Machine learning applications in convective turbulence

Jörg Schumacher
TU Ilmenau, Germany &
Tandon School of Engineering, New York University, USA

joint work with

Dimitrios Giannakis (NYU New York)
Kartik P. Iyer (NYU New York)
Dmitry Krasnov (TU Ilmenau)
Ambrish Pandey (NYU Abu Dhabi)
Sandeep Pandey (Veoneer Germany GmbH)
Janet D. Scheel (Occidental College)
Katepalli R. Sreenivasan (NYU New York)

Outline

- Large-scale flow patterns or turbulent superstructures in convective turbulence

  Example 1: Large-scale flow and turbulent transport in slender cells
  Example 2: Large-scale flow and turbulent transport in extended domains

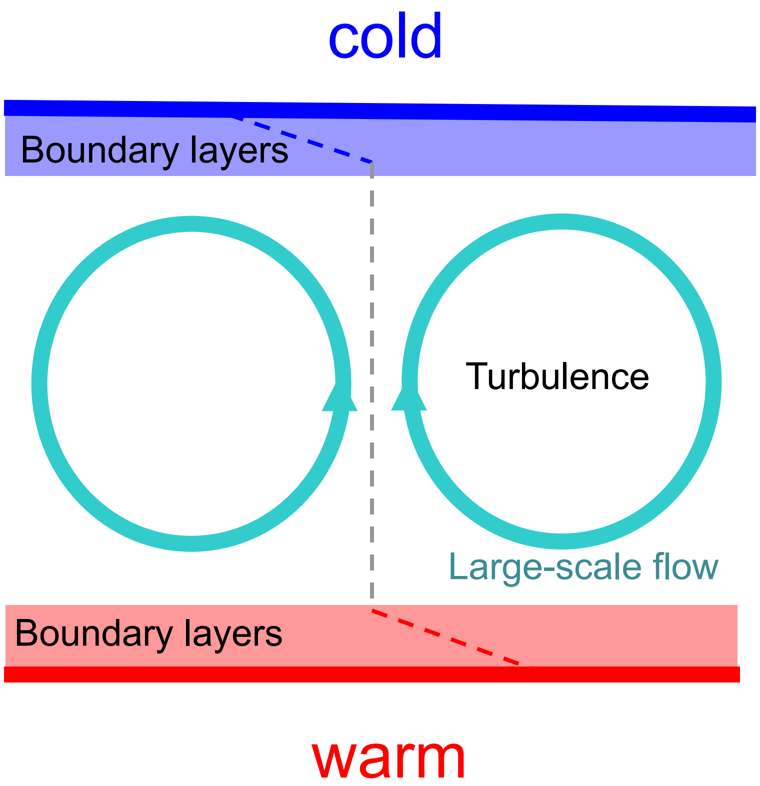
- Diffusion maps to reconstruct the large-scale flow in convective turbulence

- U-net to analyse turbulent heat transfer due to large-scale flow

- Reservoir computing to predict large-scale flow dynamics

Paradigm of convective turbulence

Chillà & JS, Eur. Phys. J. E 2012



- Input

  Temperature difference Ra
  Properties of working fluid Pr
  Geometry Γ

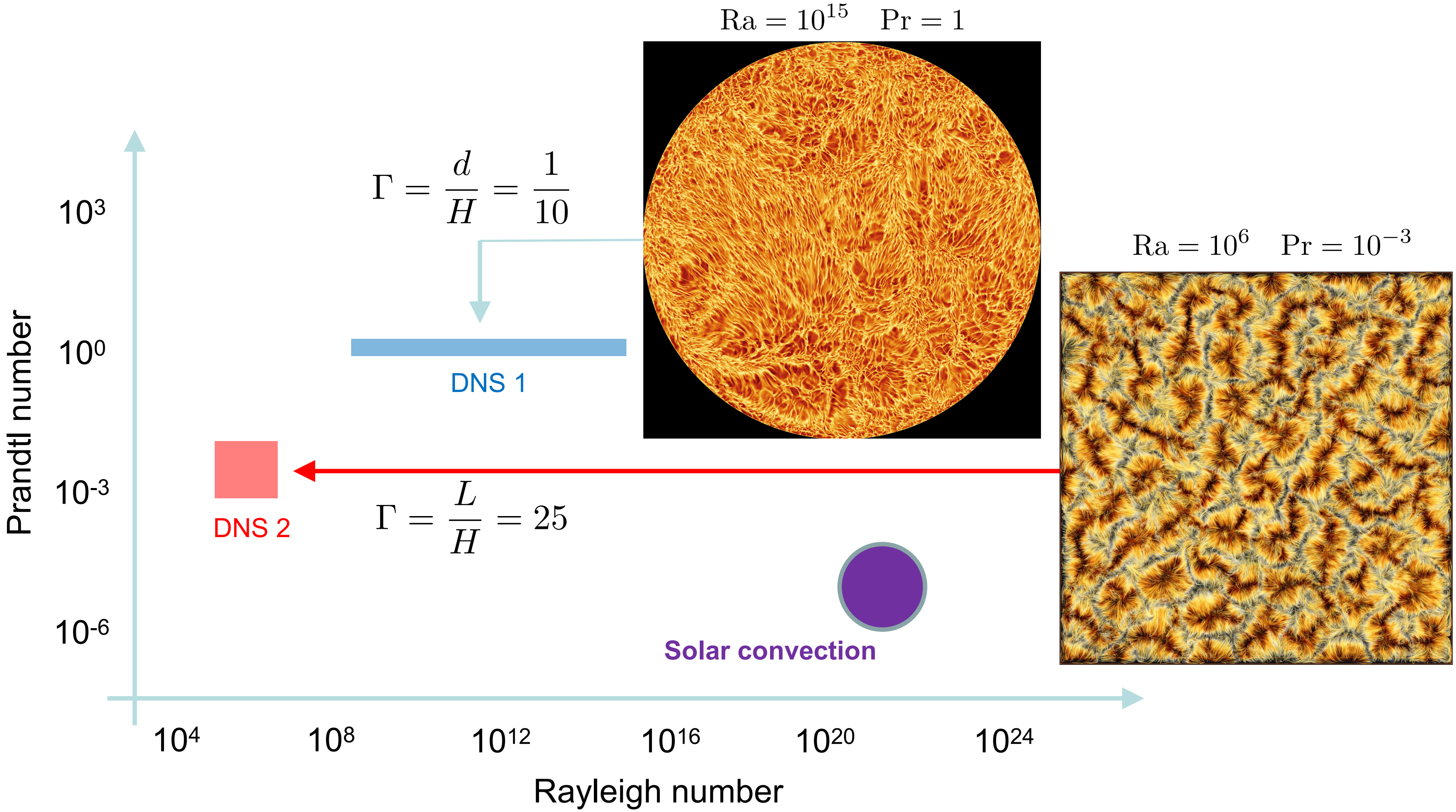
- Response

  Heat transfer Nu(Ra,Pr,Γ)
  Momentum transfer Re(Ra,Pr,Γ)

How much heat is transported from the bottom to the top?

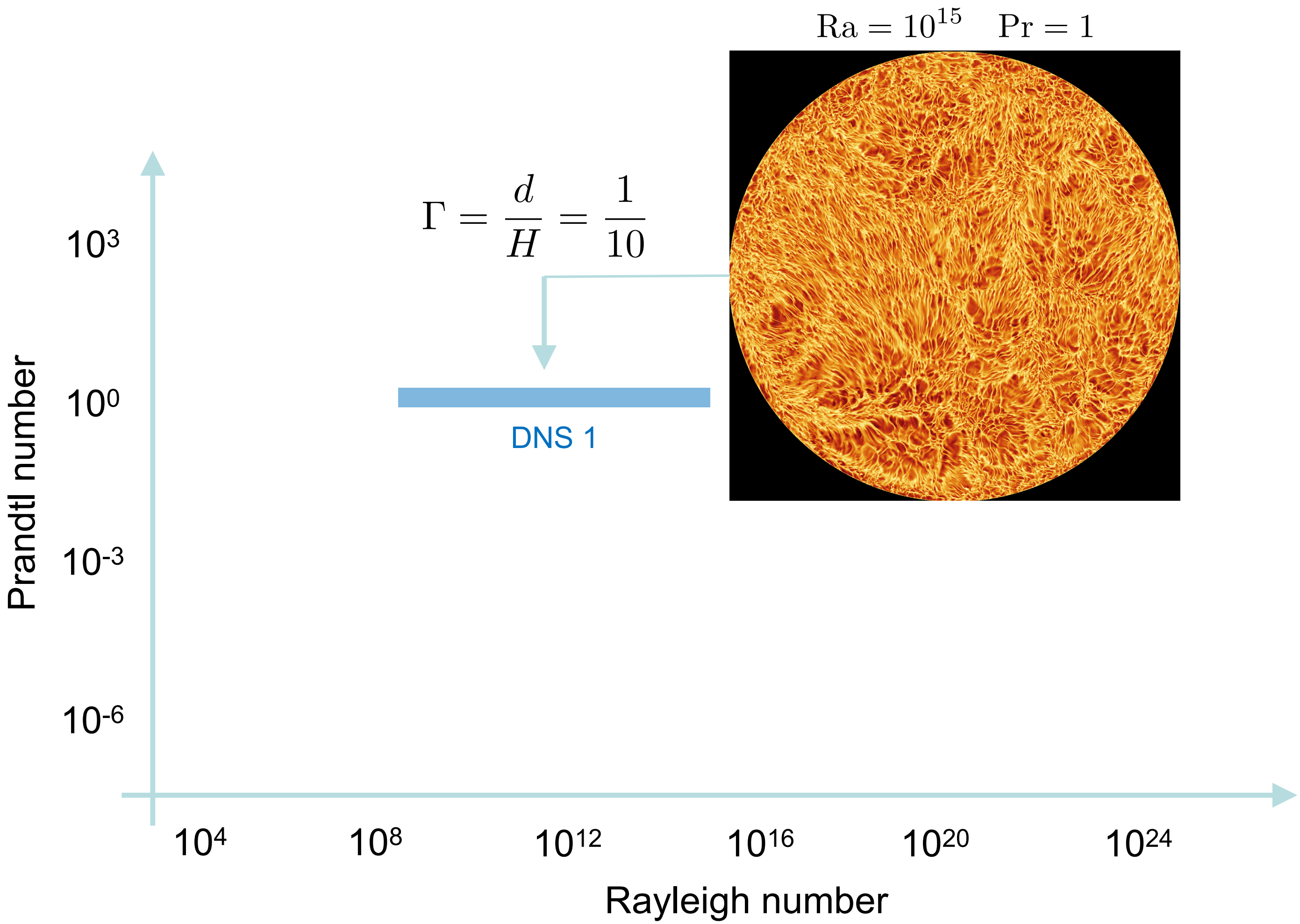
Does convection switch into an ultimate transport regime for large Ra once the boundary layers are fully turbulent?

