Supercomputer simulations of crosstalk in transmon quantum computers

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Simulation method

The simulator determines the state of the device $|\Psi(t)\rangle$ at time t by solving the time-dependent Schrödinger equation (TDSE),

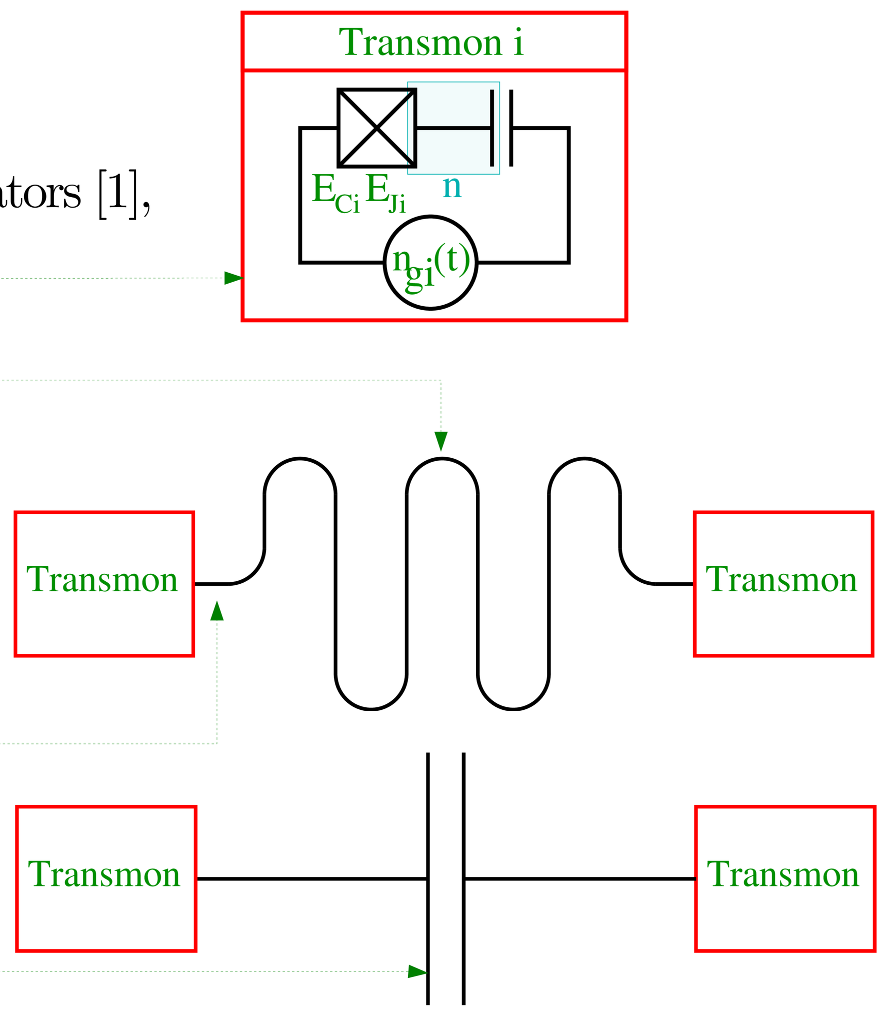
$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle,$$

where the hardware model has $N_{\rm Tr}$ transmons and $N_{\rm Res}$ resonators [1],

$$H(t) = H_0 + W(t),$$

$$H_0 = \sum_{i=0}^{N_{\rm Tr}-1} (4E_{Ci}\hat{n}_i^2 - E_{Ji}\cos\hat{\varphi}_i) + \sum_{r=0}^{N_{\rm Res}-1} \Omega_r \hat{a}_r^\dagger \hat{a}_r,$$

$$W(t) = \sum_{i=0}^{N_{\rm Tr}-1} -8E_{Ci} n_{gi}(t)\hat{n}_i + \sum_{r=0}^{N_{\rm Res}-1}\sum_{i=0}^{N_{\rm Tr}-1} G_{ri}\hat{n}_i(\hat{a}_r + \hat{a}_r^\dagger) + \sum_{0\le i<j<N_{\rm Tr}} E_{Ci,Cj}\hat{n}_i\hat{n}_j.$$



