PARALLEL INTERIOR-POINT SOLVER FOR LARGE-SCALE DOUBLY-BORDERED LINEAR ENERGY SYSTEM MODELS

BEAM-ME

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Overview

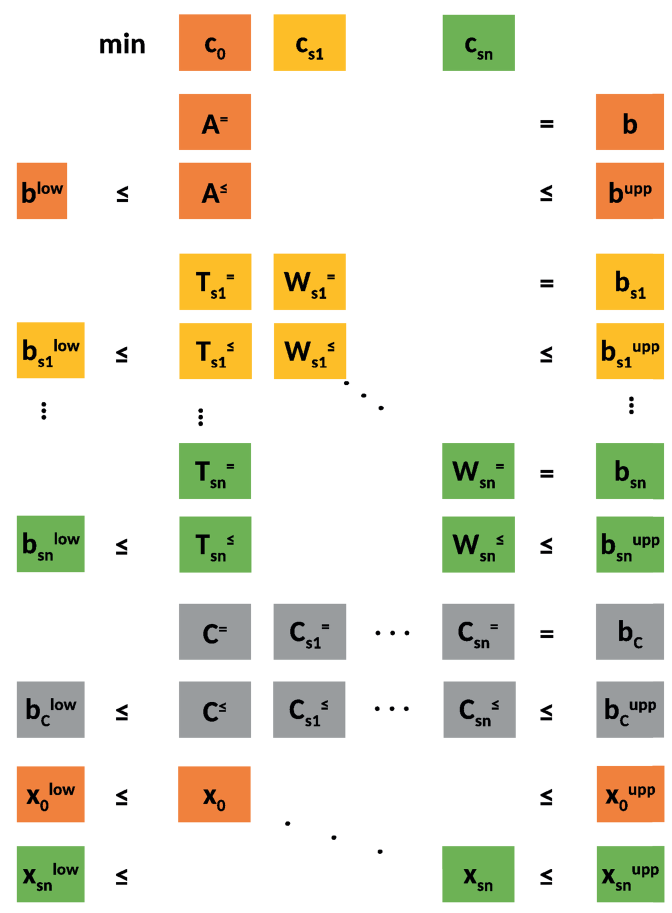
Motivation: Recent changes in the energy supply system, such as the shift to sustainable and decentralized technologies (Energiewende), and the availability of large amounts of data, have resulted in energy system models of high complexity and size. These developments bring forth immense challenges in terms of optimizing realistic energy models.
Aim: The BEAM-ME project strived to develop and implement new methods for optimizing currently intractable energy models of vast size, which are modeled as linear programs (LPs) and mixed-integer programs (MIPs), with the main focus being on LPs.

Challenges

- The energy models are too complex to solve even with the state-of-the-art commercial solvers → Solution methods need to exploit special structure of these models
- Large energy models do not even fit into a single computer memory → Parallel and distributed algorithms tailored to supercomputer architecture are necessary

Problem Structure

- The energy system problems (ESPs) considered within the BEAM-ME project have a block-diagonal structure with both linking constraints and linking variables



- The project partners have annotated their ESPs by using GAMS and an interface from GAMS to PIPS++ has been implemented

Project Partners and Funding

BEAM-ME was an interdisciplinary consortium of researchers and practitioners from system analysis, mathematics, and high performance computing.

GAMS HLRS JÜLICH Forschungszentrum DLR Deutsches Zentrum für Luft- und Raumfahrt German Aerospace Center TU berlin ZIB

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Current State

- A specialized Parallel Interior Point Solver was developed: PIPS++
  - Extended PIPS[1], which could previously solve problems with block angular structure and linking variables, to solve problems with block angular structure, and both linking variables and linking constraints
  - Linear system for finding step-direction of interior point algorithm can be reduced to the following:

$$\begin{bmatrix} K_1 & & & B_1 \\ & \ddots & & \vdots \\ & & K_N & B_N \\ B_1^T & \cdots & B_N^T & K_0 \end{bmatrix} \begin{bmatrix} \Delta z_1 \\ \vdots \\ \Delta z_N \\ \Delta z_0 \end{bmatrix} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \\ r_0 \end{bmatrix} \tag{1}$$

