Flavour Structure of the Baryon Octet

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This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 813942.

NIC Symposium 2020, 27-28th Feb. 2020, FZJ

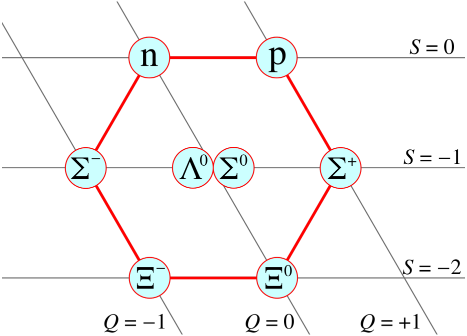
Overview

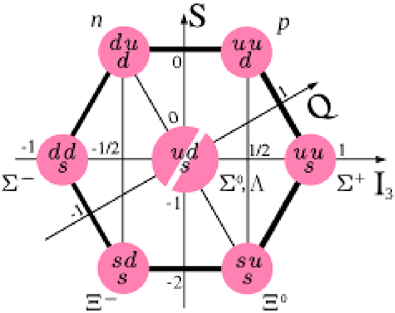
- Interest in the structure of the baryon octet and flavour symmetry (breaking) $\rightarrow$ how the quark and gluon constituents account for the properties of the baryons.
- Numerical simulations: lattice QCD.
- Octet baryon mass spectrum, πB σ terms, axial charges and form factor (nucleon) and distribution amplitudes.
- Some computational details.
- Summary and outlook.

RQCD: G Bali, V Braun, S Collins, M Göckeler, P Korcyl (Krakow), M Löffler, A Schäfer, E Scholz, J Simeth, W Söldner, A Sternbeck (Jena), P Wein, S Weishäupl, T Wurm + ...

Baryon octet

$J^P = \frac{1}{2}^+$, classified in terms of electric charge Q and strangeness (flavour QN) S.





Bound states of quarks and gluons: simple picture qqq with $q \in \{u, d, s\}$.

Flavour symmetry

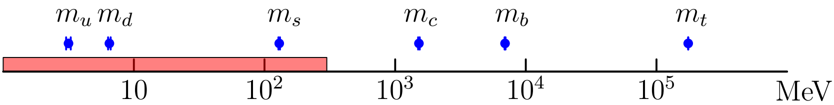
If $m_u = m_d = m_s$, properties of octet baryons related (ignore electric charge).

$$m_p = m_n = m_{\Sigma^{\pm,0}} = m_{\Xi^{-,0}} = m_{\Lambda^0}$$

Decay rates of $\beta-$decays:

$$n \to p + e^- + \overline{\nu}_e, \quad \Lambda^0 \to p + e^- + \overline{\nu}_e, \quad \Sigma^- \to n + e^- + \overline{\nu}_e,$$
$$\Xi^- \to \Lambda^0 + e^- + \overline{\nu}_e, \ldots$$

We have $m_u, m_d \ll \Lambda_{\rm int}$, m_s somewhat heavier.

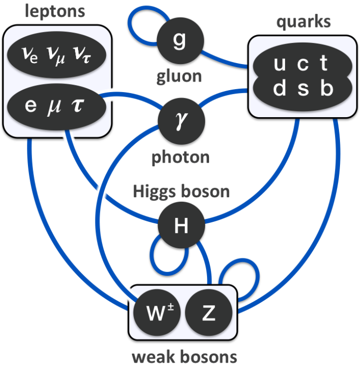


Numerical simulations (lattice QCD) to determine properties of baryon octet:

Determine pattern of symmetry breaking: $m_u = m_d \neq m_s$.

Also simulate $m_u = m_d = m_s$.

The Standard Model



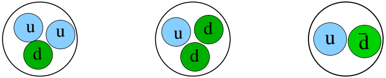
Mass generation via the Higgs mechanism.

Strong interaction (QCD):

g and quarks.

Binds quarks and gluons into hadrons.

Proton Neutron Pion



Electromagnetic interaction (QED):

γ and electrically charged particles.

Weak interaction (EWT):

$W^{\pm}$, Z, H and leptons and quarks.

Responsible for β decay $n \to p + e^- + \bar{\nu}_e$.

	Strong	Electromagnetic	Weak	**Gravity**
Range (m)	10^{-15}	∞	10^{-18}	∞
Relative Strength	1	10^{-2}	10^{-13}	10^{-38}

Shortcomings of the Standard Model

No explanation for:

Values of 25+1+2 parameters

Masses: $q \in \{u, d, s, c, b, t\}$,
$\ell \in \{e, \mu, \tau\}$, $\nu \in \{\nu_e, \nu_\mu, \nu_\tau\}$, W, H.

Couplings: α_S, α_{em}, α_w

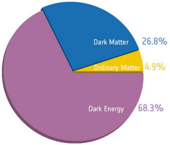
Quark mixing: 3 angles, 1 CP violating phase

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

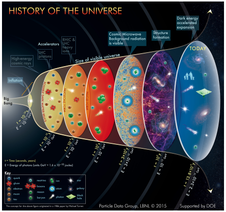
Neutrino mixing: 3 angles, 1 (3) phase(s)

Strong CP angle.

Dark matter and dark energy



Matter-antimatter asymmetry

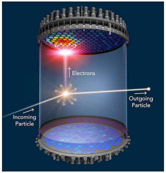


Gravity not included, ...

Experimental searches for new physics

Direct detection of new particles

e.g. Dark matter scattering with nucleons.
$\chi + p \to \chi + p$, $\chi + n \to \chi + n$ in Xe

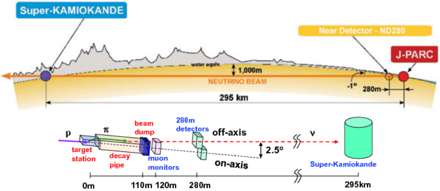


Creation of new particles in colliders

e.g. LHC $p + p$

Exploration of neutrino sector

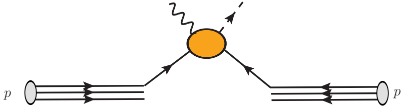
Neutrino oscillation experiments detectors
Nuclear targets: $\nu_\mu + n \to \mu^- + p$

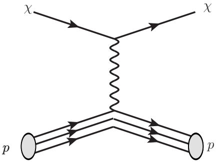


Low energy precision β experiments decay (ultra cold neutrons), neutron electric dipole moment ...