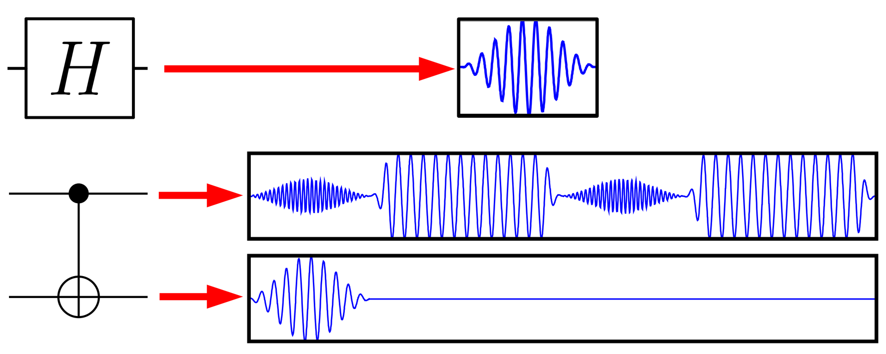
Algorithm: Suzuki-Trotter product-formula algorithm [2,3]:

$$|\Psi(t+\tau)\rangle = e^{-i\tau H_0/2}\underbrace{V e^{-i\tau\Lambda} V^\dagger}_{W} e^{-i\tau H_0/2}|\Psi(t)\rangle$$

Basis: 4 coefficients for each resonator and each transmon: $|\Psi(t)\rangle = \sum_{\rm KM} \psi_{\rm KM}(t)|{\rm KM}\rangle$

Quantum gates: implemented by optimized, time-dependent pulses [4] with the same shape as those used on the IBM Q Experience [5]. Single-qubit gates have Gaussian envelopes, and two-qubit gates use the echoed cross-resonance scheme [6]:

$$n_{gi}(t) = \sum_j \Omega_{ij}(t)\cos(2\pi f_{ij} t - \gamma_{ij})$$



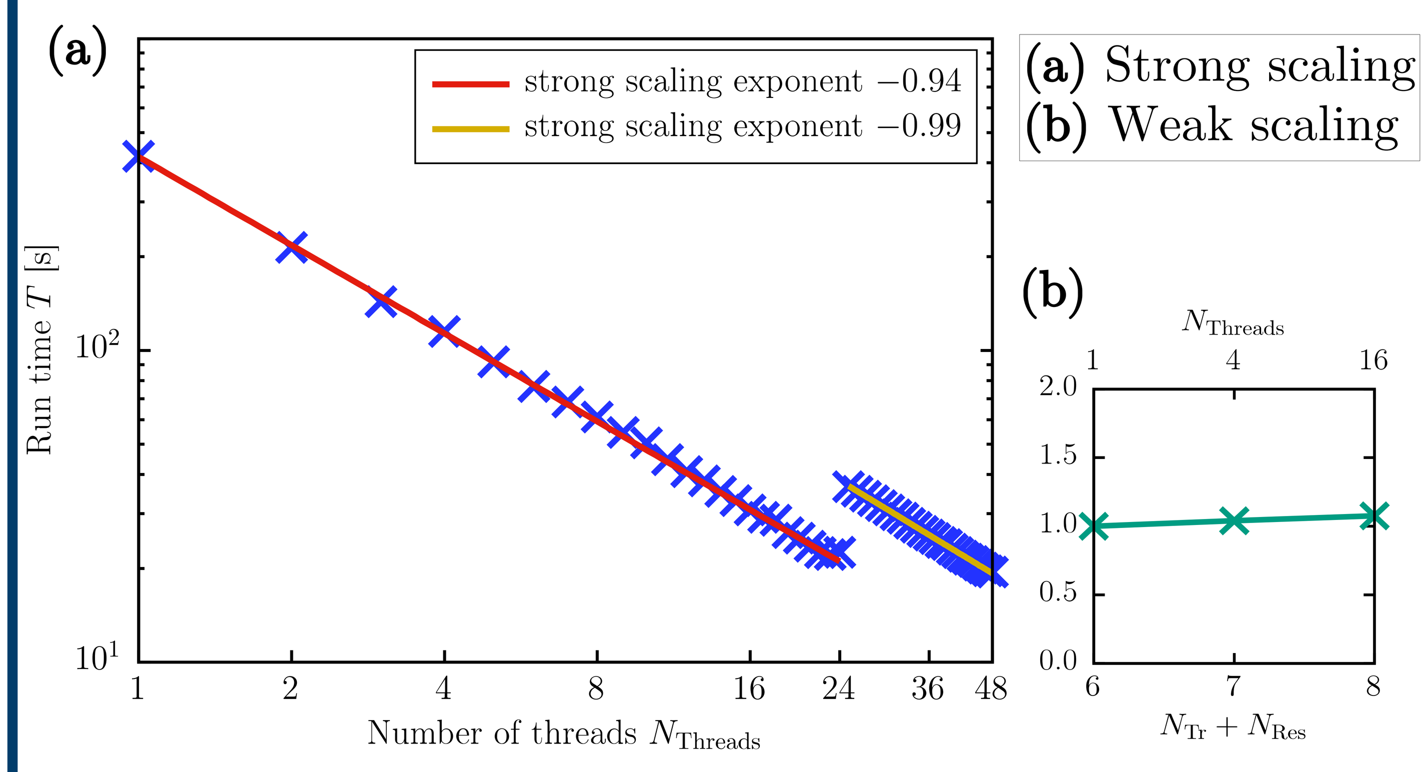
Implementation: exponentials are diagonal; V is a four-component transformation:

