phenomena, hadrons, confinement

Lattice QCD

Non-perturbative definition of QCD. Starting point for numerical calculation

Features of lattice QCD



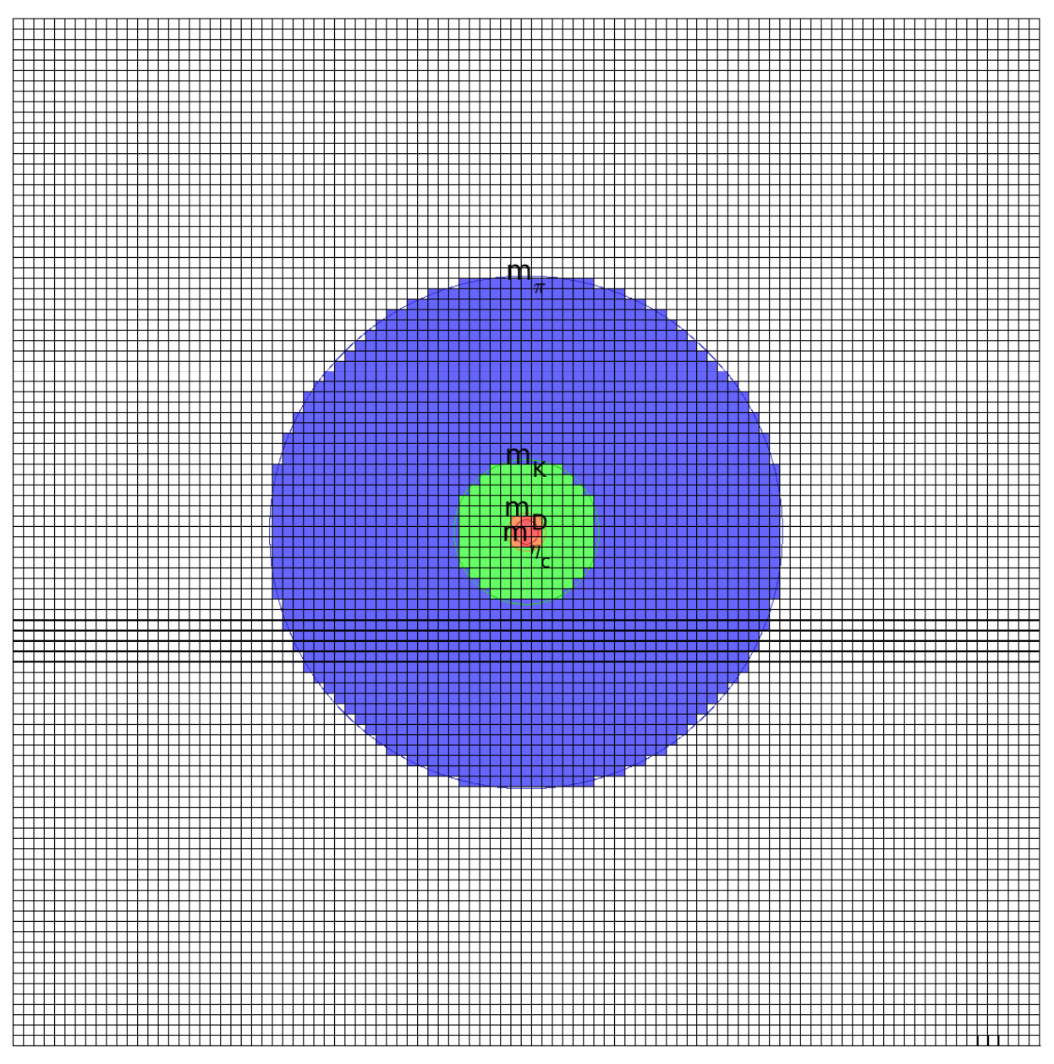
- Continuous space-time is replaced by a Euclidean lattice
- Lattice spacing a, box size $T \times L^3$
  $\Rightarrow N = L/a,\ N_t = T/a$
- Parameters and fields are dimensionless
- Fermions $\psi(x)$ live on the sites of the lattice
- Gluons described by links $U_\mu(x)$
- "Observables"
  $\langle \mathcal{O} \rangle = \int DU\, \frac{1}{Z} e^{-S_g[U]} \det[D_u] \det[D_d] \det[D_s] \det[D_c] \det[D_b] \det[D_t]\ \mathcal{O}[U]$
  - Very high dimensional integral.
    $4 \times 8 \times N^3 N_t$ dimensions
  - D_x: sparse square matrices of size $12 N^3 N_t$

$\langle \mathcal{O} \rangle \approx \frac{1}{N} \sum_{j=1}^{N_{\rm cnfg}} O[U_j]$ (MC Methods [2])

The challenge

Inverse particle masses in lattice units are correlation-lengths of suitably chosen correlation functions.