Pr-Ra parameter plane

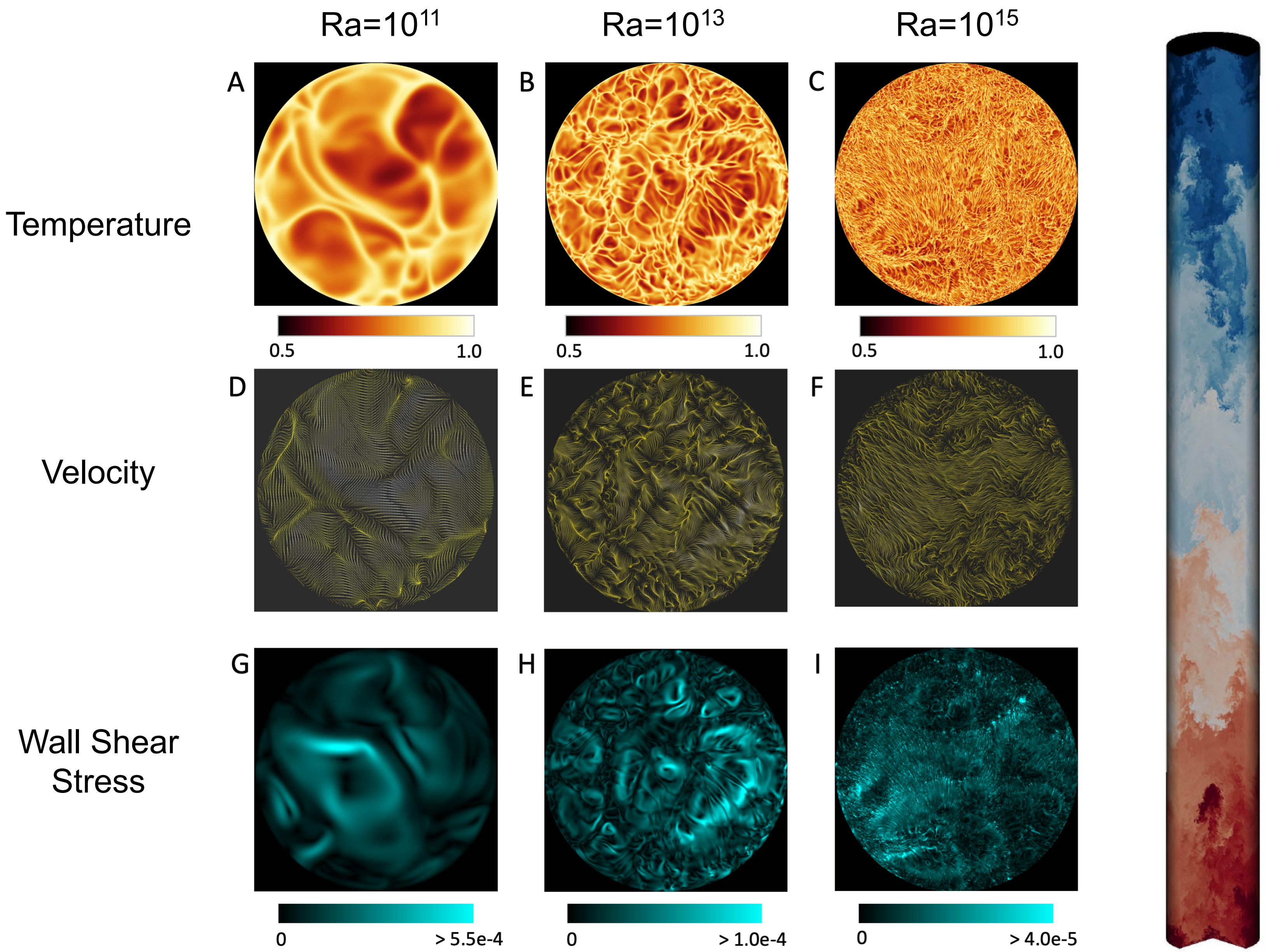


$$Ra \sim \Delta T H^3$$

Example 1: Very high Rayleigh number



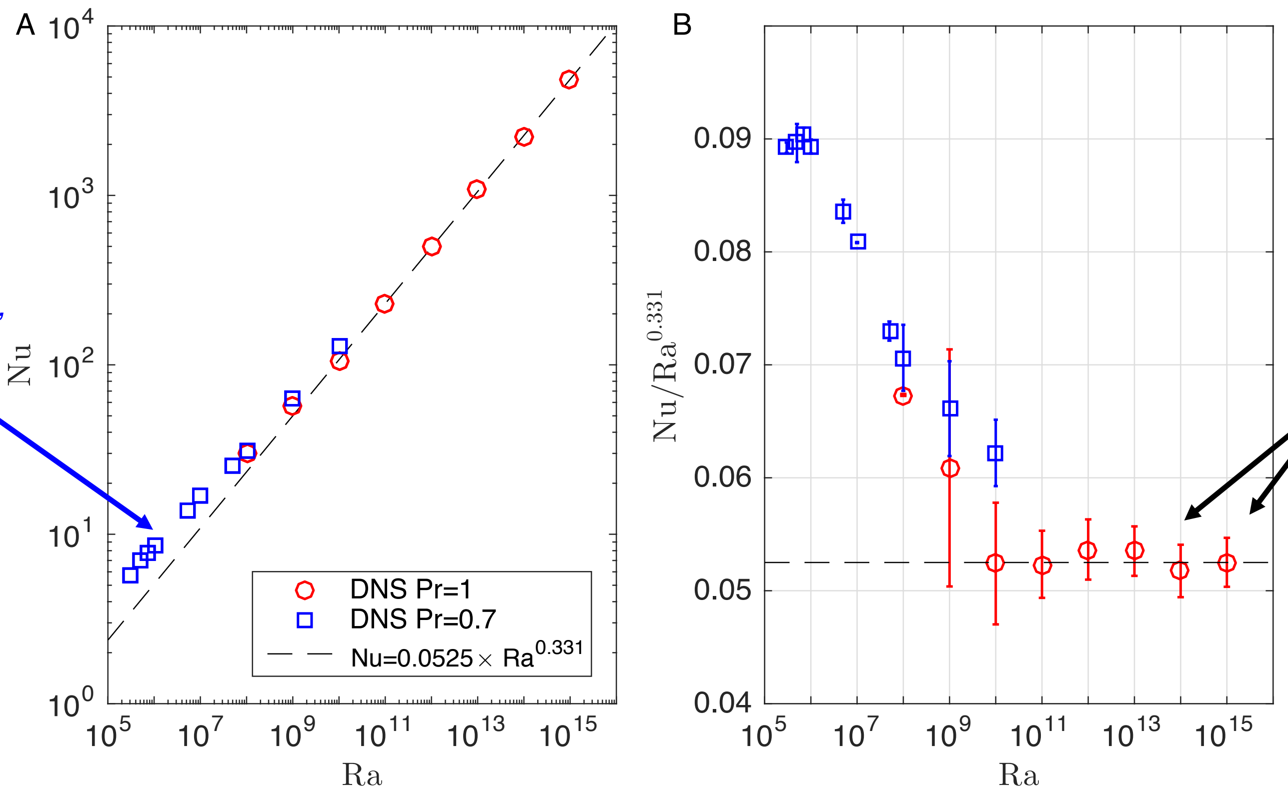
Boundary layer structure



Classical scaling of heat transfer up to Ra=10^{15}

Malkus, Proc. R. Soc. London A, 1954; Spiegel, Mécanique de la Turbulence, CNRS 1962

Scheel & JS, Phys. Rev. Fluids, 2017



Nek5000

16 million spectral elements

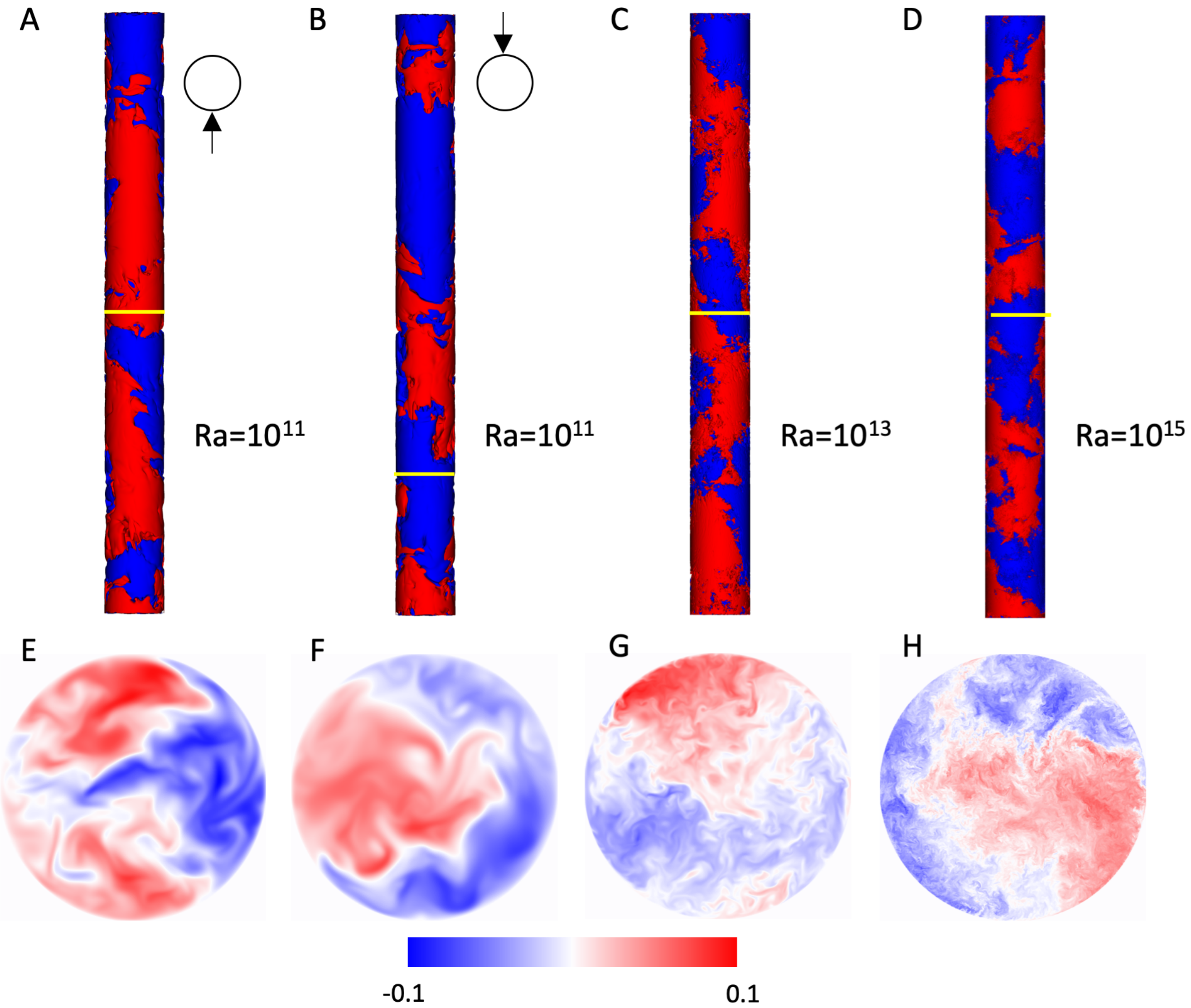
524,288 MPI tasks on BG/Q Mira (INCITE)

Classical 1/3 scaling which is based on marginally stable boundary layers

No sudden transition to an ultimate regime of convection

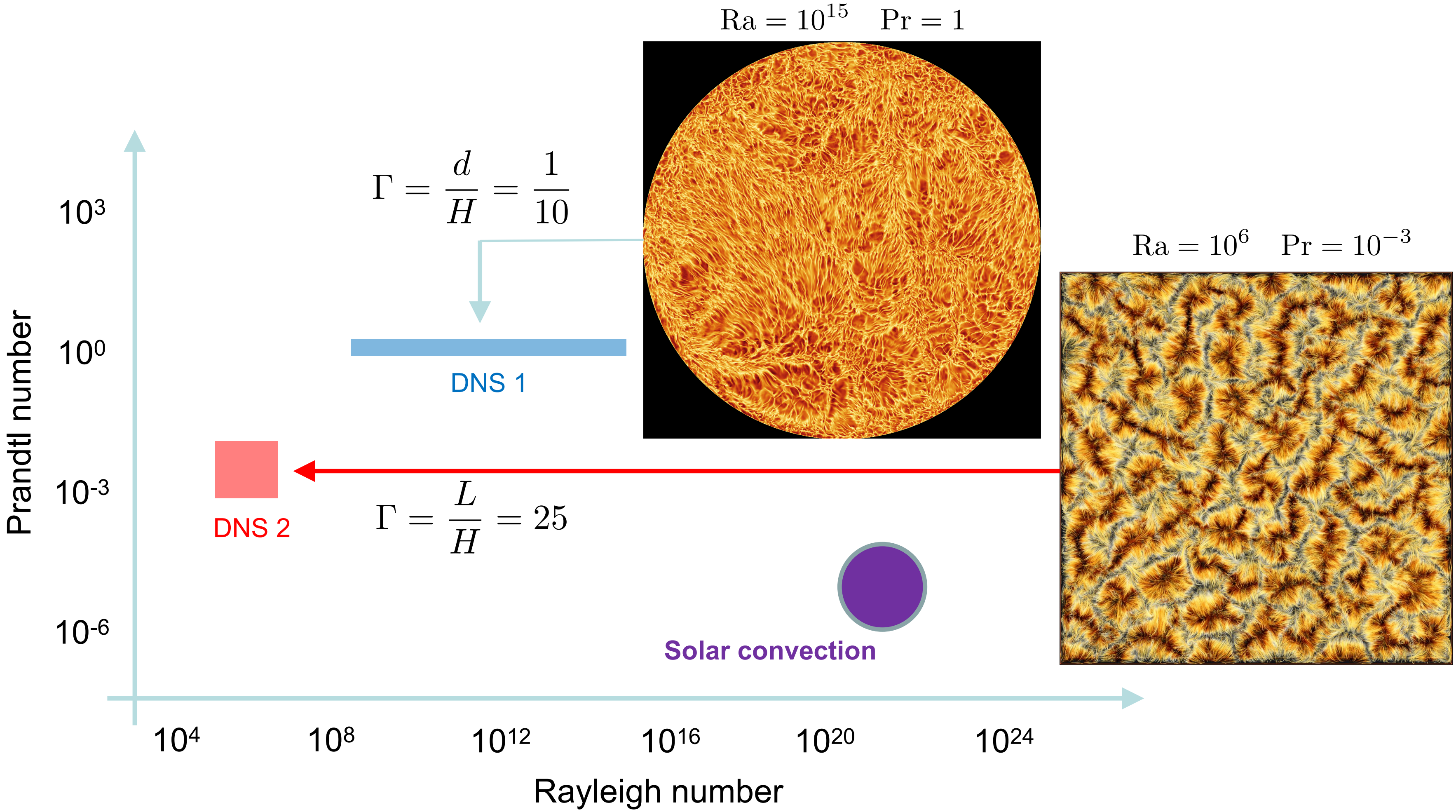
Iyer, Scheel, JS & Sreenivasan, PNAS 2020