$$\begin{pmatrix}\psi_{*\cdots*00*\cdots*}\\ \psi_{*\cdots*01*\cdots*}\\ \psi_{*\cdots*10*\cdots*}\\ \psi_{*\cdots*11*\cdots*}\end{pmatrix} \leftarrow V_{r/i}^{(a/n)} \begin{pmatrix}\psi_{*\cdots*00*\cdots*}\\ \psi_{*\cdots*01*\cdots*}\\ \psi_{*\cdots*10*\cdots*}\\ \psi_{*\cdots*11*\cdots*}\end{pmatrix}$$

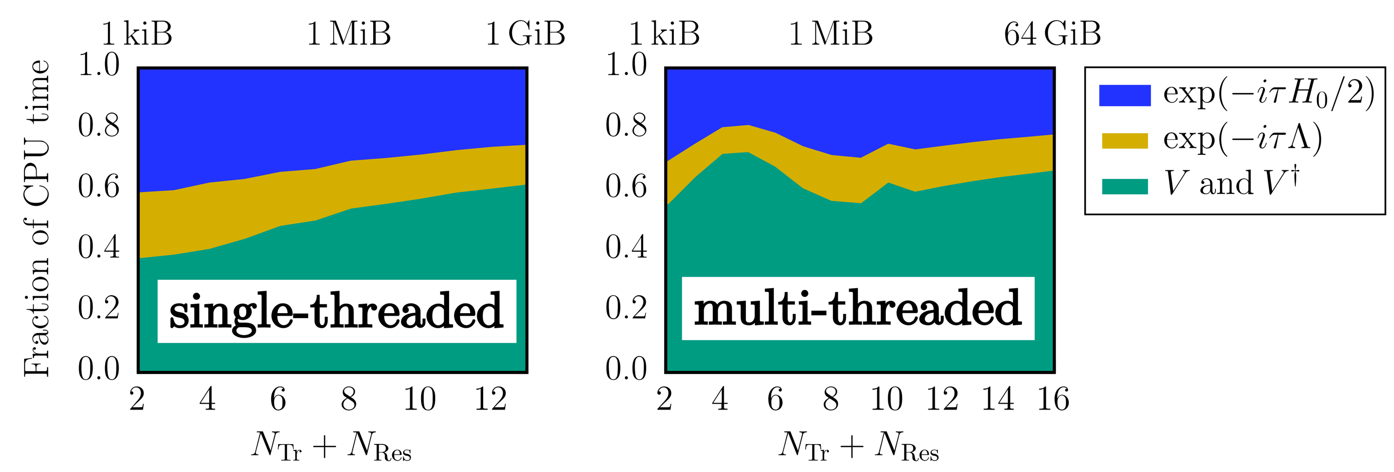
```
uint64_t inc = 1;
for(int i = NTr-1; i >= 0; --i, inc <<= 2) {
    uint64_t incl = inc << 2;
    #pragma omp for collapse(2)
    for(uint64_t K = 0; K < dim; K += incl)
        for(uint64_t M = 0; M < inc; ++M)
            mul4x4(&Vn[16*i], &psi[K | M], inc);
}
for(int r = NRes-1; r >= 0; --r, inc <<= 2) {
    uint64_t incl = inc << 2;
    #pragma omp for collapse(2)
    for(uint64_t K = 0; K < dim; K += incl)
        for(uint64_t M = 0; M < inc; ++M)
            mul4x4(&Va[16*r], &psi[K | M], inc);
}
```