- Maximal number of sites in each direction $N_{\rm max} = [L/a]_{\rm max} \approx 100$
  These would be state-of-the art calculations worth 100s of Mcoreh
- $m_\pi \approx 140$ MeV is the lightest meson. $m_\pi L > 4$ needed for manageable finite-volume effects
  $\Rightarrow a_{\rm min} \approx 0.056$ fm
- All other particles need $m \ll a^{-1}$, otherwise: big lattice artifacts
- But many are heavy, e.g.
  $m_{\eta_c} \approx 3$ GeV

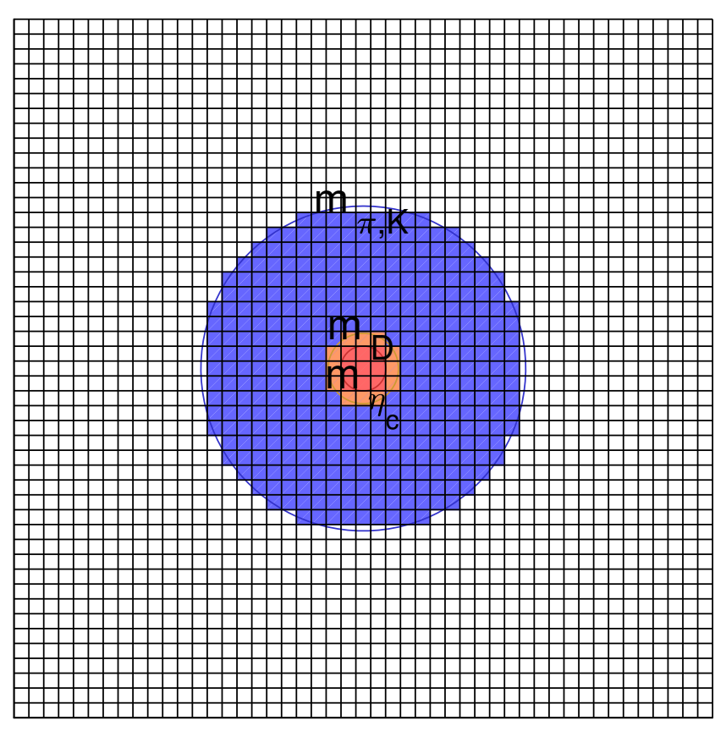


radius ≙ correlation length of meson

⇒ Lattice QCD is a difficult multi-scale problem

Our simulations

- 3 light + 1 charm quark
  up, down and strange quarks are equal
- Heavier pions than in nature
  - chiral extrapolations necessary
  - Very small a already with $N \approx 50$ possible
- Novel massive $O(a)$ improvement scheme [3]
  ⇒ Control over potential $O(am_c)$ lattice artifacts
- Open boundary conditions [2]



Generated ensembles

Simulation parameters β, $\kappa_u = \kappa_d = \kappa_s$, κ_c Chosen such that the