Large-scale flow in slender cell

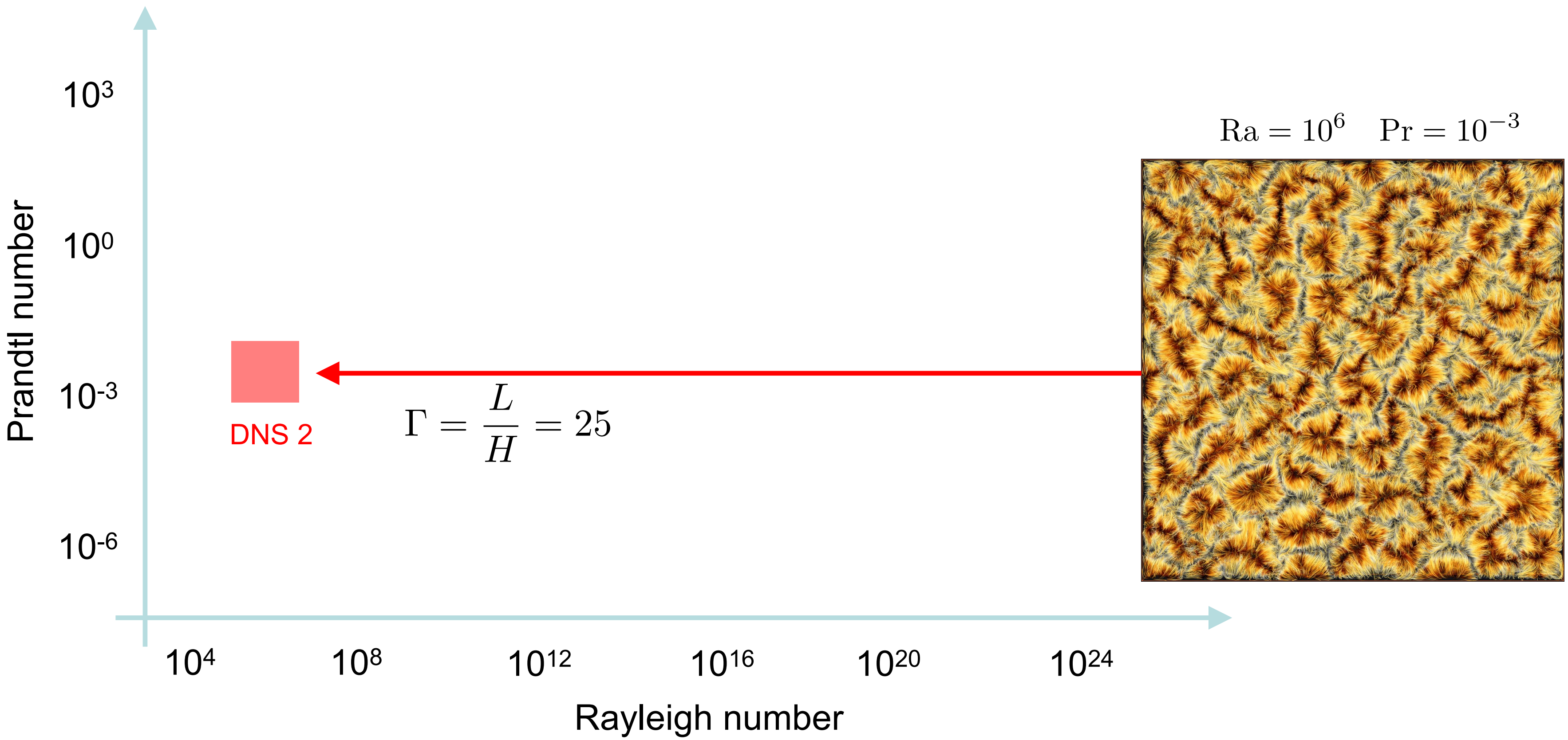


Barber pole structure affects momentum, but not heat transfer

Pr-Ra parameter plane



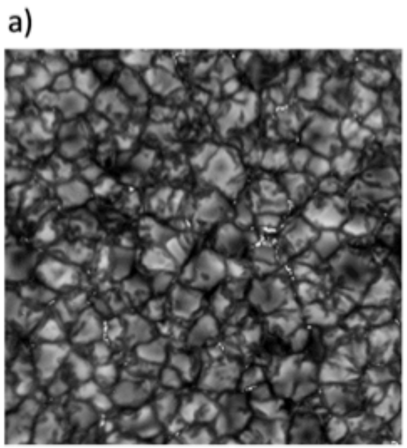
Example 2: Very low Prandtl number



Solar convection

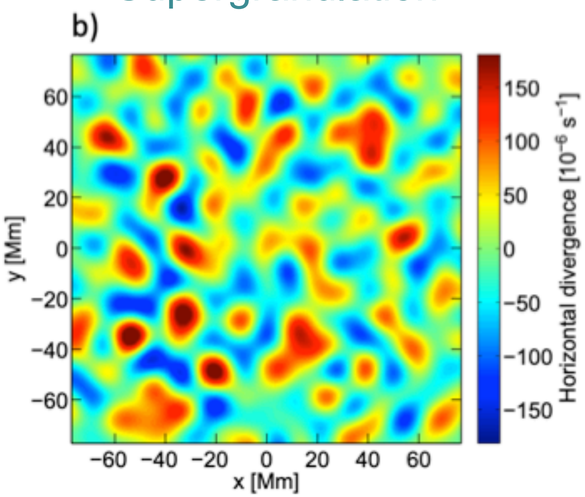
Christensen-Dalsgaard et al. Science 1996; JS & Sreenivasan, Rev. Mod. Phys., in revision, 2020

Granulation



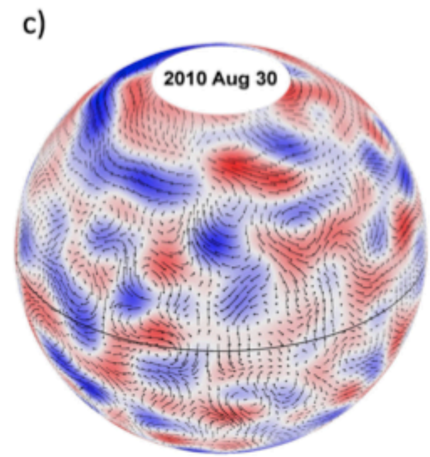
$\ell_{\mathrm{G}} \sim 10^3 \mathrm{km}$

Supergranulation

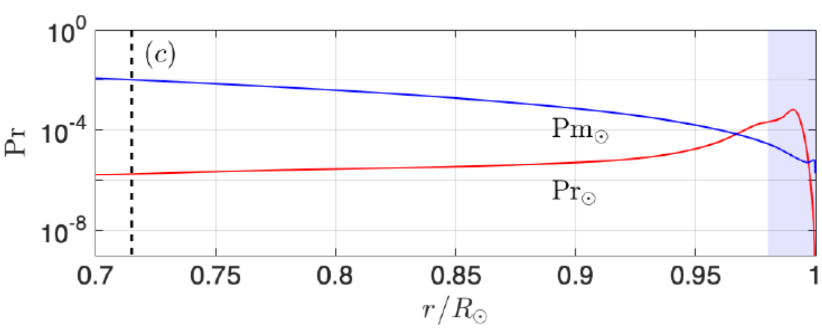


$\ell_{\mathrm{SG}} \approx 3 \times 10^4 \mathrm{km}$

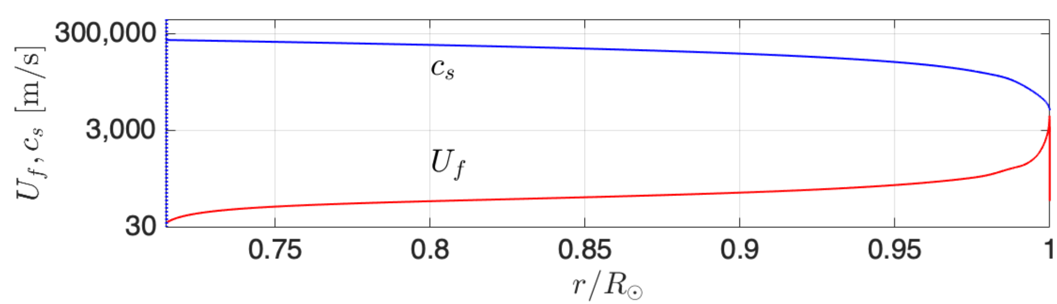
Giant cells



$\ell_{\mathrm{GC}} \sim 2 \times 10^5 \mathrm{km}$

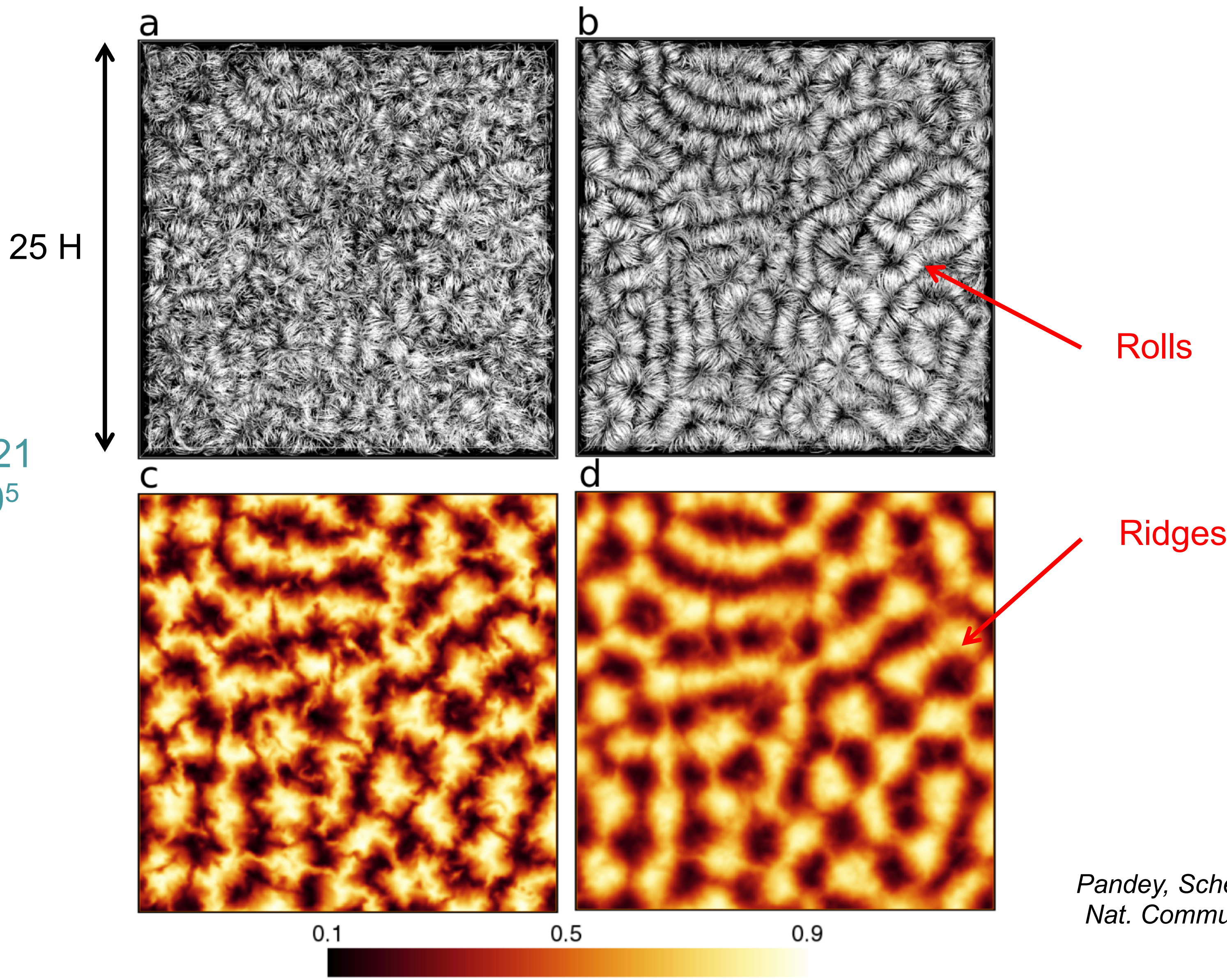


Extremely low Prandtl number $Pr = \nu/\kappa$



Compressibility important close to surface

Turbulent superstructures of convection

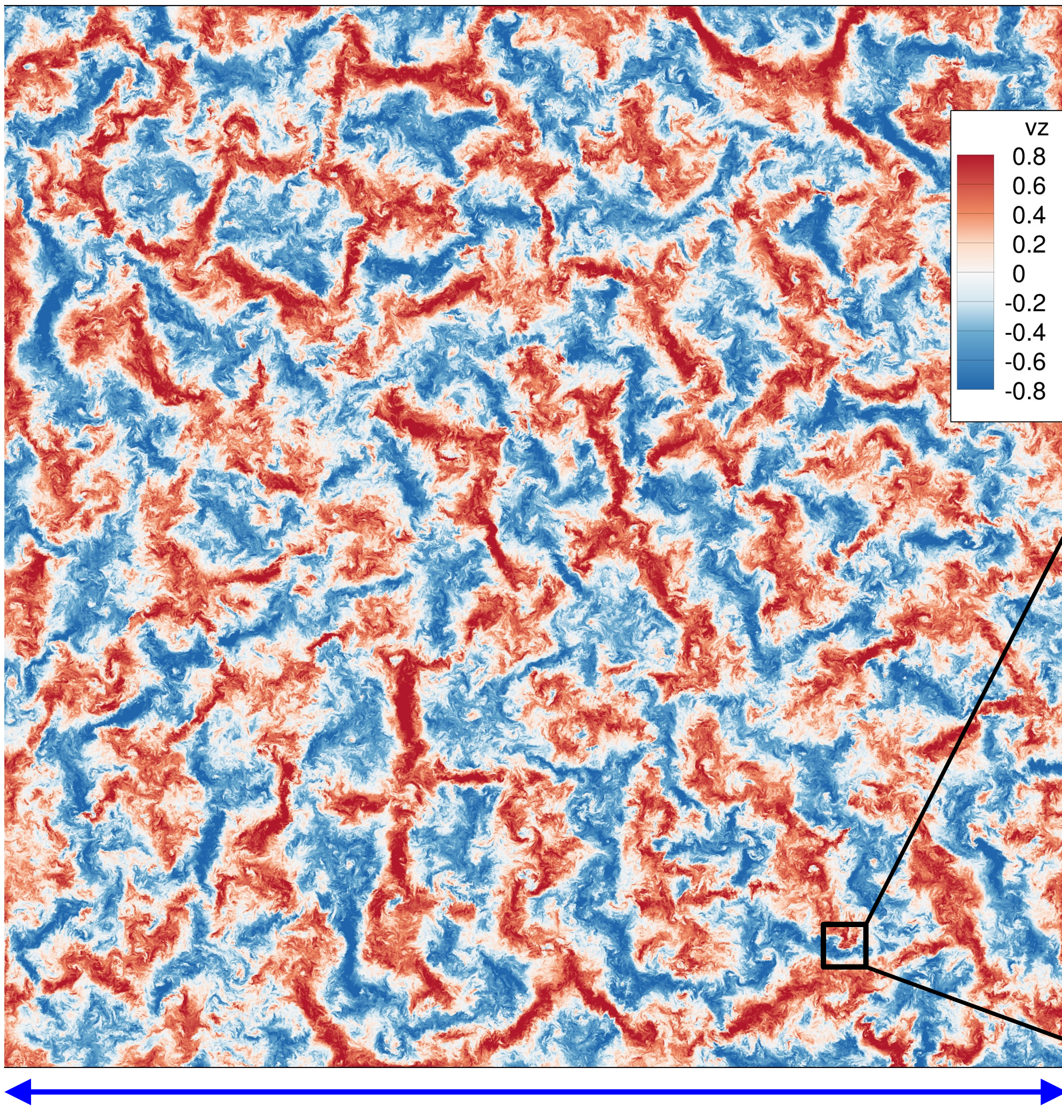


Pr=0.021
Ra=10^5

Pandey, Scheel & JS, Nat. Commun. 2018

Patterns reminiscent to those at onset of convection follow a slow dynamical evolution