Benchmark

Run times on the JURECA cluster for moderate system sizes:

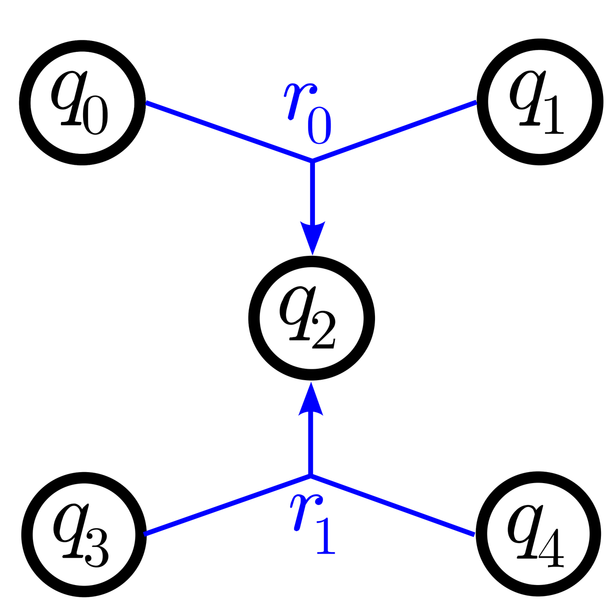


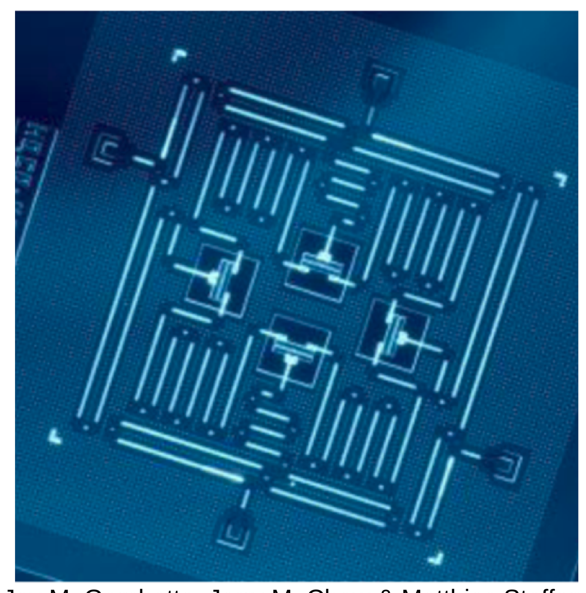
Breakdown of computation: V transformation is the bottleneck:



Transmon NISQ device

We use the `ibmqx4` processor on the IBM Q Experience [5]:

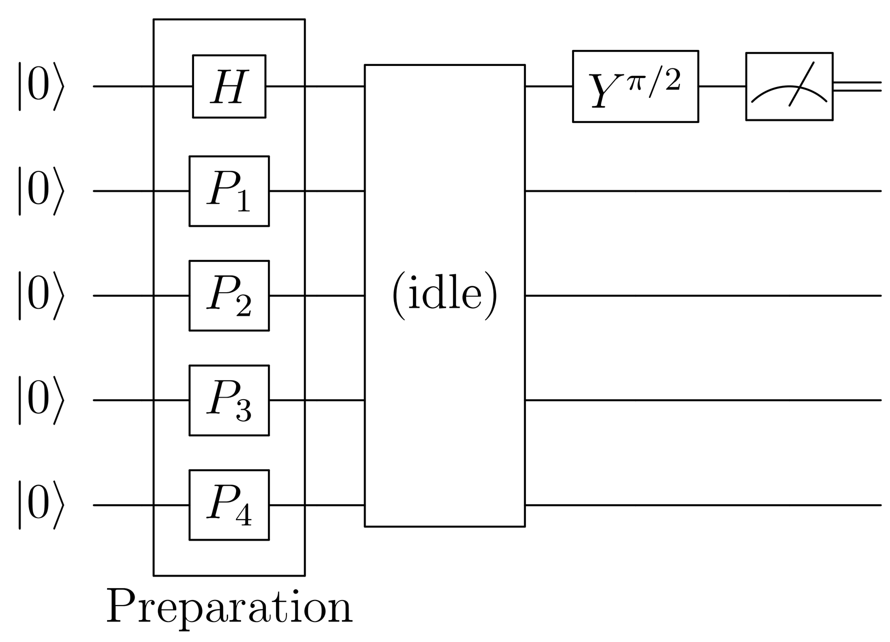




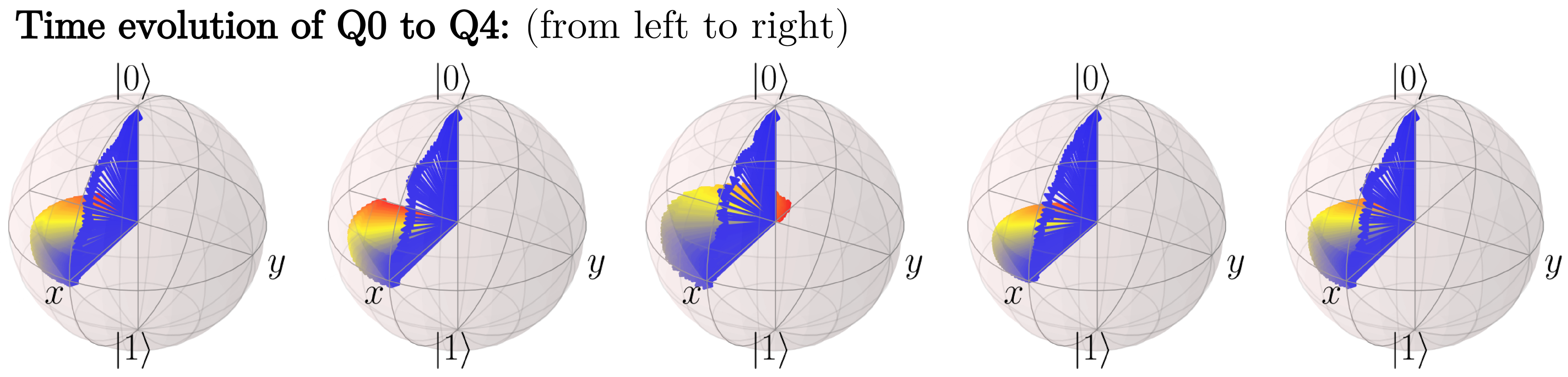