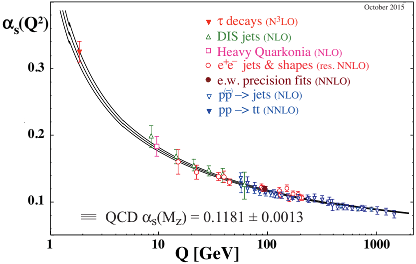
Proton structure

Interested in the fundamental interaction with the quarks and gluons





PDG 2016



Confinement: quarks and gluons bound within hadrons

$$\text{Expt} \propto \left(\begin{array}{c} \text{Behaviour of quarks} \\ \text{within the proton} \end{array} \right) \otimes \left(\begin{array}{c} \text{Fundamental} \\ \text{process} \end{array} \right)$$

Long distance/low energy information: non-perturbative.
Matrix elements $\langle p(p_f)|J|p(p_i)\rangle$ evaluated numerically.

Baryon Structure

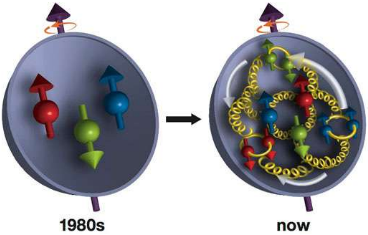
For brevity $\Delta q + \Delta\overline{q} \to \Delta q$, $\langle x\rangle_q + \langle x\rangle_{\overline{q}} \to \langle x\rangle_q$.

(Longitudinal) momentum sum rule: $$1 = \langle x\rangle_g + \sum_{q\in u,d,s,\ldots} \langle x\rangle_q$$

$\langle x\rangle_{q,g}$: fraction of the baryon momentum carried by the quark or gluon.

(Longitudinal) spin sum rule: $$\frac{1}{2} = \frac{1}{2}\sum_{q\in u,d,s,\ldots} \Delta q + L_\psi + J_g$$

Δq, contribution of the quark spin, $L_\psi = \sum_q L_q$, quark orbital angular momentum and J_g, the gluon total angular momentum.



Simple *uud* picture of proton:
$\langle x\rangle_u + \langle x\rangle_d = 1$ and $\Delta u + \Delta d = 1$

Expt:
$\langle x\rangle_u + \langle x\rangle_d \sim 0.32$ at $\mu^2 = 10$ GeV^2
$\Delta u + \Delta d \sim 0.35$ at $\mu^2 = 4$ GeV^2.

Flavour symmetry

Focus on the properties of the proton/neutron (nucleons), however, flavour separation can sometimes be difficult.

Example: determination of Δq, $q \in \{u, s, d\}$ for the proton/neutron.

Access a combination $(a_8 \pm 3a_3)C_{NS} + 4a_0 C_s$ from deep-inelastic scattering experiments ($N + e^- \to X + e^-$, $\pm$ for $N = p, n$).

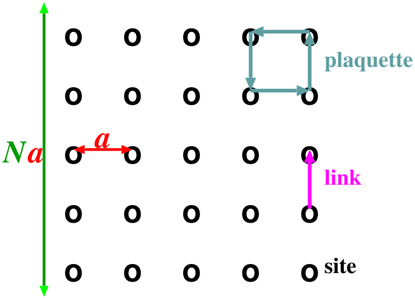
$$\begin{aligned} g_A = a_3 &= \Delta u - \Delta d \\ a_8 &= \Delta u + \Delta d - 2\Delta s \\ a_0(\mu^2) &= \Delta u + \Delta d + \Delta s \end{aligned}$$

Axial charge, $g_A = a_3$, is known very precisely from neutron β decay.

Assume SU(3) flavour symmetry (in $q \in \{u, d, s\}$) to extract a_8 from hyperon β-decays.

Hyperon: baryon containing one or more s quarks and u or d: Λ^0, $\Sigma^{\pm,0}$, $\Xi^{-,0}$, ...

Lattice QCD



typical values:

$a^{-1} = 2{-}5$ GeV, $Na = 2{-}7$ fm

continuum limit: $a \to 0$, Na fixed

infinite volume: $Na \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\, [d\psi][d\bar{\psi}]\, O[U] e^{-S[U,\psi,\bar{\psi}]}$$

"Measurement": average over a *representative* ensemble of gluon configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

Input: discretized $\mathscr{L}_{QCD} = \frac{1}{16\pi\alpha_s(a)} FF + \sum_f \bar{q}_f(\not{D} + m_f(a))q_f$

$$m_\Xi^{\text{latt}} = m_\Xi^{\text{phys}} \quad \longrightarrow \quad a$$
$$M_\pi^{\text{latt}}/m_\Xi^{\text{latt}} = M_\pi^{\text{phys}}/m_\Xi^{\text{phys}} \quad \longrightarrow \quad m_u(a) \approx m_d(a)$$
$$\cdots$$

Output: hadron masses, matrix elements, decay constants, etc...

Required:

1. Volume, $L = Na \to \infty$: FSE suppressed with $\exp(-LM_\pi) \Rightarrow LM_\pi \gtrsim 4$.
2. Lattice spacing, $a \to 0$: functional form known: $\mathcal{O}(a^2), \mathcal{O}(\alpha_s a) \Rightarrow \approx 4$ lattice spacings.
3. Quark mass, $m_q^{\text{latt}} \to m_q^{\text{phys}}$: chiral perturbation theory (χPT) helps for m_{ud} but m_{ud}^{latt} must be sufficiently small to start with. $M_\pi^2 \propto m_{ud}$.

Computational challenges

Cost of simulation is proportional to

- number of points: $\sim N^4 = (L/a)^4$
- condition number of linear system: $1/M_\pi^2$
- $L^{1/2}/M_\pi$ in (Omelyan) time integration within hybrid Monte Carlo
- $1/a^{\geq 2}$ critical slowing down (autocorrelations)

Adjusting $L \propto 1/M_\pi$ this means:

$$\mathrm{cost} \propto \frac{1}{a^{\geq 6}\, M_\pi^{7.5}}$$

For baryonic observables at small M_π additional noise/signal problems.