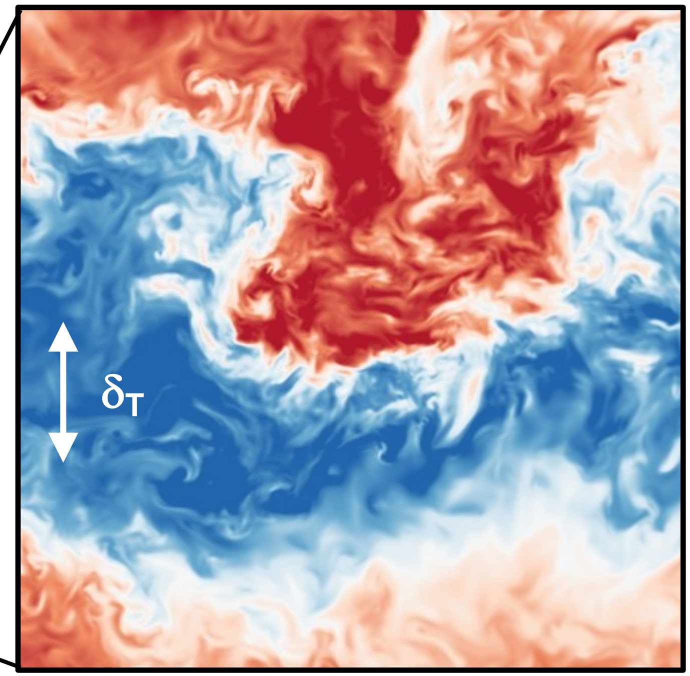
Turbulent convection at Pr=0.001



Ra=10^6 Pr=0.001   AR=25

Re=19880   Nu=2.49

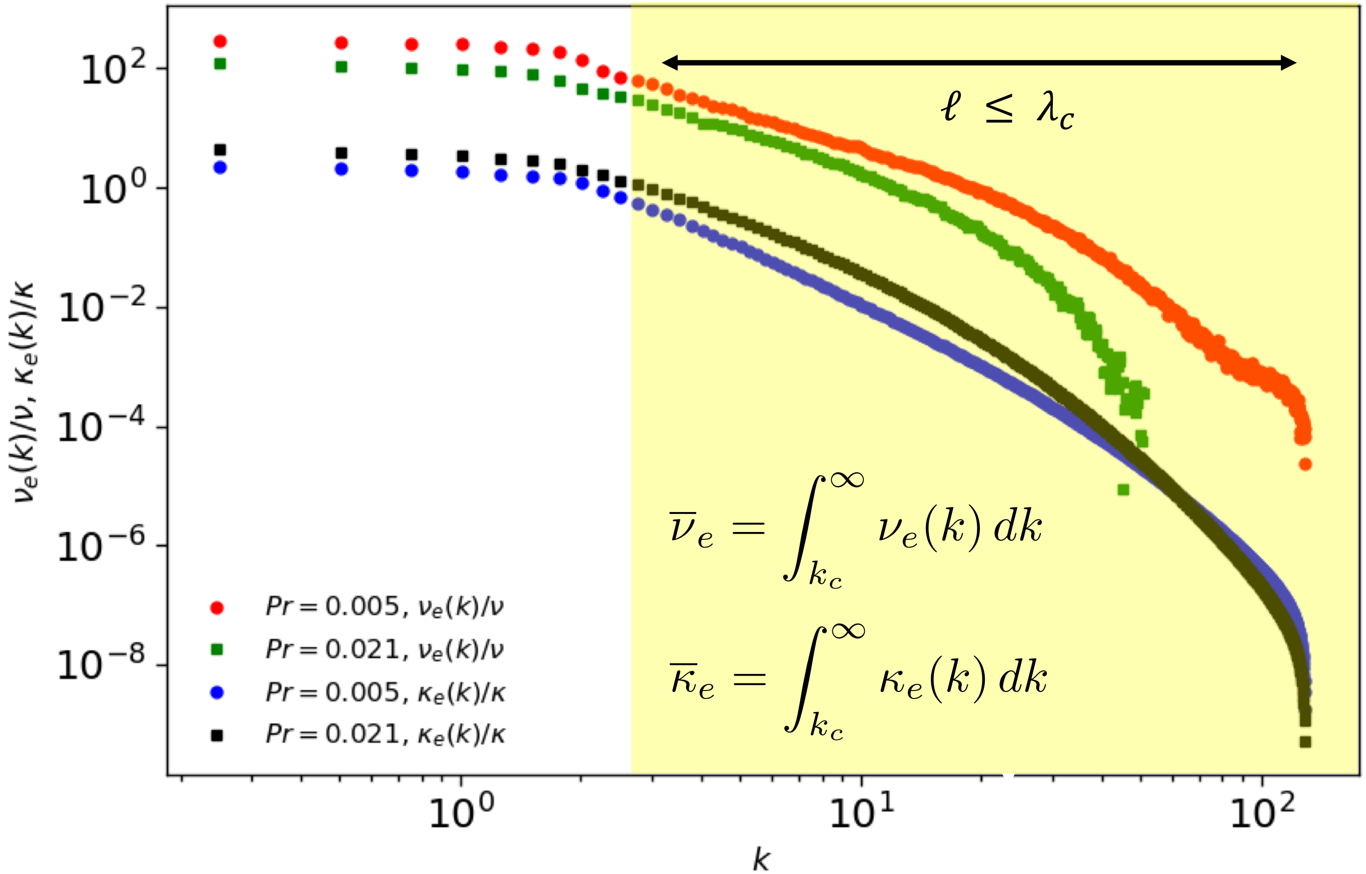
$$\delta_T = \frac{1}{2\mathrm{Nu}} \approx 1000\,\delta_v$$



12800 × 12800 × 800 = 131 billion grid points

Turbulent viscosity and diffusivity

Emran & JS, JFM 2015; Bekki, Hotta & Yokoyama, ApJ 2017



$$\mathrm{Pr}_t = \frac{\overline{\nu}_e/\nu}{\overline{\kappa}_e/\kappa}\mathrm{Pr} \approx 10^2\,\mathrm{Pr} \sim 0.5 \qquad \mathrm{Ra}_t = \frac{\nu\kappa}{\overline{\nu}_e\overline{\kappa}_e}\mathrm{Ra} \approx 10^{-2}\,\mathrm{Ra} \sim \mathrm{Ra}_c$$

Large-scale dynamics = „renormalized“ high-Prandtl-number convection at lower Rayleigh number

Machine learning

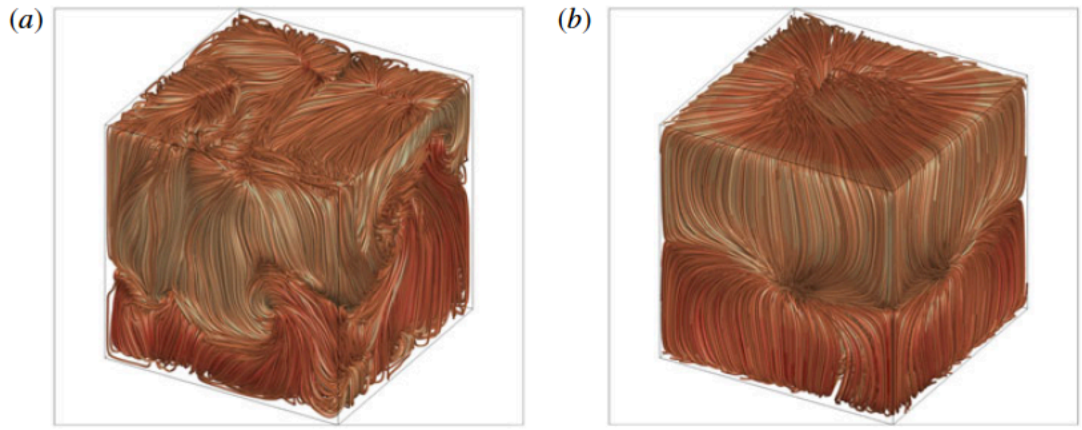
How do turbulent superstructures in convection evolve in time?
How do they contribute to global turbulent transport?

↓

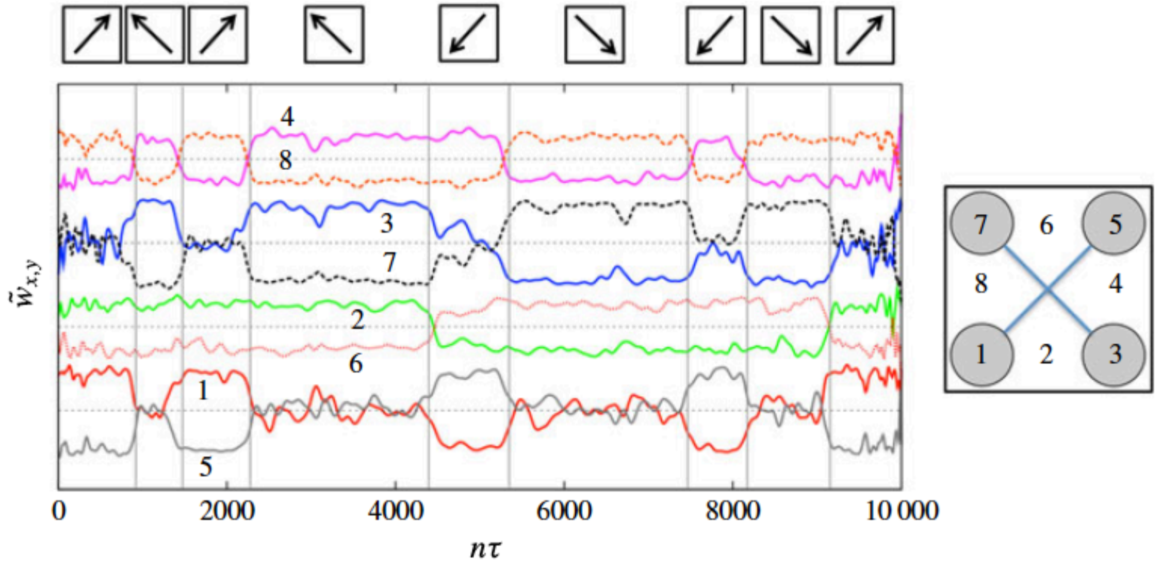
Unsupervised ML	**Supervised ML**
Lagrangian coherent sets by spectral clustering of trajectories in RBC *Schneide et al., Phys. Rev. Fluids (2018)* *Schneide et al., Phys. Rev. E (2019)*	Reduction of turbulent superstructure to a planar network by Deep Convolutional Neural Networks *Fonda et al. PNAS (2019)*
Large-scale RBC flow by Koopman eigenfunctions from diffusion maps *Giannakis et al., J. Fluid Mech. (2018)*	Learning large-scale flow dynamics and statistics in RBC flow by Reservoir Computing *Pandey & JS, Phys. Rev. Fluids, submitted (2020)*

Large-scale flow by unsupervised geometric learning

Ra=10^7
Pr=0.7



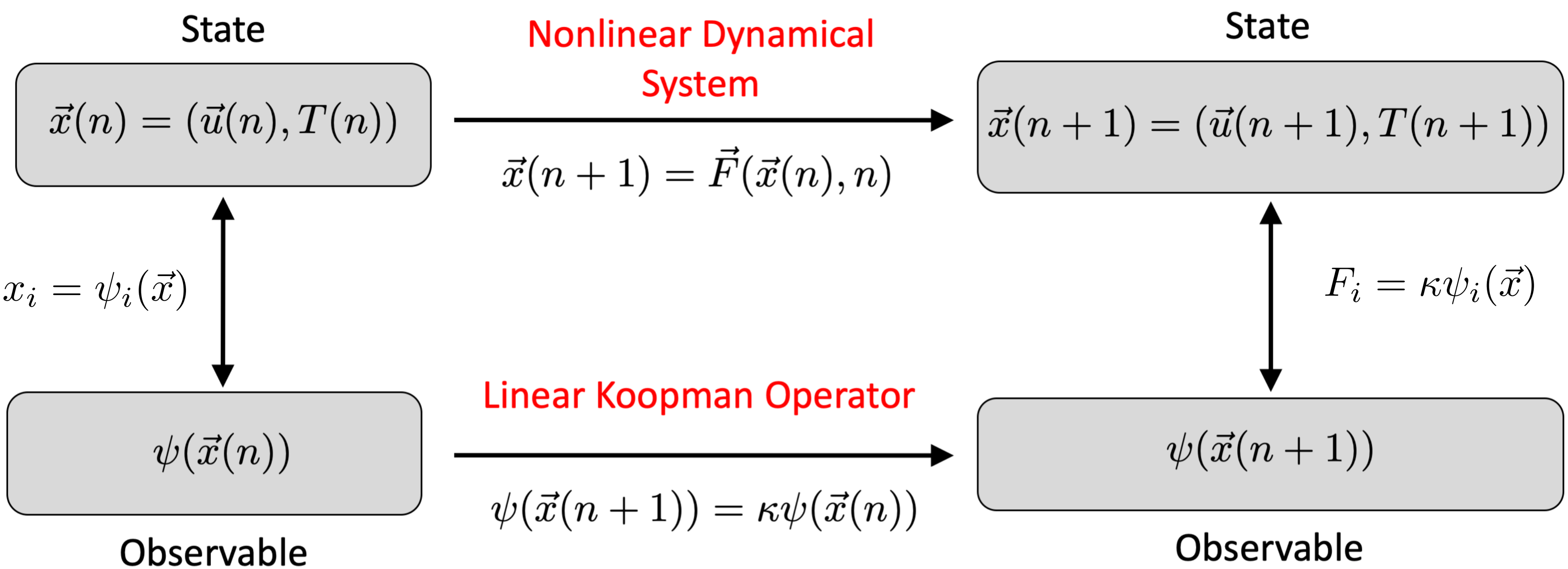
Diagonal large-scale flow



Switching between four diagonal large-scale flow states via four metastable states

