Quantum annealing with anneal path control and catalyst Hamiltonian

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Introduction

Quantum annealing is a quantum version of classical simulated annealing, but using quantum fluctuations instead of thermal fluctuations, to explore the energy landscape of an optimization problem. This approach has received enormous interest in the last two decades, and is regarded as a second model of quantum computing [1], which is quite distinct to the gate-based model.

The standard form of quantum annealer, such as the one that D-Wave Systems Inc. uses, can be described by a transverse-field Ising model. This type of quantum annealer is designed for solving quadratic unconstrained binary optimization problems. While researches on the universality and hypothetical quantum speed up for this type of quantum annealer are still ongoing, here we investigate two different variations of qantum annealer, namely, adding anneal path control and catalyst Hamiltonian to the standard form, respectively.

Standard form

The Hamiltonian of the quantum annealing can be written as a time-dependent linear combination of an initial Hamiltonian H_I and a final Hamiltonian H_P encoding the problem to be solved:

$H(t) = A\left(\frac{t}{t_a}\right)H_I + B\left(\frac{t}{t_a}\right)H_P,$

$H_I = -\sum_{i=1}^{N}\sigma_i^x,$

$H_P = -\sum_{i=1}^{N}h_i^z\sigma_i^z - \sum_{i,j=1}^{N}J_{ij}\sigma_i^z\sigma_j^z,$

where t_a denotes the total annealing time, $\sigma_i^{x,y,z}$ are Pauli matrices. During the annealing process, $A(t/t_a)$ starts from 1 and slowly decreases to 0, and $B(t/t_a)$ starts from 0 to 1. For a linear scheme, $A\left(\frac{t}{t_a}\right) = 1 - \frac{t}{t_a}$ and $B\left(\frac{t}{t_a}\right) = \frac{t}{t_a}$.

Adding anneal path control

The anneal path control is implemented by modifying the $A\left(\frac{t}{t_a}\right)$ and $B\left(\frac{t}{t_a}\right)$ for each qubit i [2], resulting in the Hamiltonian:

$H(t) = -\sum_{i=1}^{N}A_i(\frac{t}{t_a})\sigma_i^x - \sum_{i=1}^{N}B_i(\frac{t}{t_a})h_i^z\sigma_i^z - \sum_{i,j=1}^{N}\sqrt{B_i(\frac{t}{t_a})B_j(\frac{t}{t_a})}J_{ij}\sigma_i^z\sigma_j^z,$

where $A_i\left(\frac{t}{t_a}\right) = 1 - \left(\frac{t}{t_a}\right)^{1+\gamma_i}$ and $B_i\left(\frac{t}{t_a}\right) = \left(\frac{t}{t_a}\right)^{1+\gamma_i}$.

Adding catalyst Hamiltonian

A third term, called catalyst Hamiltonian H_C, is added into the standard form, which disappears at both the beginning and end of the annealing process, i.e.,

$H(t) = A\left(\frac{t}{t_a}\right)H_I + B\left(\frac{t}{t_a}\right)H_P + C\left(\frac{t}{t_a}\right)H_C,$

where $C\left(\frac{t}{t_a}\right) = g\left(1 - \frac{t}{t_a}\right)\frac{t}{t_a}$, and g control the strength of H_C.
Two types of Hamiltonian are considered [3], which are given by
$H_C^F = -\sum_{i,j=1}^{N}\sigma_i^x\sigma_j^x,$

$H_C^A = +\sum_{i,j=1}^{N}\sigma_i^x\sigma_j^x.$

Method

The whole system evolves according to the time-dependent Schrödinger equation (TDSE),
$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle.$
For small systems, the TDSE can be solved by exact diagonalization of the Hamiltonian. For large systems, we use the second-order Suzuki-Trotter product formula to approximate the time-evolution of the TDSE.
The spectrum of the system is solved by exact diagonalization for small systems and Lanczos method for large systems.