State of the art: 192×96^3 sites, corresponding to $\approx (2 \times 10^{10})^2$ (sparse) complex matrices.

Tremendous progress in Hybrid Monte Carlo, solver, noise reduction.

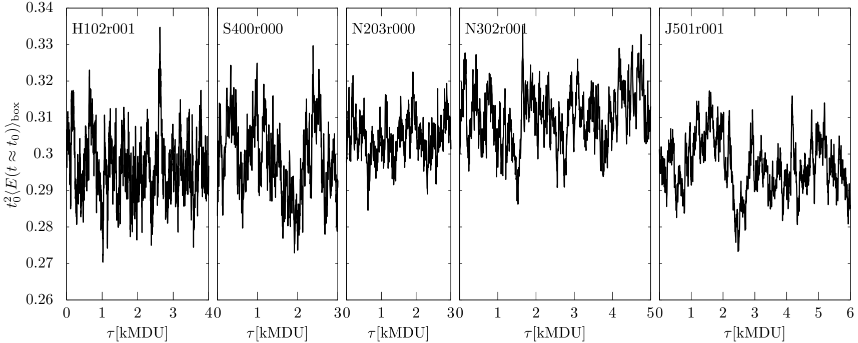
$N_f = 2 + 1$ CLS ensembles

Coordinated Lattice Simulations (CLS): Berlin, CERN, Mainz, UA Madrid, Milano Bicocca, Münster, Odense, Regensburg, Rome I and II, Wuppertal, DESY-Zeuthen.

⋆ Non-perturbatively improved clover fermion action and tree-level Lüscher-Weisz gauge action.

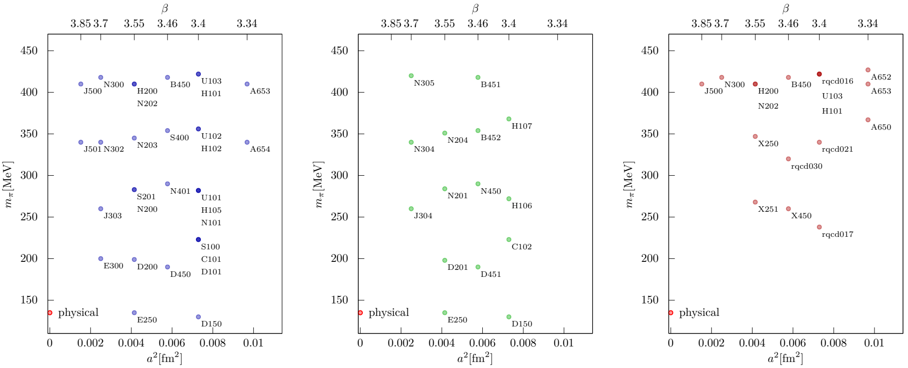
⋆ Mostly open boundary conditions in time.

Wilson flow action density, $t_0^2 E(t \approx t_0)$, $M_\pi \approx 340$ MeV, averaged over ≈ 1 fm slice.



CLS ensembles: M_π vs a^2

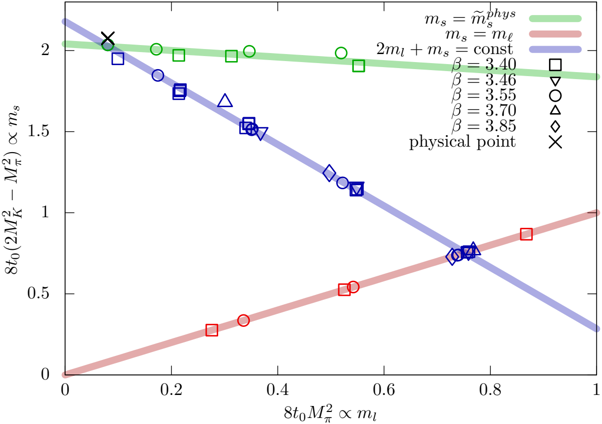
- ⋆ Three trajectories, physical point ensembles.
- ⋆ Typically 6000–10000 MDUs.



$2m_\ell + m_s = \text{const.}$ $\qquad$ $m_s = \text{const.}$ $\qquad$ $m_\ell = m_s$

E: $192 \cdot 96^3$, J: $192 \cdot 64^3$, D: $128 \cdot 64^3$, N: $128 \cdot 48^3$, C: $96 \cdot 48^3$,
S: $128 \cdot 32^3$, H: $96 \cdot 32^3$, B: $64 \cdot 32^3$, U: $128 \cdot 24^3$.

CLS ensembles: m_ℓ-m_s plane



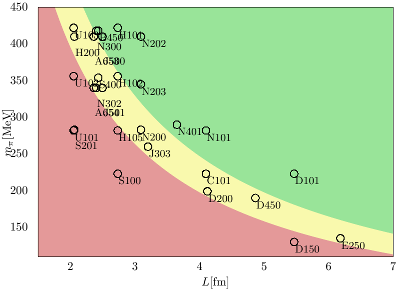
Approach to the physical point, $m_s + 2m_\ell = \text{const}$ [QCDSF+UKQCD: 1003.1114], **and** $\widehat{m}_s \approx \text{const}$ [RQCD, 1606.09039; 1702.01035],

Investigate flavour symmetric theory $m_{u,d} = m_s$ (red line).

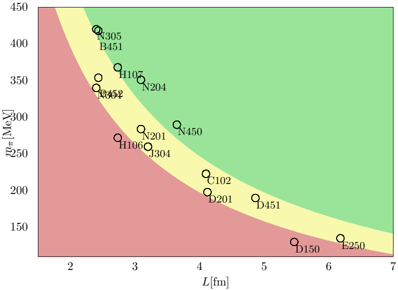
Investigate flavour symmetry breaking (blue line).