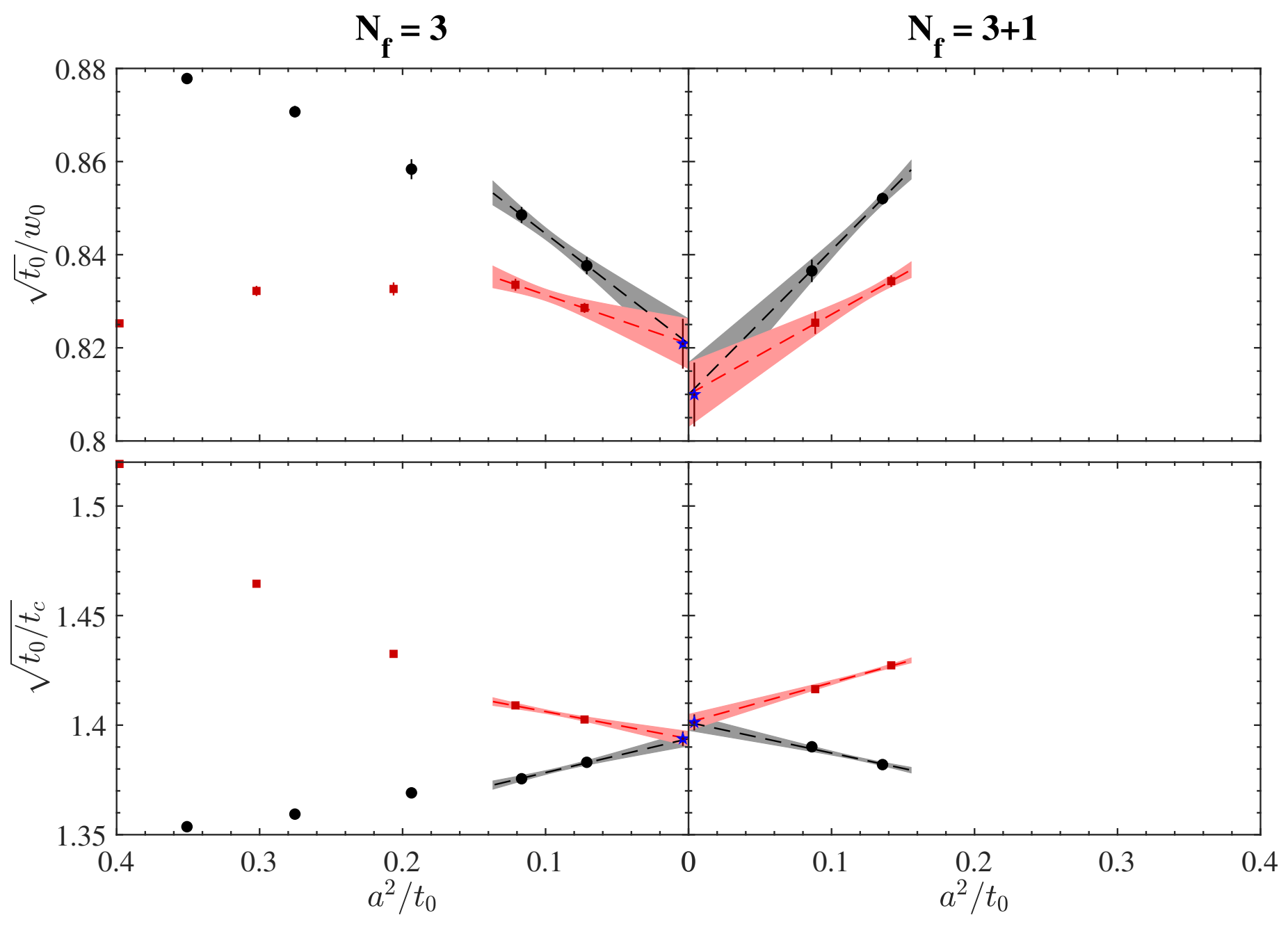
- Sum of renormalized light quark masses as in nature
- Renormalized charm quark mass as heavy as in nature
- Lattice spacing $a \approx 0.055$ fm or $a \approx 0.04$ fm

ens.	$\frac{T}{a} \times \frac{L^3}{a^3}$	β	κ_l	κ_c	a[fm]	$Lm_\pi^\star$	N_{traj} (MDUs)
A0	96×16^3	3.24	0.13440733	0.12784	0.054	1.77	1400 (2800)
A1	96×32^3	3.24	0.134402(35)	0.12805(62)	0.0531(11)	3.536(2)	3908 (7816)
A2	128×48^3	3.24	0.134396(14)	0.12798(19)	0.0536(11)	5.354(13)	3868 (7736)
B	144×48^3	3.43	0.13599(1)	0.13090(25)	0.0428(7)	4.282(14)	4000 (8000)

Decoupling of heavy quarks

Expectation: low energy physics is well described by QCD without charm quarks, as long as $E \ll m_c$. We measure 3 well defined, very precise low energy quantities with dimension "length": $\sqrt{t_0}$, $\sqrt{t_c}$, w_0 [4,5] in QCD with 3 light quarks and in QCD with 3 light and a charm quark