Koopman operator for large-scale flow

Williams et al., J. Nonlinear Sci. 2015



$$F_i = \kappa\psi_i(\vec{x}) = \sum_{k=1}^{N_k} v_{ik}(\kappa\varphi_k)(\vec{x}) = \sum_{k=1}^{N_k} \lambda_k v_{ik}\varphi_k(\vec{x})$$

Complementary analysis method for dynamical systems

Eigenvalue problem of Koopman by construction of a data-driven basis in feature Hilbert space

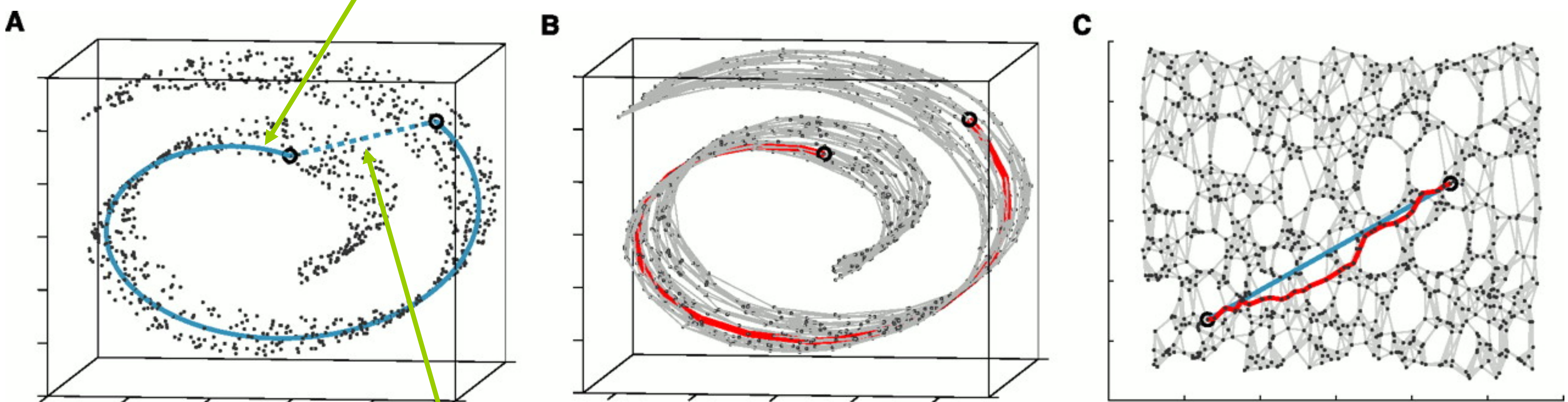
Diffusion process for manifold reconstruction

Coifman and Lafon, Appl. Comput. Harmon. Anal. 2006

What is the intrinsic geometry of the large-scale flow manifold?

Example of „Swiss roll“ (nonlinear 2D manifold in a 3D space)

Geodesic path

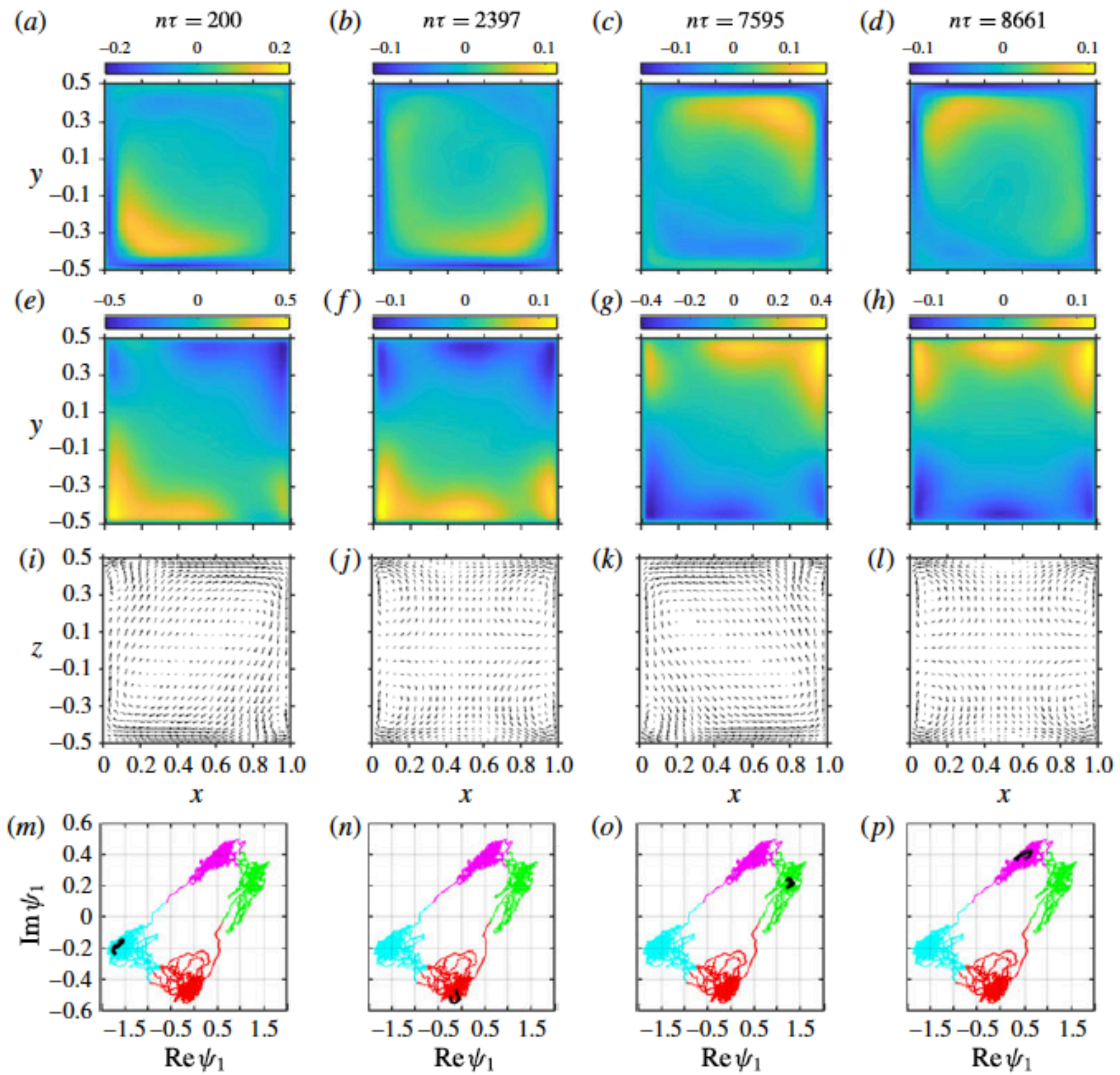


Euclidean distance fails to describe intrinsic geometry

Diffusion maps = Diffusion kernel connect points on a manifold by a diffusion process

$$K_{ij}(\mathbf{u}_i, \mathbf{u}_j) = \exp\left(-\frac{1}{Q}\sum_{q=0}^{Q-1}\frac{\|\mathbf{u}_{i-q} - \mathbf{u}_{j-q}\|^2}{\varepsilon}\right)$$

Large-scale flow and clusters in phase space



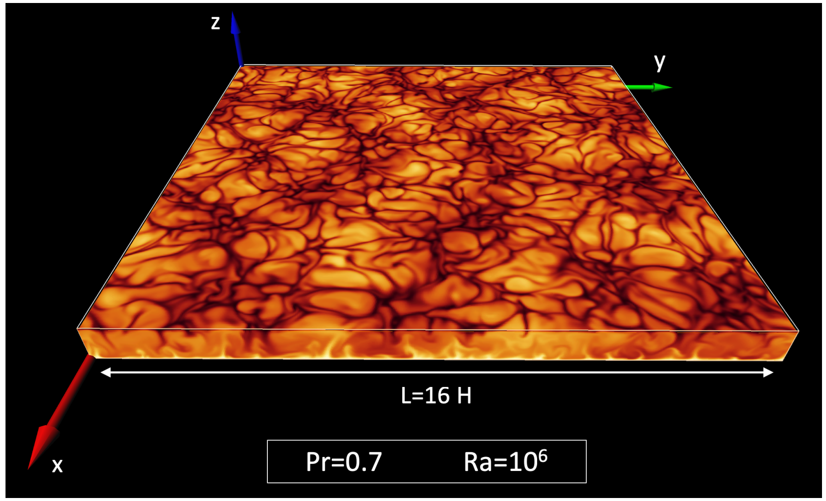
Vertical velocity near top

Vertical velocity in midplane

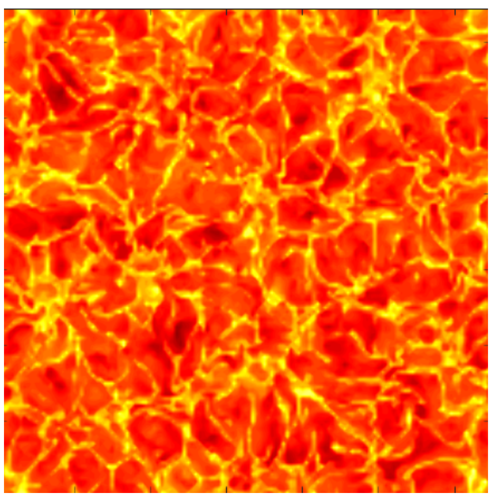
Velocity in vertical plane

Reconstructed velocity field from primary Koopman eigenfunctions

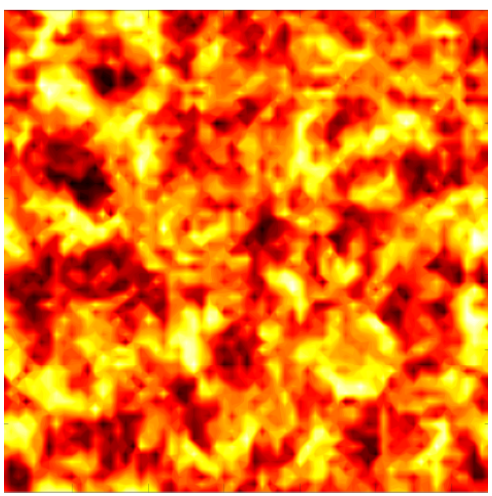
Application to RBC at large aspect ratio