CLS ensembles: spatial volume

$2m_\ell + m_s = \text{const.}$



$m_s = \text{const.}$



$\mathbf{LM_\pi < 4}$ $\quad$ $\mathbf{4 \leq LM_\pi < 5}$ $\quad$ $\mathbf{LM_\pi \geq 5}$

Octet baryon spectrum: $B \in \{N, \Lambda, \Sigma, \Xi\}$, **Preliminary**

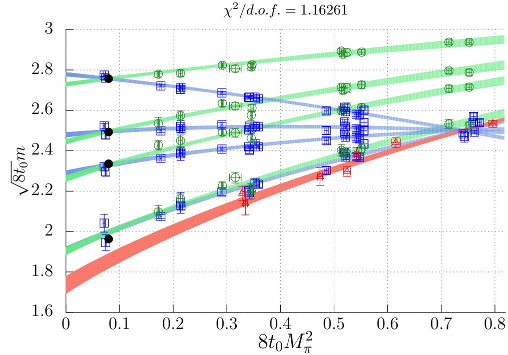
Simultaneous fit to octet masses: (12 parameters)

$$\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, \mathrm{a}) = \mathbf{m_B}(\mathbb{M}_\pi, \mathbb{M}_\mathbf{K}, \mathbf{0}) \left[1 + c\mathrm{a}^{\mathbf{2}} + \bar{c}\mathrm{a}^{\mathbf{2}}\overline{\mathbb{M}}^2 + \delta c_B \mathrm{a}^{\mathbf{2}}\delta\mathbb{M}^2\right]$$

$\mathrm{m}_B = \sqrt{8t_0} m_B$, $\mathrm{a} = a/\sqrt{8t_0^*}$, $\overline{\mathbb{M}}^2 = (2\mathbb{M}_K^2 + \mathbb{M}_\pi^2)/3$, $\delta\mathbb{M}^2 = 2(\mathbb{M}_K^2 - \mathbb{M}_\pi^2)$

W. Söldner

Agreement with expt.



Finite volume: $LM_\pi > 3.4$. $O(a^2)$ effects removed using fit. Similarly, data shifted to $\overline{\mathbb{M}}^2 = \overline{\mathbb{M}}^{2,\mathrm{phys}}$, $\overline{\mathbb{M}}_{\bar{s}s}^2 = \overline{\mathbb{M}}_{\bar{s}s}^{2,\mathrm{phys}}$, where $\overline{\mathbb{M}}_{\bar{s}s}^2 = 2\mathbb{M}_K^2 - \mathbb{M}_\pi^2$.

Mass extrapolations

NNLO covariant SU(3) baryon ChPT: ($\mathbb{M}_{\eta_8}^2 = \overline{\mathbb{M}}^2 + \delta\mathbb{M}^2/3$)

$$\mathrm{m}_B(\mathbb{M}_\pi, \mathbb{M}_K, 0) = \mathrm{m}_0 + \overline{\mathbb{b}}\overline{\mathbb{M}}^2 + \delta\mathbb{b}_B\delta\mathbb{M}^2 + \mathbb{g}_{B,\pi} f_O\left(\frac{\mathbb{M}_\pi}{\mathrm{m}_0}\right) + \mathbb{g}_{B,K} f_O\left(\frac{\mathbb{M}_K}{\mathrm{m}_0}\right) + \mathbb{g}_{B,\eta_8} f_O\left(\frac{\mathbb{M}_{\eta_8}}{\mathrm{m}_0}\right)$$

EOMS regularisation*: $f_O(x) = -2x^3[\sqrt{1-x^2/4}\,\mathrm{arccos}(x/2) + x\ln(x)/2]$

SU(3) constraints:

⋆ $\delta\mathbb{b}_B = \delta\mathbb{b}_{N,\Lambda,\Sigma,\Xi}$ determined by 2 parameters.

⋆ $\mathbb{g}_{B,\pi,K,\eta_8} = g_{B,\pi,K,\eta_8} m_0^3/(4\pi F_0)^2$ known quadratic functions of the LECs F, D.

$m_\Xi^{\mathrm{latt}} = m_\Xi^{\mathrm{ph}}$ imposed to fix the scale: $\sqrt{8t_{0,\mathrm{ph}}} = 0.4128(22)$(??) fm.
Compatible with $\sqrt{8t_{0,\mathrm{ph}}} = 0.413(6)$ fm from $F_\pi + 2F_K$ [ALPHA,1608.08900].

* Extended on mass shell scheme.

σ terms: $\sigma_{q,B} = m_q \langle B | q\bar{q} | B \rangle$

Feynman-Hellmann theorem

$\frac{\partial E(\lambda)}{\partial \lambda} = \langle \psi(\lambda) | \frac{\partial H(\lambda)}{\partial \lambda} | \psi(\lambda) \rangle$, $\quad H(\lambda) = H_0 + \lambda H_1$ with $\lambda H_1^{QCD} \to m_q \bar{q} q$.

$$\sigma_{\pi B} = m_u \frac{\partial m_B}{\partial m_u} + m_d \frac{\partial m_B}{\partial m_d} \approx M_\pi^2 \frac{\partial m_B}{\partial M_\pi^2}, \qquad \sigma_{sB} = m_s \frac{\partial m_B}{\partial m_s} \approx \frac{1}{2} M_{\bar{s}s}^2 \frac{\partial m_B}{\partial M_{\bar{s}s}^2}$$

$\sigma_{\pi B} = \sigma_{uB} + \sigma_{dB}$.

σ_{sB} is not well determined as m_s is not varied near the physical point.

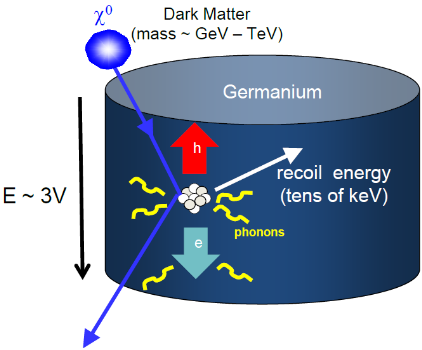
Pion sigma terms: **Preliminary**

$$\sigma_{\pi N} = 41(2)(2)(??)\ \mathrm{MeV} \qquad \sigma_{\pi \Lambda} = 29(2)(1)(??)\ \mathrm{MeV}$$
$$\sigma_{\pi \Sigma} = 23(1)(1)(??)\ \mathrm{MeV} \qquad \sigma_{\pi \Xi} = 13(1)(0)(??)\ \mathrm{MeV}$$

Compatible with [FLAG,1902.08191] average (FH+direct) for $N_f = 2+1$ of $\sigma_{\pi N} = 39.7(3.6)$ MeV dominated by [BMW-c,1510.08013] $\sigma_{\pi N} = 38(3)(3)$ MeV.