Crosstalk experiment

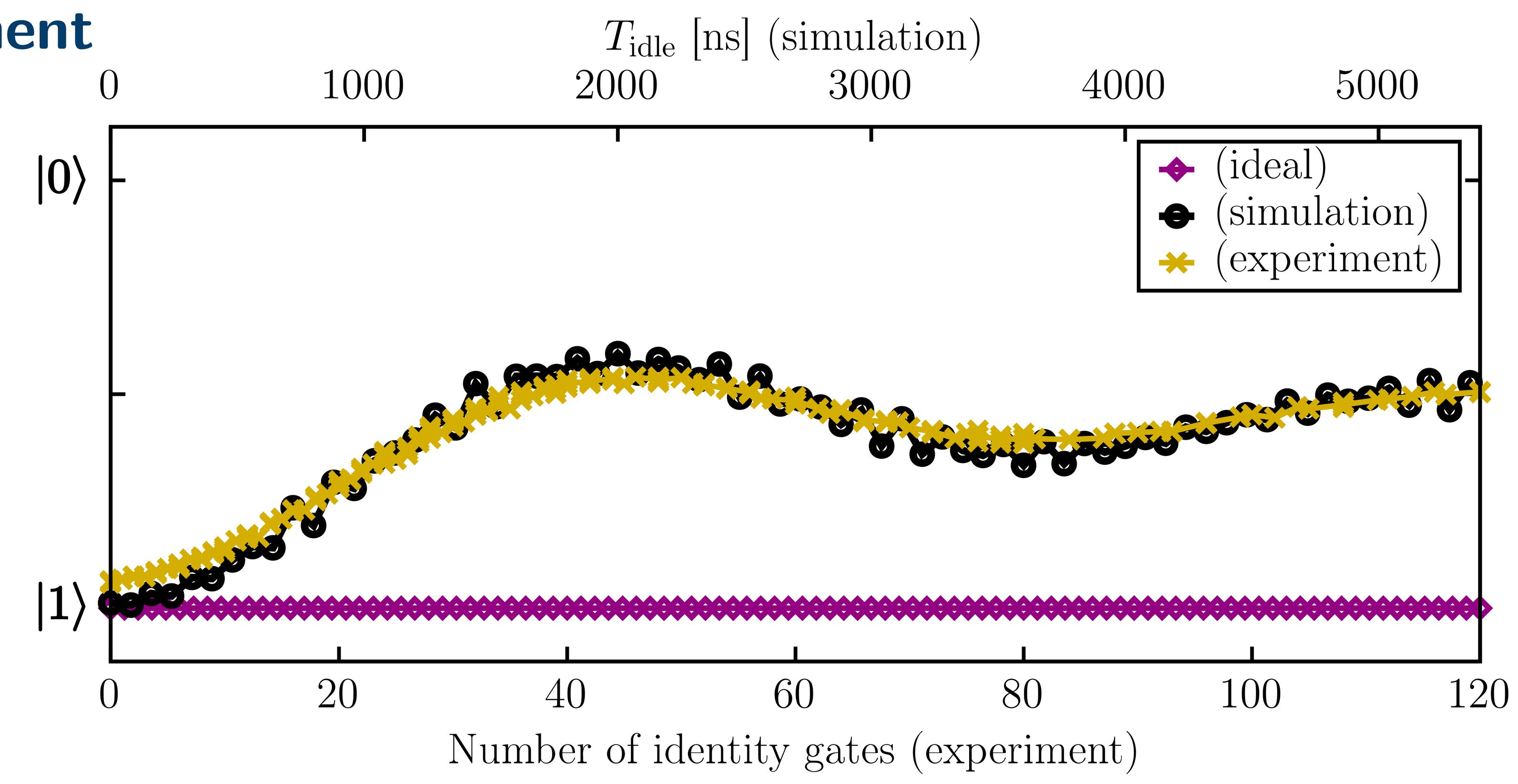
Quantum circuit:

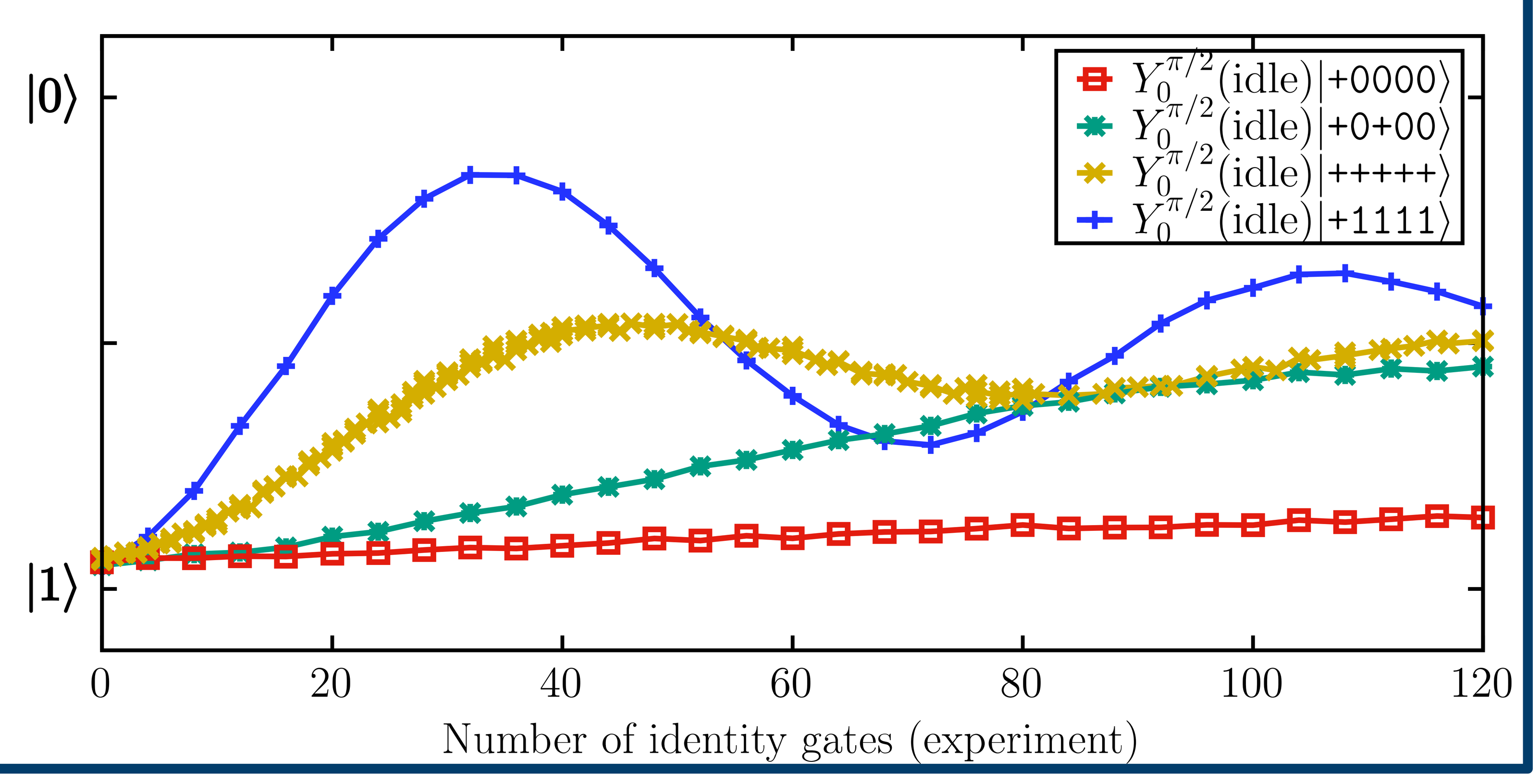
1. prepare Q0 in $|+\rangle$
2. prepare Q1 to Q4 in $|+\rangle$ or $|0\rangle$ or $|1\rangle$
3. wait for some "idle" time
   → ideally: nothing happens
   → **in practice**: crosstalk moves qubits!
4. rotate Q0 around y axis by π/2
   → ideally: $|+\rangle \rightarrow |1\rangle$
   → **in practice**: depends on "idle" time!
5. measure Q0



Time evolution of Q0 to Q4: (from left to right)







Conclusions:

- Simulation and experiment match excellently
- Both are far from the ideal result (diamonds)
- Errors are generic for transmon processors [3,7]

We conclude that the simulation model captures the errors in the NISQ device very well.

References:

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[4] Willsch et al. *Phys. Rev. A* **96**, 062302 (2017)
[5] IBM Q, https://www.research.ibm.com/ibm-q
[6] Sheldon et al. *Phys. Rev. A* **93**, 060302 (2016)
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