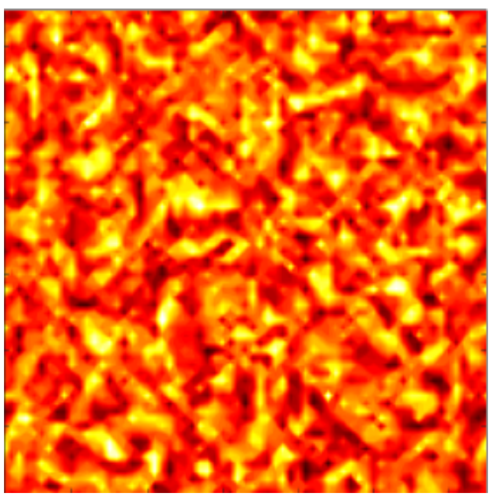
Original



Eigenfunction 2

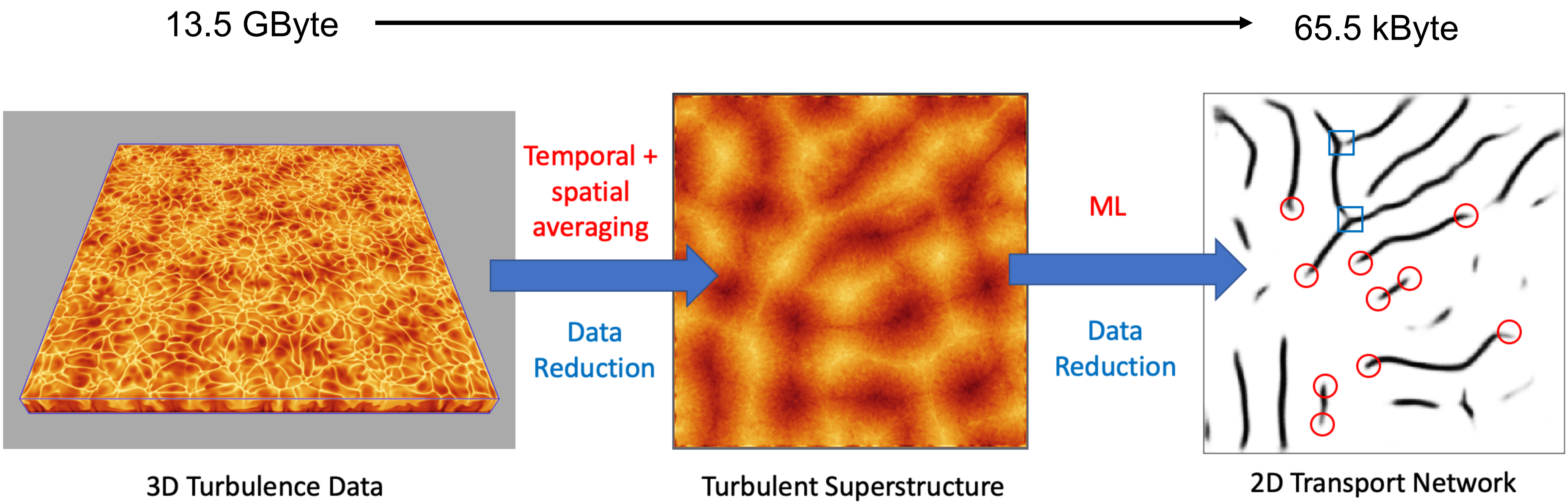


Eigenfunction 95



Koopman eigenfunctions = spatial patterns temperature at different scale

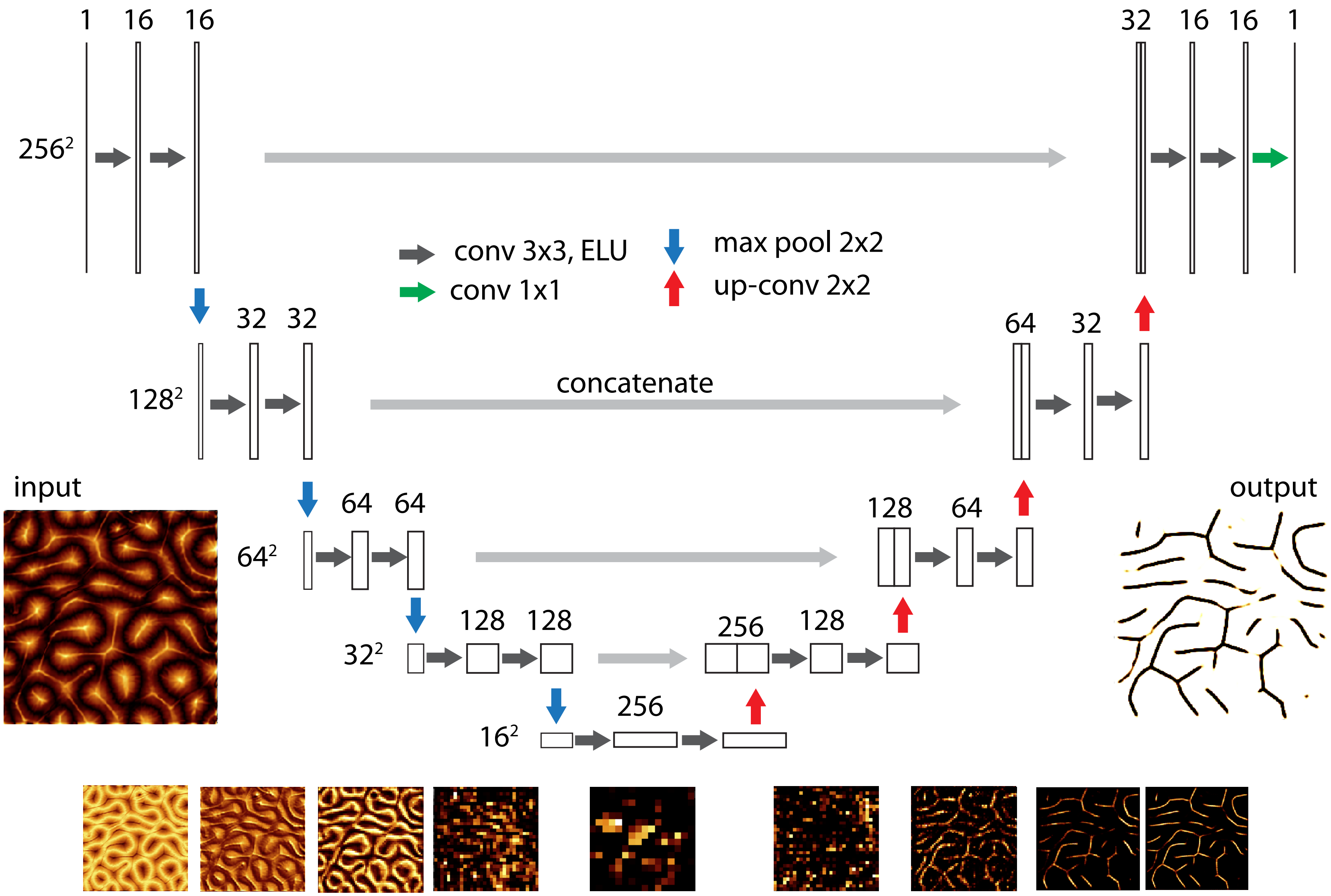
Deep learning in turbulent convection



From 3D turbulent convection to a convective heat transfer network

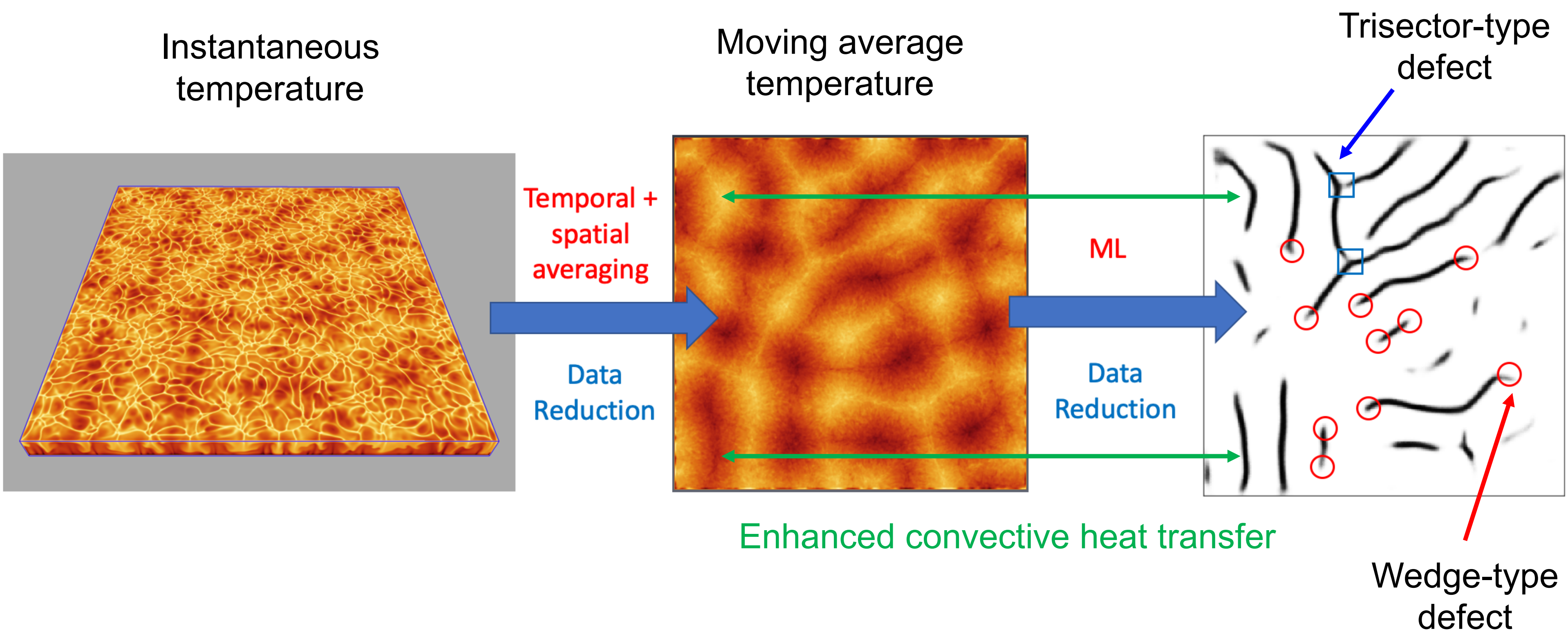
U-shaped deep neural network

Ronneberger et al., LNCS, 2015



Slow evolution and role in turbulent heat transfer

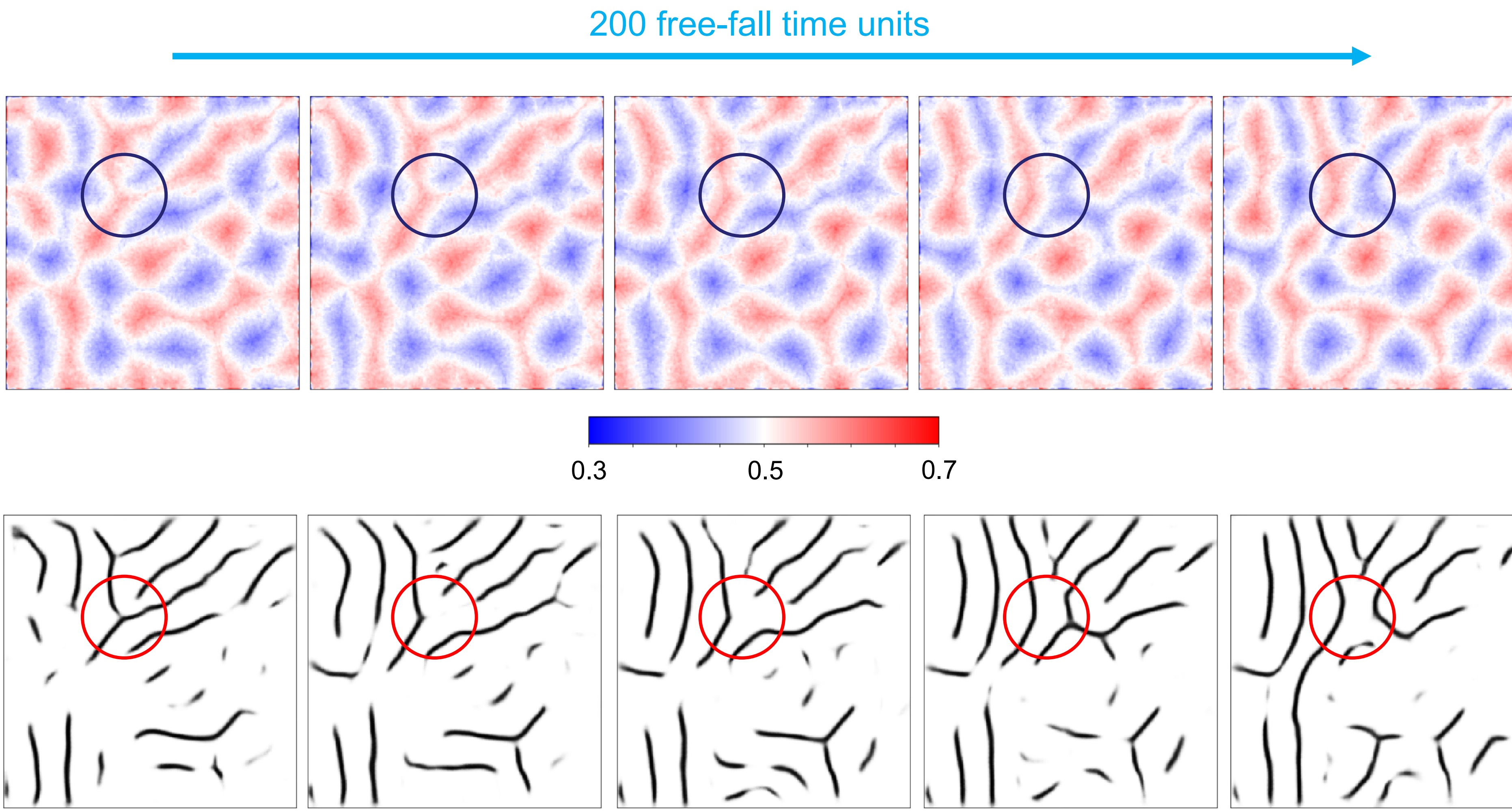
Fonda, Pandey, JS & Sreenivasan, PNAS 2019



Slowly evolving planar network with changing defect topology over few hundred convective time units

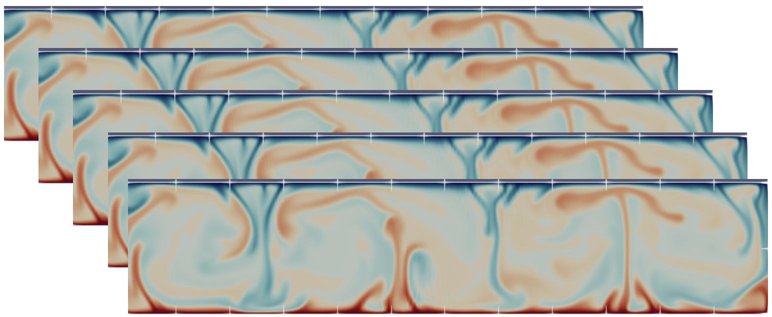
Quantification of heat transfer due to network: remains intact as a contributer to turbulent transport with increasing Rayleigh number