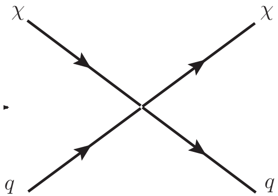
Phenomenological estimate from an analysis of $\pi - N$ scattering data: [Hoferichter,1506.04142] $\sigma_{\pi N} = 59.1(3.5)$ MeV.

Direct dark matter detection



Scattering of DM (WIMPs) off nuclei, near zero recoil.

Spin-independent effective interaction $\sim \bar{\chi}\chi\bar{q}q$

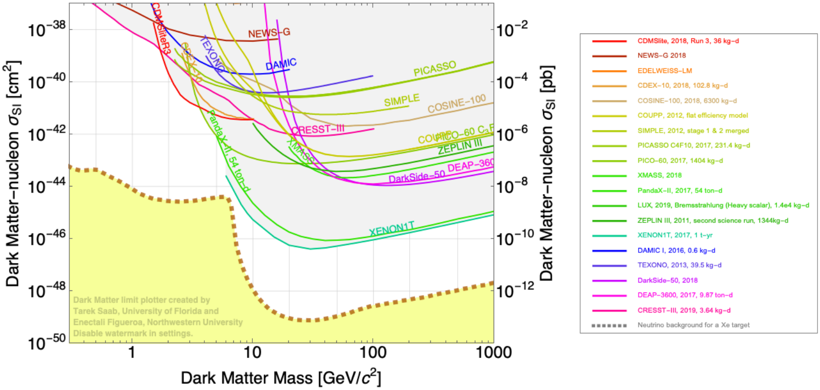


$$\sigma^{SI} \propto \left[Z f_p + (A - Z) f_n\right]^2 , \qquad \frac{f_N}{m_N} = \sum_q f^N_{T_q} \frac{\alpha_q}{m_q}, \quad f^N_{T_q} = \frac{1}{m_N} \sigma^N_q$$

$N = p, n$

Dark matter-nucleon spin independent cross-section

Dark Matter Limit Plotter 2019.



Yellow: neutrino background.

Also spin dependent cross-section, depends on $\langle N|\bar{q}\gamma_\mu\gamma_5 q|N\rangle$, less well constrained.

Use σ_q for predictions of σ_{SI} $\rightarrow$ rule out parameter space of models.

Axial charges of the baryon octet: $m_{u,d} = m_s$

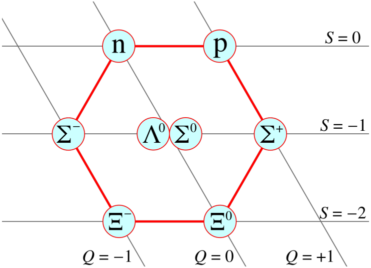
For neutron β-decay, axial charge:

$$g_A = a_3 = \Delta u - \Delta d = \langle n|(\overline{u}\gamma_0\gamma_5 d)|p\rangle = \langle p|(\overline{u}\gamma_0\gamma_5 u - \overline{d}\gamma_0\gamma_5 d)|p\rangle = \langle n|(\overline{d}\gamma_0\gamma_5 d - \overline{u}\gamma_0\gamma_5 u)|n\rangle$$

Define axial charges for other members of the octet.

Characterize weak decays: $\Sigma \to \Sigma$, $\Xi \to \Xi$, $\Xi \to \Sigma$, $\Xi \to \Lambda$, $\Lambda \to N$ ....

Isospin $I = 1, \frac{1}{2}$ and hypercharge $Y = S + 1$.



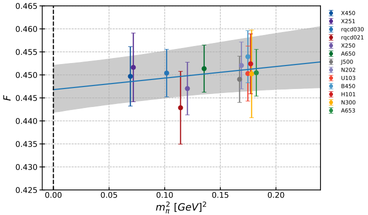
$$\begin{aligned} g_A^B &= (\Delta u - \Delta d)^B \\ &= \langle B|(\overline{u}\gamma_0\gamma_5 u - \overline{d}\gamma_0\gamma_5 d)|B\rangle \\ &= 2I_3(F + YD) \end{aligned}$$

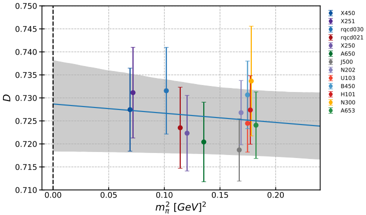
$I = 0$

$$\begin{aligned} g_A^\Lambda &= \langle \Lambda|(\overline{u}\gamma_0\gamma_5 d)|\Sigma\rangle \\ &= 2D \end{aligned}$$

Axial charges of the baryon octet: $m_{u,d} = m_s$

$g_A^p = F + D$, $g_A^{\Sigma^+} = 2F$, $g_A^{\Xi^0} = F - D$.





Simple finite V, finite a, m_q fit so far.

$$F = C_0 + C_M m_\pi^2 + C_L m_\pi^2 e^{-m_\pi L} / \sqrt{m_\pi L} + C_a a^2$$

In the future higher order ChPT will be considered.

Cf. $D/F > 2$ from m_B extrapolation.

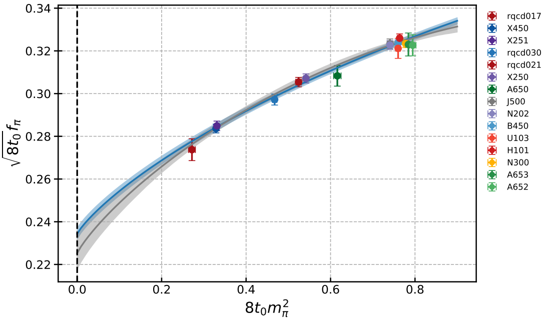
Expt. hyperon semi-leptonic decays + SU(3) symmetry [Cabbibo,hep-ph/0307298]
$F = 0.463(8)$ and $D = 0.804(8)$ at the physical point (not at $m_\pi = 0$).

Pion decay constant: $m_{u,d} = m_s$

Weak decay of the pion:

$$\pi^+ \to \mu^+ \nu_\mu, \qquad \langle 0 | \bar{u} \gamma_0 \gamma_5 d | \pi \rangle = i m_\pi f_\pi$$

$m_q \to 0$, $f_\pi \to \sqrt{2} F_0$



Finite V, finite a, SU(3) ChPT NLO (blue) and NNLO (grey).