The considered optimization problems are 2-SAT problems with 12 Boolean variables with a known unique ground state and a highly degenerate first excited state. More information about the chosen problem can be found in Ref. [4].
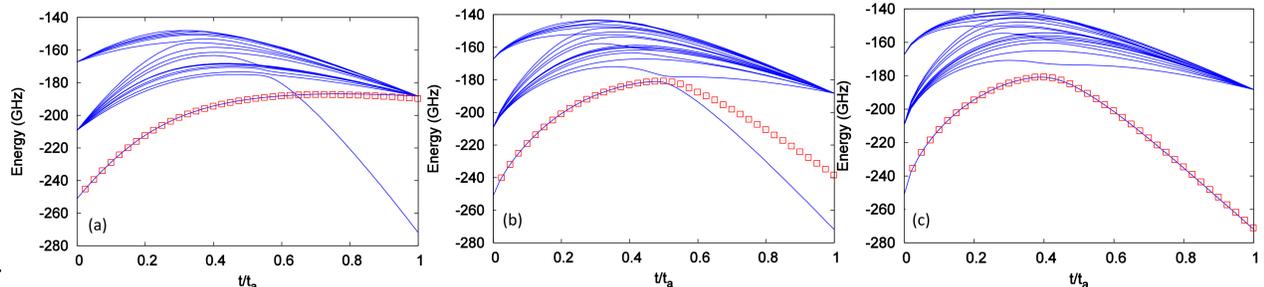

Fig. 1: Energy spectrum for 2-SAT problem 487 for a linear annealing scheme (a), and annealing process with anneal path control (b, c). The squares denote the average energy of the system during the annealing process. The annealing time $t_a = 5ns$.

Simulation results for adding anneal path control

We solved the TDSE for the standard form with linear annealing scheme and with a scheme allowing control for individual qubit, respectively, for a given 2-SAT problem. Some of the results are shown in Fig. 1.

The average energy obtained from the standard form follows the ground state energy up to the critial point, then makes a Landau-Zener transition to end up close to the energy of the first excited state of the problem. The control/offset parameters used where anneal path control are applied are obtained according to certain iterative method which depends on the probability of floppiness [5]. From the results, it is clearly seen that by adding the anneal path control, the minimum gap is largely increased, and the average energy of the system is nicely following the ground state energy for the whole spetrum for certain choices of the control parameters (see Fig. 1 (c)).
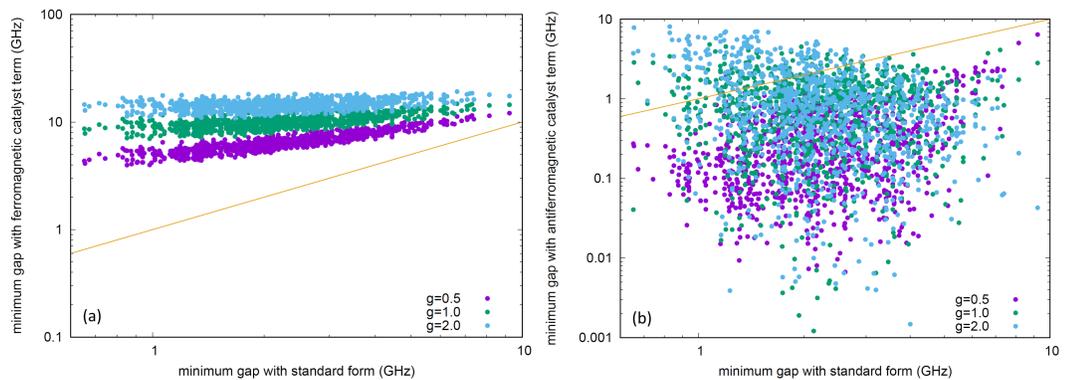

Fig. 2: Scatter plot of the minimum gap during the annealing process obtained from the standard form and the form with ferromagnetic (a) and antiferromagnetic (b) catalyst term.

Simulation results for adding catalyst Hamiltonian

We obtained the spectra for the standard form without and with a catalyst Hamiltonian, respectively, for given 2-SAT problems. Linear annealing scheme is used.
Some of the results are shown in Fig. 2.

It is seen that adding ferromagnetic catalyst term increases the minimum gap for all the cases, while adding antiferromagnetic term decreases the minimum gaps for almost all of the cases for $g = 0.5$, increases the minimum gaps for some cases for $g = 1.0$ and 2.0. This leads to interesting physical dynamics of the state of the system as a function of the annealing time, for solving the optimization problems [6].

Summary

Adding anneal path control to quantum annealing is a practical method for enhancing the performance of the annealing method for systems for which perturbative anticrossings dominate the slow-down mechanism of quantum annealing [5]. The performance upon adding extra catalyst term to quantum annealing largely depends on the type of catalyst. For ferromagnetic catalyst Hamiltonian, we observed that the minimum gaps are opened up for all the cases we studied, and therefore the performance is enhanced. For antiferromagnetic term, the minimum gaps are mostly reduced for small coupling strength $g = 0.5$; the number of anti-crossings is increased for large $g = 1.0$, and 2.0, which for certain cases might enhance the performance greatly [6].

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