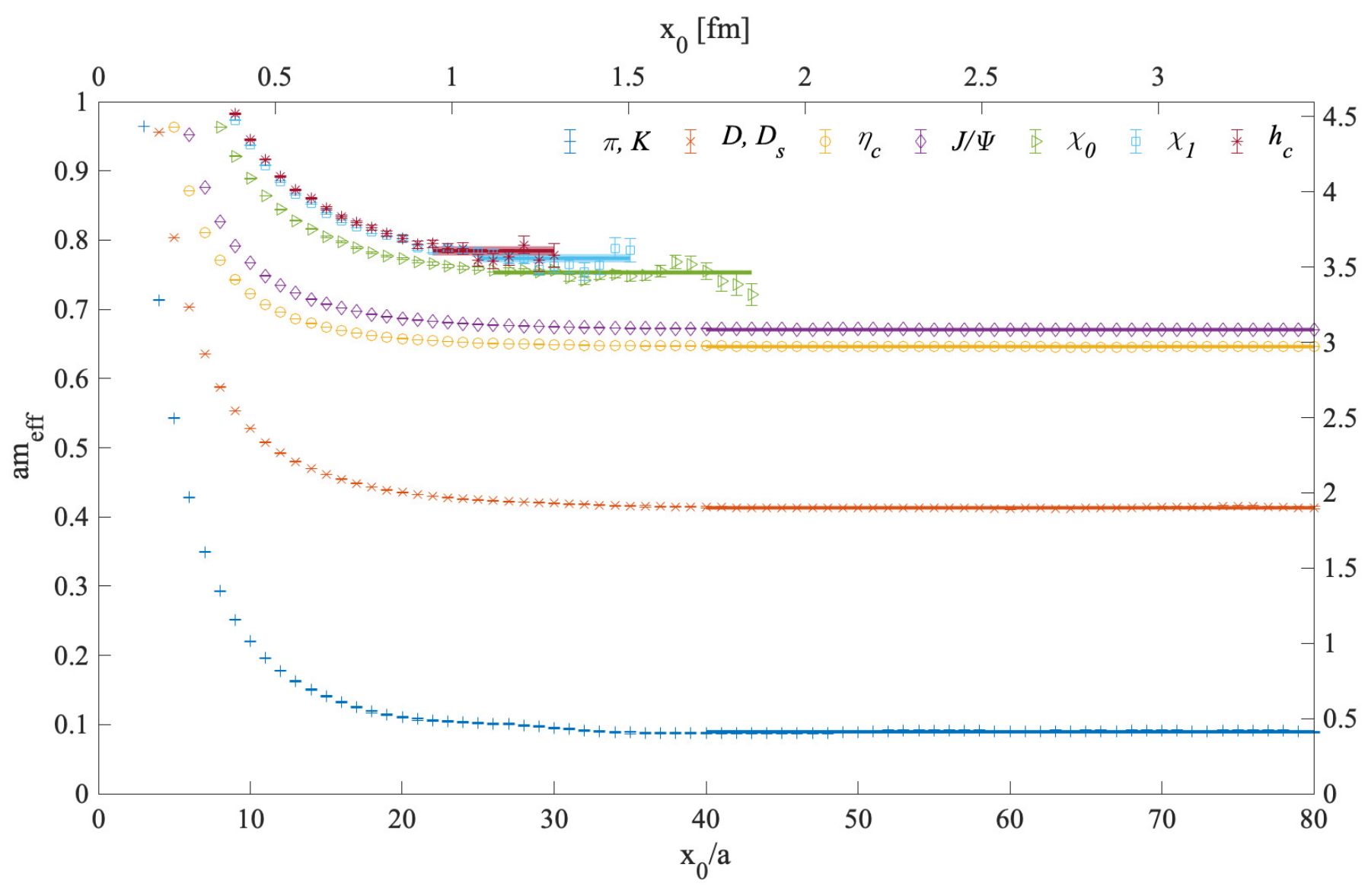
- Effect in the continuum limit: small as expected
- Fine lattice spacings are crucial
- Despite high precision: difficult to resolve



Charmonium spectrum

- Charmonia: composite particles made from a charm and an anti-charm quark
- In nature [6]: $\eta_c(1S)$, $J/\psi(1S)$, $h_c(1P)$, $\chi_{c0}(1P)$, $\chi_{c1}(1P)$, $\chi_{c2}(1P)$, $\eta_c(2S)$, $\psi(2S)$, $\eta_{c2}(1D)$, $\psi(3770)$, $X(3872)$, $Y(4260)$ observed, more predicted

⇒ major ongoing experimental efforts: Belle II, LHCb, BES III, PANDA
Charmonia can be studied on our ensembles with a high precision



Effective masses on ens. B

meson	experiment	this work
η_c	2.9834(5)	2.95(20)
J/ψ	3.096900(6)	3.06(21)
χ_{c_0}	3.4148(3)	3.48(29)
χ_{c_1}	3.51066(7)	3.43(34)
h_c	3.52538(11)	3.60(34)
$\frac{J/\psi - \eta}{\eta}$	0.038(2)	0.0376(11)

Meson masses in GeV

Outlook

- Simulations closer to the continuum
- Simulations closer to the physical mass point
- More physics observables, e.g.
  - Excited charmonia
  - Quark-disconnected contributions
  - Charmed mesons, D, D_s, ...
- Fundamental parameters of the SM
  The "scale-setting" done here is crucial for
  - The strong coupling
  - The renormalized quark masses

References

[1] R. Höllwieser et al., arXiv:2002.02866
[2] M.Lüscher and S. Schaefer, CPC 184 (2013)
[3] P. Fritzsch et al., JHEP 1806 (2018)
[4] M. Lüscher, JHEP 1403 (2014)
[5] S. Borsanyi et al., JHEP 1209 (2012)
[6] PDG, PRD 98, 030001 (2018)