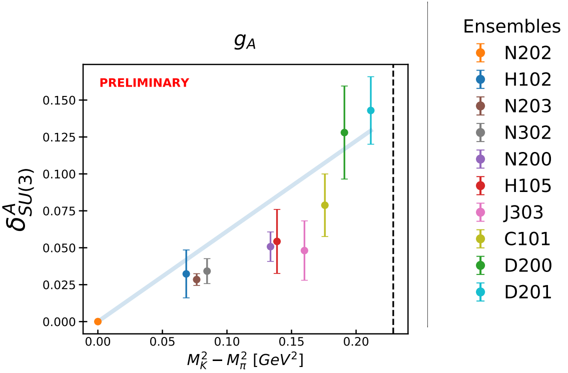
Analysis in progress for F_π for $m_{u/d} \neq m_s$ ensembles.

SU(3) flavour symmetry breaking in the baryon octet

$m_{u/d} = m_s$: $g_A^p = F + D$, $g_A^{\Xi^0} = F - D$, $g_A^{\Sigma^+} = 2F \Rightarrow (g_A^p + g_A^{\Xi^0})/g_A^{\Sigma^+} = 1$.

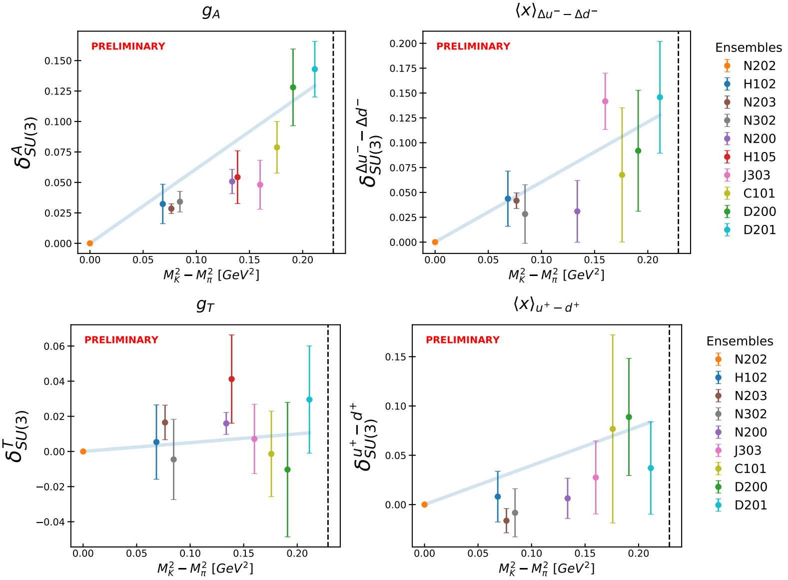
Away from this limit: corrections start at $O(\delta m_l)$, $\delta m_l = m_s - m_{u/d} \propto M_K^2 - M_\pi^2$.

Shown: $\delta^A_{SU(3)} = (g_A^p + g_A^{\Xi^0})/g_A^{\Sigma^+} - 1$



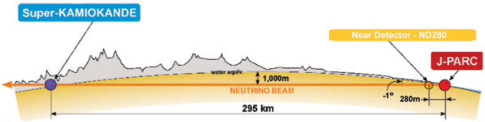
Line not a fit. Results shown from both m_s =constant and $m_{u,d} + m_s$ =constant ensembles.

SU(3) flavour symmetry breaking in the baryon octet



Flavour symmetry breaking is mild.

Long base-line neutrino oscillation experiments

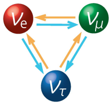


T2K: Tokai to Super-Kamiokande,

$E = 0.6$ GeV, $L/E \approx 500$ **km/GeV.**

Also, e.g. NOνA USA, $L/E \approx 400$ km/GeV, DUNE USA $L/E \approx 520$ km/GeV.

Muon neutrino beam: protons on nuclei $\to$ pions and kaons $\to \mu^+ \nu_\mu$



Neutrino of definite flavour, ν_μ: combination of mass eigenstates $\to$ neutrino mixing in the time evolution $|\nu_j(t)\rangle = e^{-im_j^2 L/(2E)} |\nu_j(0)\rangle$

$$P_{\alpha \to \beta} = \left| \langle \nu_\beta(t) | \nu_\alpha \rangle \right|^2 = |\sum_j \mathbf{U}^*_{\alpha \mathbf{j}} \mathbf{U}_{\beta \mathbf{j}} e^{-i\mathbf{m}_\mathbf{j}^2 L/2E}|^2, \quad \text{PMNS matrix } \mathbf{U}$$

Near and far detectors.

$$\mathbf{N}^\mu_{\text{far}}(\mathbf{E}_\nu) = \mathbf{N}^\mu_{\text{near}}(\mathbf{E}_\nu) \times [\text{flux(L)}] \times [\text{detector}] \times [1 - \sum_\beta \mathbf{P}_{\mu \to \beta}(\mathbf{E}_\nu)]$$

E_ν, must be reconstructed from the momentum of the detected charged lepton.

Trivial for $\nu_\mu + n \to \mu^- + p$ if the initial momentum of n and of ν_μ are known.

But: The neutrino beam is not monochromatic. Neutron $p_{\text{Fermi}} \approx 200$ MeV is washed out since bound in nucleus. The lepton momentum reconstruction is often incomplete. Misidentification of inelastic scattering as elastic scattering.

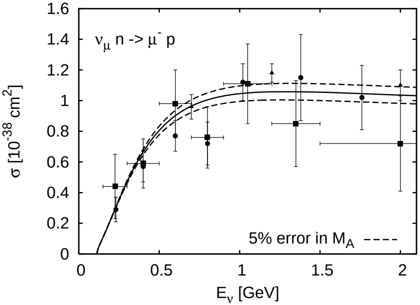
Nucleon axial form factor $G_A(Q^2)$

Monte Carlo simulations performed to predict E_ν. These require the differential cross-section which depends on nuclear models and

$\langle n(\vec{p}_f)|(\overline{u}\gamma_0\gamma_5 d)|p(\vec{p}_i)\rangle \to G_A(Q^2),\ Q^2 = -q^2,\ q = p_f - p_i.$

Pre 1990 information on $\mathbf{G_A(Q^2)}$ extracted from $\bar{\nu}$-p and ν-d scattering.