Slow large-scale dynamics



Slow dynamical evolution with defect point generation and annihilation

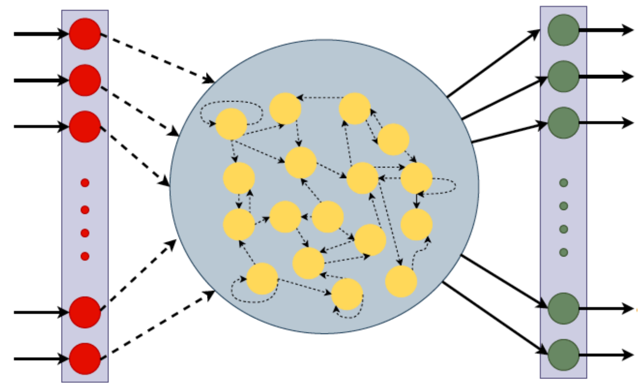
Deep learning of turbulent convection



2D turbulence data
$Ra=10^7$ & $Pr=7$

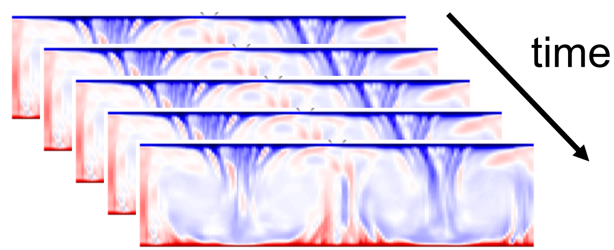
POD analysis

Step 1:
Data
Reduction



Step2:
Neural Network
Run

Prediction of
large-scale
dynamics

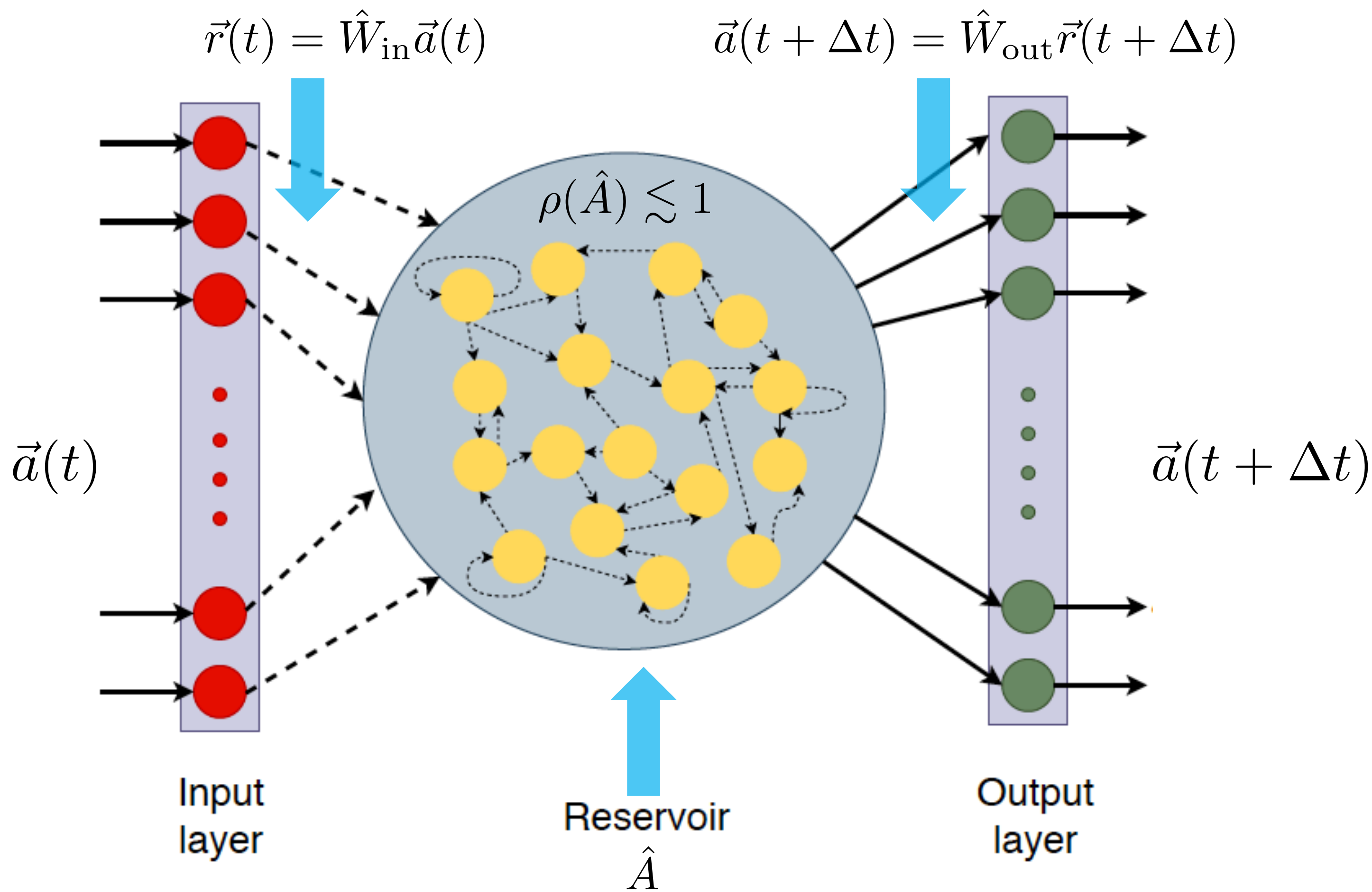


Equation-free and data-driven reduced-order model for the dynamics of turbulent superstructures

Recurrent Neural Networks = Neural Networks with a time memory
→ Long short-term memory network
→ Reservoir computing model

Reservoir computing – Training phase

Jaeger & Haas, Science 2004, Pathak et al., Chaos, 2017

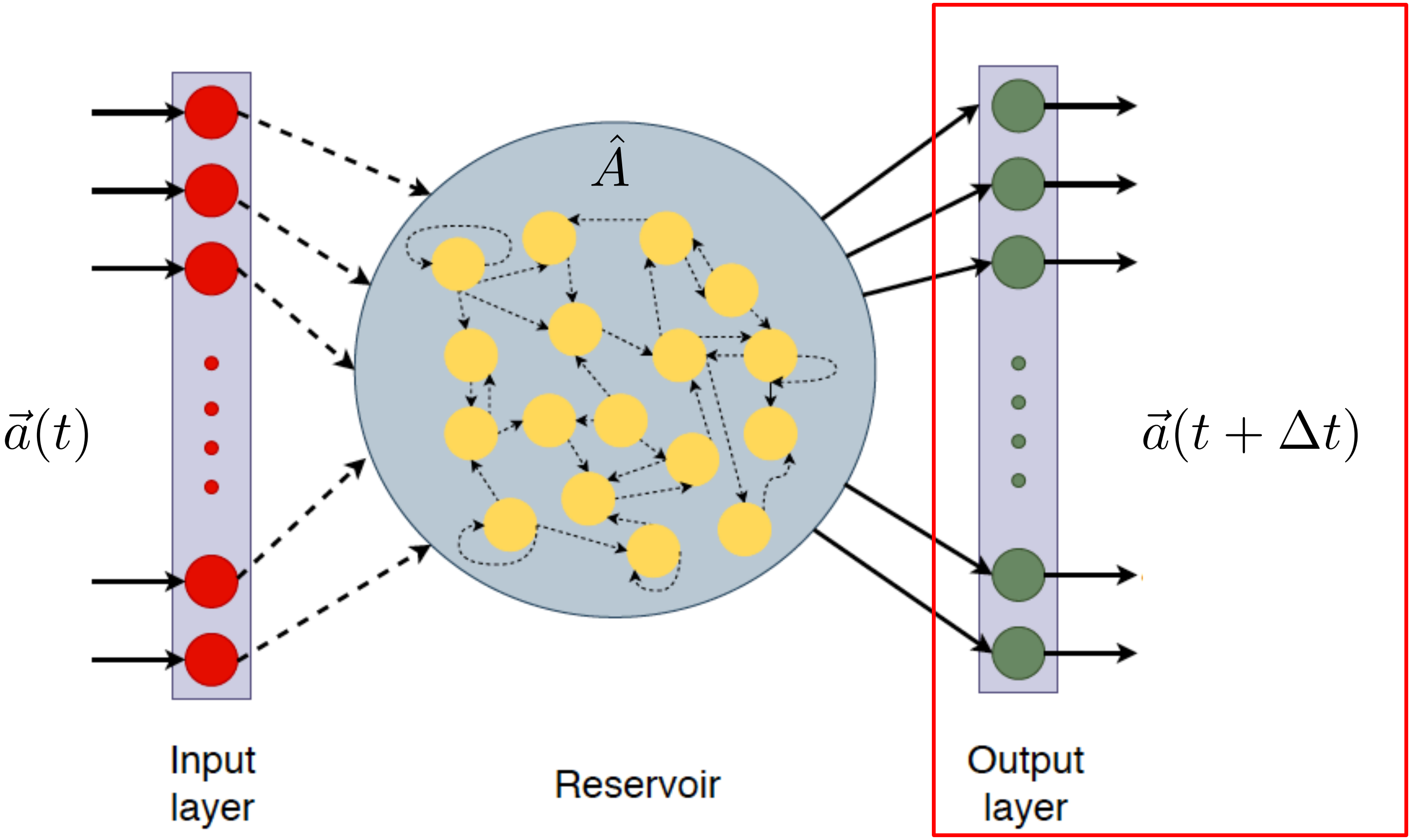


$$\vec{r}(t+\Delta t) = (1-\alpha)\vec{r}(t) + \alpha \tanh[\hat{A}\vec{r}(t) + \hat{W}_{\rm in}\vec{a}(t)]$$

$\hat{W}_{\rm in}, \hat{A}, \hat{W}_{\rm out}$ are sparse random matrices initially with $\hat{A} \in \mathbb{R}^{N_r \times N_r}$

Reservoir computing – Optimization

Jaeger & Haas, Science 2004, Pathak et al., Chaos, 2017



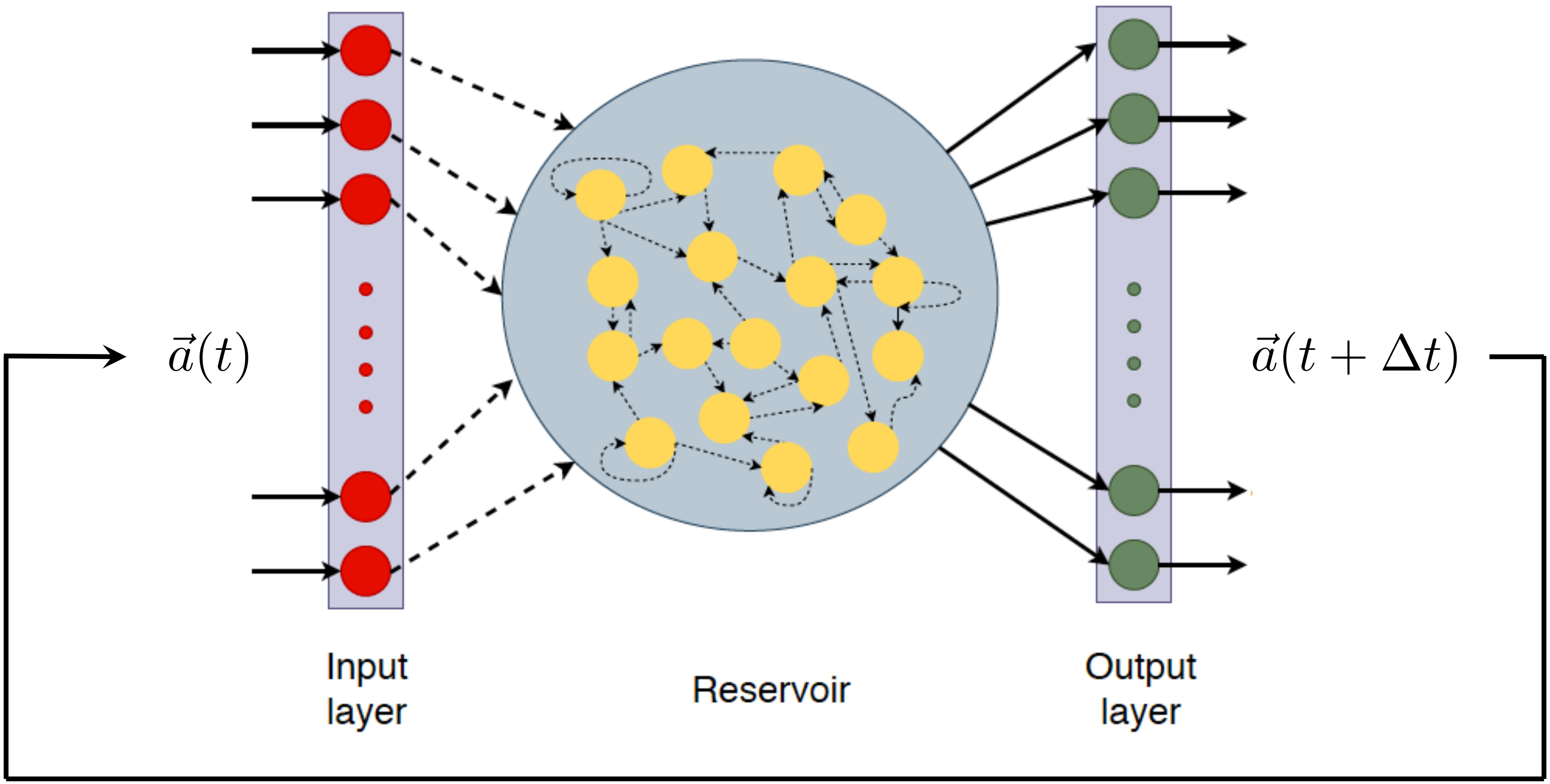
Optimization at output layer only!

No backpropagation !