1. Multiply (multi-) rows $i = 1, \ldots, N$ by $-B_i^T K_i^{-1}$
2. Solve $\left(K_0 - \sum_{i=1}^N B_i^T K_i^{-1} B_i\right)\Delta z_0 = r_0 - \sum_{i=1}^N B_i^T K_i^{-1} r_i$ (Schur complement)
3. Compute $\Delta z_i, i = 1, \ldots, N$

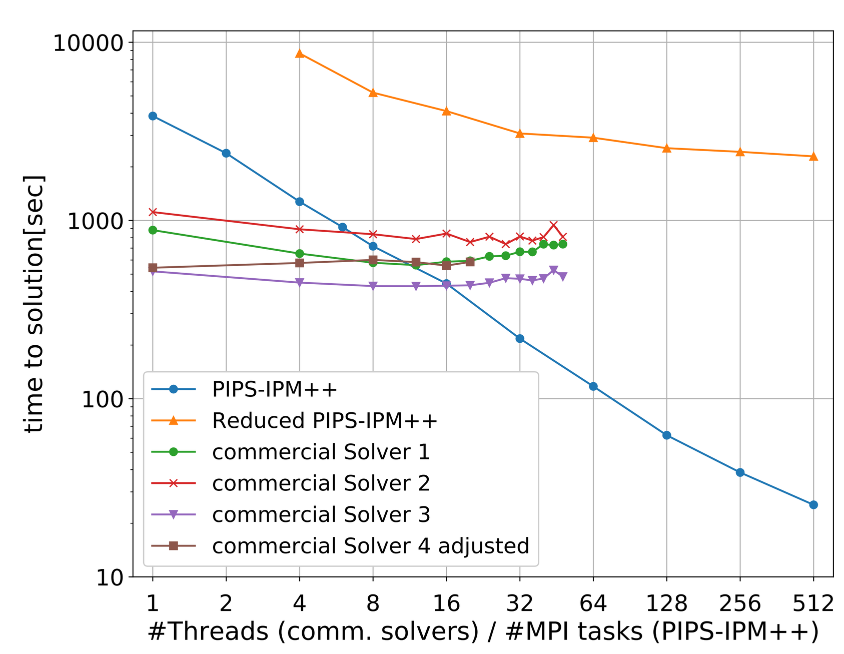
  - Blocks are distributed among MPI-processes to avoid memory bottlenecks
- Further enhancements
  - Designed and implemented parallel, structure preserving presolving
  - Improved convergence behavior of interior-point algorithm, e.g. by multiple corrector steps and by using (parallel) scaling
  - Substructures within the linking blocks are detected and exploited for solving Schur complement (step 2. above)
  - Designed and implemented parallel preconditioner for Schur complement

Overall already more than 60,000 lines of code added or modified, but development still ongoing.

[1] *Petra, C.G., Schenk, O., Anitescu, M.: Real-time Stochastic Optimization of Complex Energy Systems on High Performance Computers. Computing in Science and Engineering (CiSE) 16(5), pages 32-42.*

Current Results

- PIPS++ achieves very good results on benchmark instances "SIMPLE" created at the beginning of the project by GAMS and DLR.
- Scaling plot (right): Results of PIPS(++) (previous and latest) and four leading commercial solvers on SIMPLE instance with 5.1 million variables and 5.6 million constraints



- So far largest real-world ESP: LP with around 226 million variables, 256 million constraints, and 717 million non-zeros; solved in 29 minutes by using 1776 CPU cores at the JUWELS supercomputer (Jülich). Commercial solvers run out of memory.

Future work

- Implement new nested approach, which recursively replicates the structure of system (1) within each block K_i. Splits Schur complement in several smaller ones and thus allows one to handle problems with more linkage.
- Make the new solver PIPS++ publicly available.
- Further development and work done in follow-up project UNSEEN