[Mosel,1602.00696]



$$G_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{\mathbf{M_A}^2})^2}$$

Shown $M_A = 1.03(5)$ **GeV**

MiniBooNE: [Aguilar-Arevalo,1002.2680]

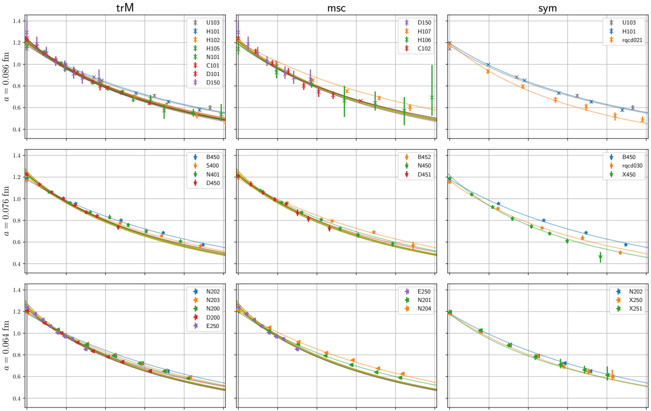
$M_A = 1.35(17)$ **GeV**

Errors of the new experiments will be dominated by these systematics.

Nucleon axial form factor $G_A(Q^2)$

$G_A(Q^2)$ from $Q^2 = 0$ to 1.0 GeV^2. Three of five lattice spacings shown.

[RQCD,1911.13150]



Fit: finite V, finite a, m_q, Q^2 dependence.

Consider dipole form $G_A = g_A/(1 + Q^2/M_A)^2$ and less constrained z-expansion.

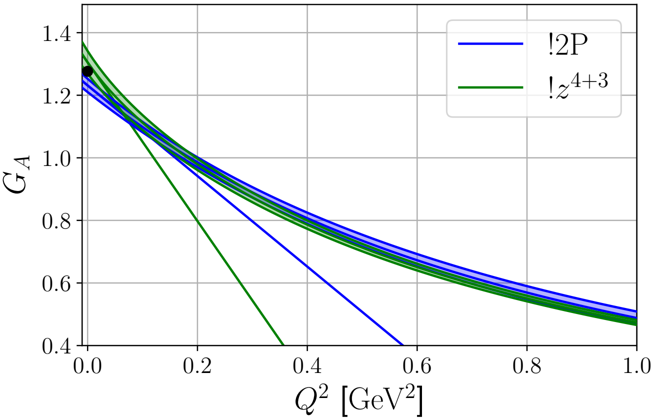
(Form factor $G_A(Q^2)$ fitted together with $G_P(Q^2)$ and $\tilde{G}_P(Q^2)$)

Nucleon axial form factor $G_A(Q^2)$

curves: fits with dipole and z-expansion forms.

lines: slope at $Q^2 = 0$, $G_A^{-1}(0) dG_A/dQ^2 = -\langle r_A^2 \rangle/6$, $\langle r_A^2 \rangle = 12/M_A^2$

$M_A = 1.30(7)$ GeV (dipole), $M_A = 1.01(7)$ GeV (z-exp.) [RQCD,1911.13150]

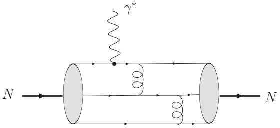


Similar dependence of M_A on fit form extracted from experimental results.
However, form factor $Q^2 \sim 0.1 - 1$ GeV interesting for the neutrino expts..

Octet baryon distribution amplitudes

⋆ Describe the distribution of the longitudinal momentum amongst the baryon constituents in the light cone (infinite momentum) frame. Like quantum mechanical wavefunction of baryon.

⋆ Provide the nonperturbative input for the theoretical description of hard (large Q^2) exclusive processes: e.g. $p \to p$ em elastic form factor.



⋆ Transition amplitude factorizes into (hard) transition amplitude and (non-perturbative) distribution amplitude

$$G(Q^2) = \int dxdy \Phi(y, Q^2) T_H(x, y, Q^2) \Phi(x, Q^2)$$

DAs are universal: involved in many processes

Electric and magnetic nucleon form factor $Q^2 \sim 14$ GeV ^{2} (**JLab, FAIR**)

Electric neutron form factor $Q^2 \sim 8$ GeV ^{2} (**JLab, FAIR**)

Electroproduction of nucleon resonances at large $Q^2 \sim 14$ GeV^2 (**JLab**)

. . .

Octet baryon distribution amplitudes

The DAs can be expanded in terms of orthogonal polynomials $P_{nk}(x_1, x_2, x_3)$: $x_1 + x_2 + x_3 = 1$

$$\Phi_+^B = 120 x_1 x_2 x_3 \left(\varphi_{00}^B \mathcal{P}_{00} + \varphi_{11}^B \mathcal{P}_{11} + \dots\right), \quad \Phi_-^B = 120 x_1 x_2 x_3 \left(\varphi_{10}^B \mathcal{P}_{10} + \dots\right),$$
$$\Pi^{B \neq \Lambda} = 120 x_1 x_2 x_3 \left(\pi_{00}^B \mathcal{P}_{00} + \pi_{11}^B \mathcal{P}_{11} + \dots\right), \quad \Pi^{\Lambda} = 120 x_1 x_2 x_3 \left(\pi_{10}^{\Lambda} \mathcal{P}_{10} + \dots\right)$$

The lower order coefficients ϕ_{nk}^B and π_{nk}^B can be computed on the lattice.

[RQCD,1903.12590] 40 ensembles: $a \to 0$, $m_q \to m_q^{\mathrm{phys}}$, finite V investigated.