$\hat{W}^*_{\mathrm{out}}$

Reservoir computing – Prediction Phase

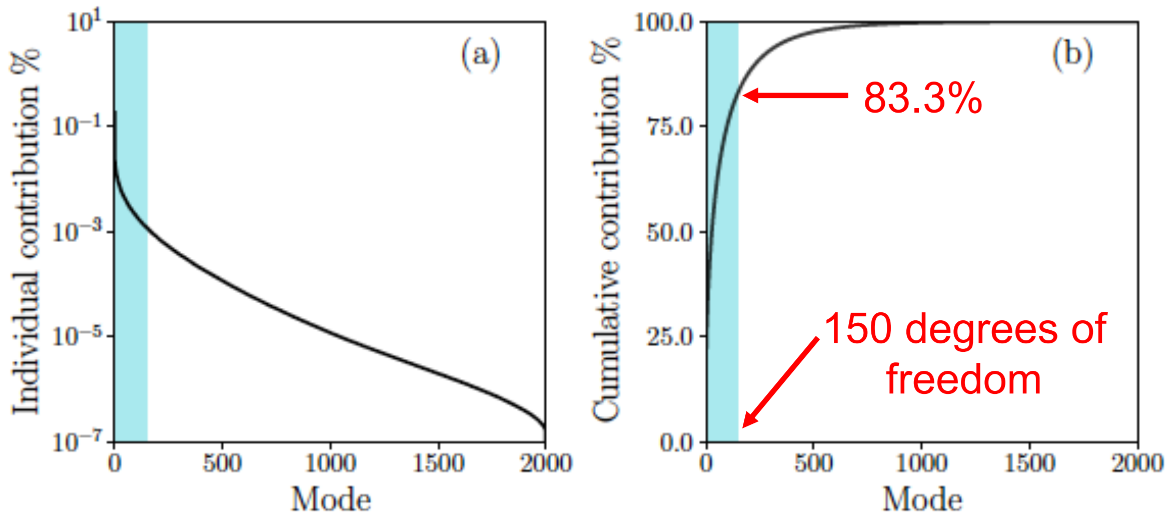
Jaeger & Haas, Science 2004, Pathak et al., Chaos, 2017



$$\vec{r}(t+\Delta t) = (1-\alpha)\vec{r}(t) + \alpha \tanh[\hat{A}\vec{r}(t) + \hat{W}_{\rm in}\hat{W}^*_{\rm out}\vec{r}(t)]$$

Step 1: POD snapshot analysis

Sirovich & Park, Phys. Fluids 1990; Bailon-Cuba & JS, Phys. Fluids 2011



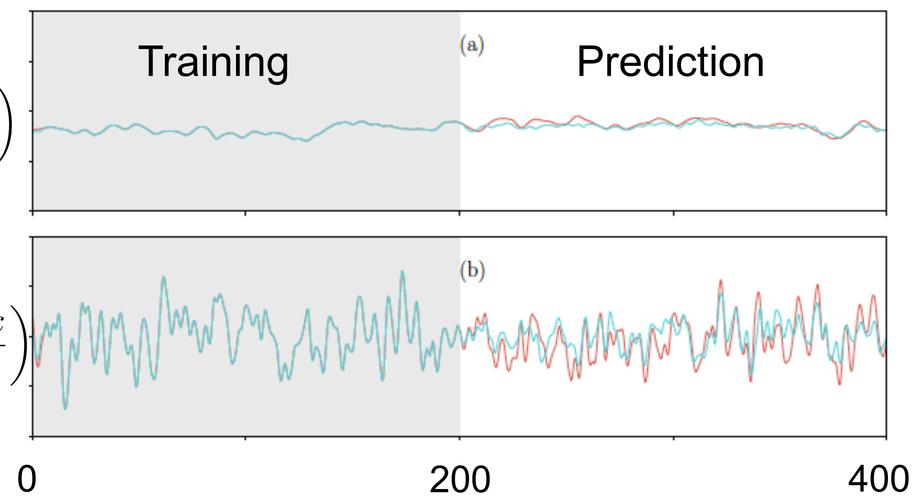
2D RBC turbulence data

Ra=10^7 & Pr=7

$$\vec{v} = (\vec{u}, \theta = T - T_{\rm lin}(z)) \qquad \vec{v}(x,z,t) = \sum_{j=1}^{M} \vec{v}_j(x,z,t) = \sum_{j=1}^{M} \sum_{n=-N/2}^{N/2} a_j^n(t) \exp\left(i\frac{2\pi n x}{L}\right) \vec{\phi}_j(z)$$

$$a_1(x,z,t) = \sum_n a_1^n(t) \exp\left(i\frac{2\pi n x}{L}\right)$$

$$a_{50}(x,z,t) = \sum_n a_{50}^n(t) \exp\left(i\frac{2\pi n x}{L}\right)$$



$N_r = 2100$

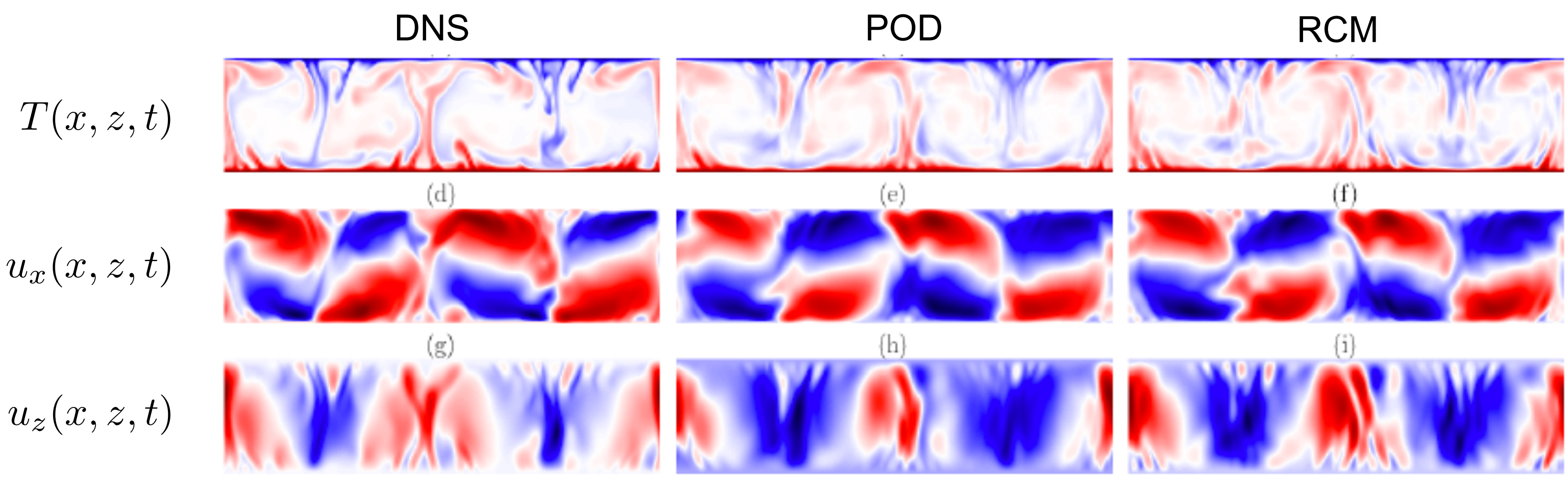
$\rho(\hat{A}) = 0.95$

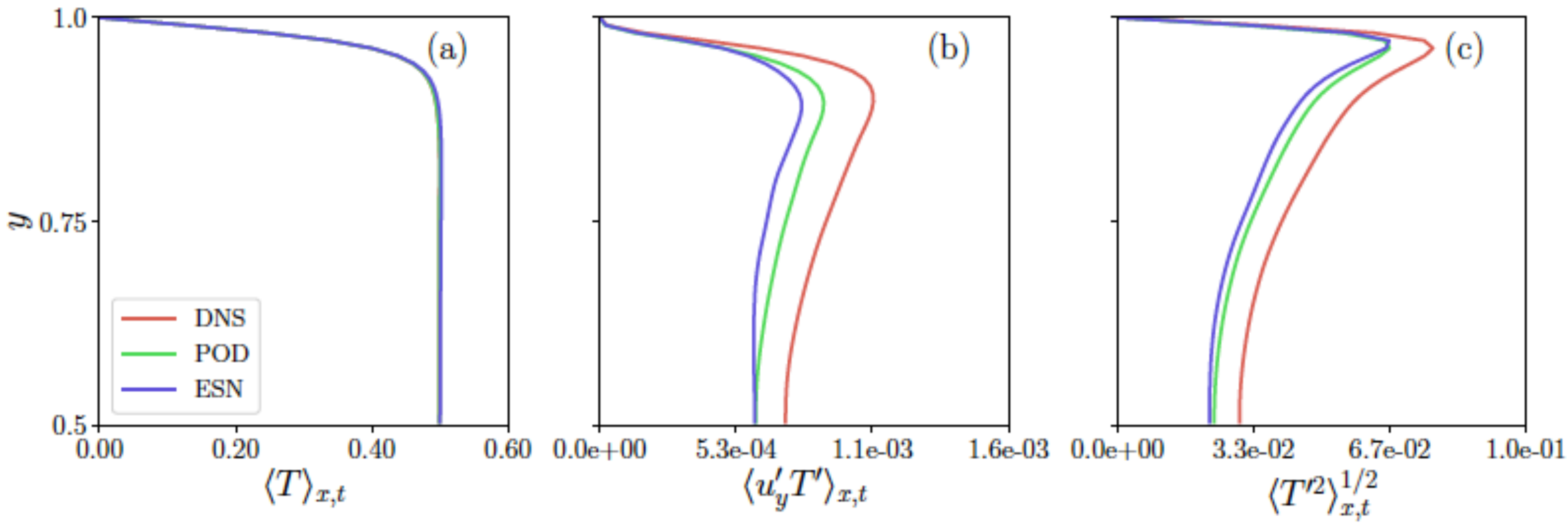
$\alpha = 0.95$

$D = 0.2$

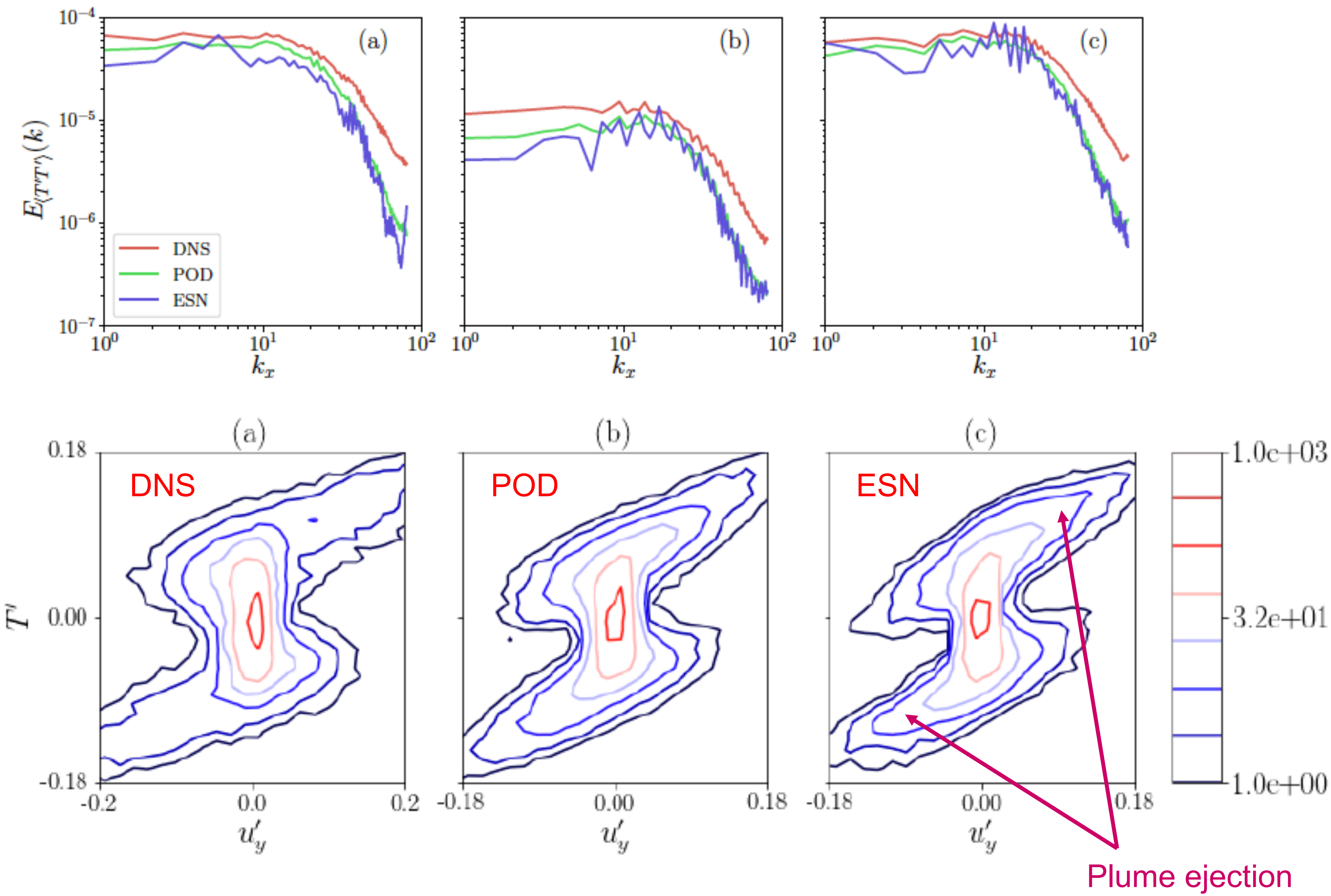
Time in free-fall units

Step 2: Dynamics from RCM





Step 2: Statistics from RCM



Summary

- Unsupervised ML in convection: Reconstruction of large-scale flow by eigenfunctions of the Koopman operator
- DL in convection: Reduction of 3d TSS to slowly evolving planar transport network by a DCNN to analyse turbulent heat transfer
- DL of convection: Reduced-order equation-free model of large-scale flow on basis of Reservoir Computing to predict large-scale dynamics and statistics

D. Giannakis, A. Kolchinskaya, D. Krasnov, JS, J. Fluid Mech. **847**, 735 (2018)
C. Schneide, A. Pandey, K. Padberg-Gehle, JS, Phys. Rev. Fluids **3**, 113501 (2018)
A. Pandey, J.D. Scheel, JS, Nat. Commun. **9**, 2118 (2018)
E. Fonda, A. Pandey, JS, K.R. Sreenivasan, PNAS **116**, 8867 (2019)
K.P. Iyer, J.D. Scheel, JS, K.R. Sreenivasan, PNAS, in press (2020)
S. Pandey, JS, Phys. Rev. Fluids, submitted (2020)

NIC
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Carl Zeiss Stiftung
GCS Gauss Centre for Supercomputing
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