B	N	Σ	Ξ	Λ
f^B	$3.54^{+6}_{-4}(2)$	$5.31^{+5}_{-4}(5)$	$6.11^{+7}_{-6}(14)$	$4.87^{+7}_{-4}(6)$
f_T^B	$3.54^{+6}_{-4}(2)$	$5.14^{+5}_{-4}(4)_m$	$6.29^{+8}_{-7}(16)$	
φ_{11}^B	$0.118^{+6}_{-5}(22)$	$0.195^{+4}_{-6}(41)$	$-0.014^{+4}_{-3}(18)$	$0.243^{+8}_{-7}(47)$
π_{11}^B	$0.118^{+6}_{-5}(22)$	$-0.090^{+3}_{-2}(25)$	$0.399^{+7}_{-9}(81)$	
φ_{10}^B	$0.182^{+20}_{-14}(7)$	$0.090^{+11}_{-31}(4)_m$	$0.350^{+18}_{-20}(16)$	$0.610^{+23}_{-28}(24)$
π_{10}^B				$0.214^{+33}_{-26}(9)$

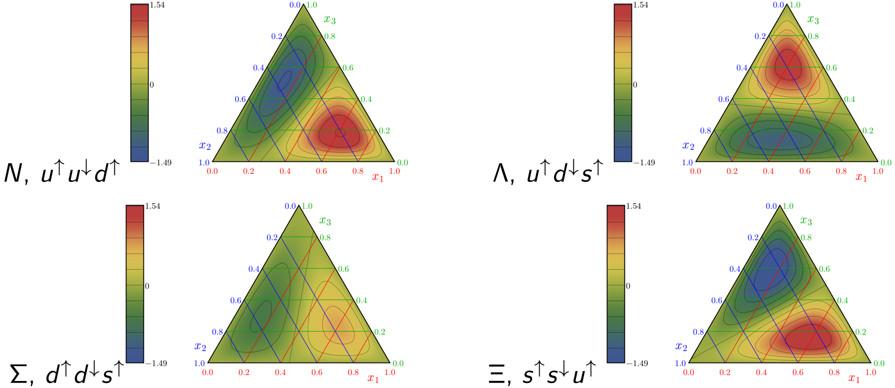
$f^B = \phi_{00}^B$ and $f_T^{B \neq \Lambda} = \pi_{00}^B$

Strong SU(3) flavour breaking observed, e.g. normalisation constants f^B, $f_T^{B \neq \Lambda}$.

Octet baryon distribution amplitudes

Barycentric plots ($x_1 + x_2 + x_3 = 1$) showing deviation from asymptotic shape $\phi^{as} \equiv 120 x_1 x_2 x_3$.

[RQCD,1903.12590] 40 ensembles: $a \to 0$, $m_q \to m_q^{\mathrm{phys}}$, finite V investigated.

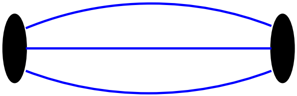


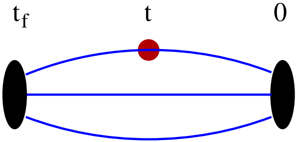
Compute $\langle x_i \rangle$ w.r.t the distribution amplitude and cf. $\langle x_i \rangle^{as} = 1/3$.

Find N, Σ, Ξ: q_1 carries larger momentum fraction compared to (q_2, q_3) di-quark, strange quarks carry more momentum.

Some computational details

Evaluate correlation functions:





Example of baryon at rest:

LHS: $C_{2pt} = Z_B e^{-m_B t_f} + \ldots$, RHS: $C_{3pt} = Z_B^2 \langle B | \overline{q} \Gamma q | B \rangle e^{-m_B t_f} + \ldots$

Lines: quark "propagators" M_{xy}^{-1}, $M = \not{D} + m_q$, $12V \times 12V$ matrix.

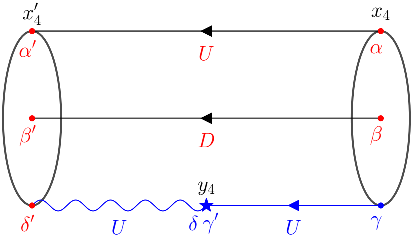
Use translational invariance $\rightarrow$ only require 12 (spin $\times$ colour) rows of M^{-1}.

Inversion: Adaptive algebraic multigrid solver (DD-α AMG) of Frommer et al [1303.1377].

C_{3pt} requires full M^{-1} between current and sink.

Standard approach involves fixing B and $\vec{p}_f \rightarrow$ computationally expensive to study baryon octet.

Some computational details



Compute M^{-1} (wavy line) stochastically $\rightarrow$ introduces additional noise.

Diagram factorises into two parts, leave spin indices open.

$$C_{3pt}(\mathbf{p}', \mathbf{q}, x_4', y_4, x_4)^{\alpha'\,\alpha\,\beta'\,\beta\,\delta'\,\delta\,\gamma'\,\gamma}_{U\,D\,U\,U} = \frac{1}{N_{\rm sto}} \sum_{n=1}^{N_{\rm sto}} \sum_{k=1}^{3} \left(S_{UD}(\mathbf{p}', x_4', x_4)^{\alpha'\,\alpha\,\beta'\,\beta\,\delta'}_{nk} \cdot I_{UU}(\mathbf{q}, y_4, x_4)^{\delta\,\gamma'\,\gamma}_{nk} \right)$$

Advantage: no additional cost for $\vec{p}_f$, moderate cost for different B.

However, large data!

Some computational details

Use HDF cloud and on JUST in /largedata/ + archive.

⋆ Typical size of data $\sim$ 50 TB.

⋆ Store in Jülich.

⋆ Strip data for current analysis on 2 virtual machines (16 CPUs with 32 GB each).

⋆ Transfer to Regensburg for analysis (>2 PB local storage).

Summary

- Lattice QCD input is essential to achieve the full potential of many current and future experiments searching for hints of beyond the Standard Model interactions.
- Certain limits need to be taken in Lattice QCD:
  $m_q \to m_q^{\mathbf{phys}}$, $V = a^4 N_t N_s^3 \to \infty$, $a \to 0$.
- Full exploration of these limits is achieved by employing CLS ensembles.
- In particular, the quark mass dependence: simulate both $m_{u,d} = m_s$ and $m_{u,d} \neq m_s$.
- Many results for the baryon octet: mass spectrum, sigma terms, axial charges, scalar and tensor charges, quark momentum fractions, axial form factors, electromagnetic form factors, distribution amplitudes.
- Determine the size of SU(3) flavour breaking $\to$ test assumptions made in phenomenological analyses.
- Investigating flavour symmetric theory $\to$ determination of ChPT LECs.