The Columbia Plot of the QCD Phase Transition

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Introduction

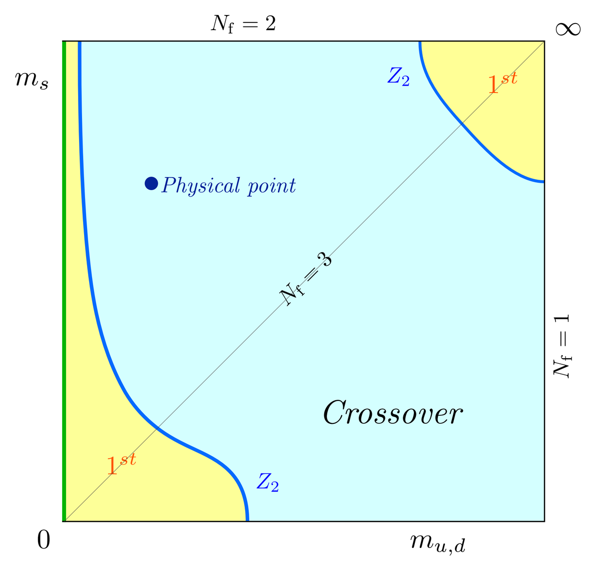
- Quantum Chromodynamics (QCD) is the theory of the strong interactions
- quarks and gluons are fundamental degrees of freedom of QCD

The QCD phase diagram

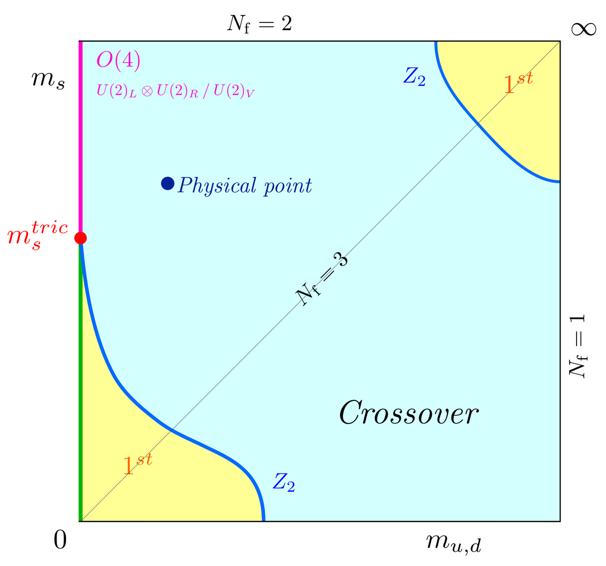
- at large temperature or densities: quarks and gluons form **quark-gluon plasma** due to weak coupling
- at low temperature and density: large coupling leads to numerous **bound hadron states** consisting of quarks and gluons
- QCD phase diagram: form of matter as function of temperature and (baryon) matter density
- non-perturbative approach: Monte Carlo simulations of **lattice QCD** to study the transition between hadronic phase and quark-gluon plasma
- **sign problem** prohibits direct simulations at real baryonic chemical potential
- investigate the **thermal transition** also for unphysical parameters

The Columbia plot

- at physical quark masses and zero baryonic chemical potential: thermal transition is **analytic, smooth crossover** [1]
- order of thermal transition depends on masses of the degenerate u, d quarks and the s quark
- two possible scenarios predicted from **linear sigma models** [2] (see figures below)
- infinite and zero quark masses: non-analytic phase transition due to spontaneous breaking of $Z(3)$-**center** or **chiral symmetry**, respectively
- first order region in heavy quark mass corner persists in continuum limit [3]
- first order region for small quark masses recedes strongly with decreasing lattice spacing [4]
- **Is the chiral transition of first or second order in the continuum limit?**
- our group presents **evidence for the continuum Columbia plot** in the heavy quark mass corner and for the chiral transition



(a) First order scenario in the $m_s - m_{u,d}$ plane



(b) Second order scenario in the $m_s - m_{u,d}$ plane

Computational Strategy

Lattice setup

- discretization of continuum QCD action: unimproved Wilson gauge action, Wilson and staggered fermions actions (for expressions see [5])
- bare parameters:
  - lattice gauge coupling $\beta = 6/g^2$
  - quark mass am / hopping parameter $\kappa = \frac{1}{2am+8}$
- lattice of size $N_\tau \times N_\sigma^3$ with lattice spacing a
- temperature $T = 1/(a(\beta)N_\tau)$
- continuum limit: $N_\tau \to \infty$ for fixed T

Numerical Tools

- LQCD code: CL^{2}QCD [6] based on OpenCL
- handle thousands of simulations: `BaHaMAS` [7]
- analysis: python scripts bundled in PLASMA

Analysis of transitions in finite volumes

- locate phase transitions and identify their nature by finite size scaling analyses of standardized moments of appropriate order parameter $\mathcal{O}$

$$B_n(\beta, am, N_\sigma) = \frac{\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^n\rangle}{\langle(\mathcal{O} - \langle\mathcal{O}\rangle)^2\rangle^{n/2}}$$

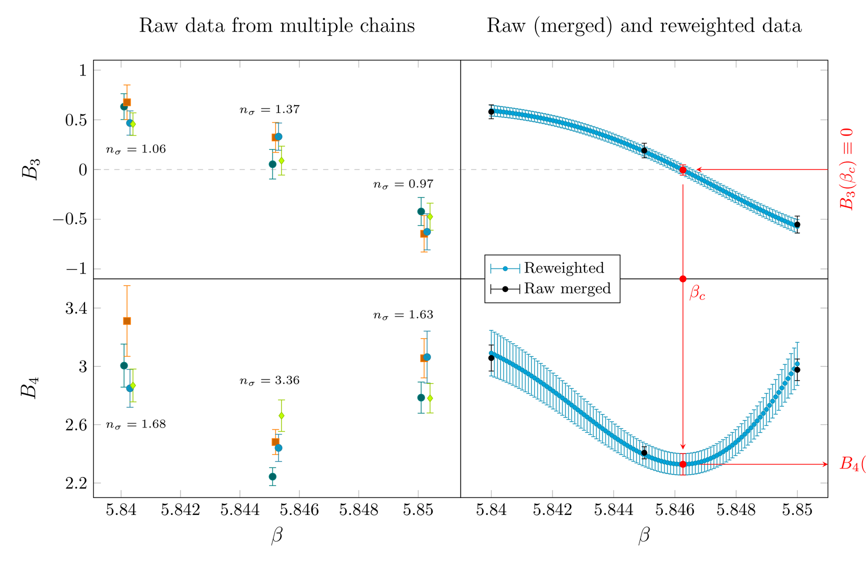
- phase boundary β_{pc}: $B_3(\beta_{pc}, am, N_\sigma) = 0$
- order of the transition: $B_4(\beta_{pc}, am, N_\sigma)$
- B_4 values in the thermodynamic limit

1. order	$Z(2)$ 2. order	crossover
1	1.604	3

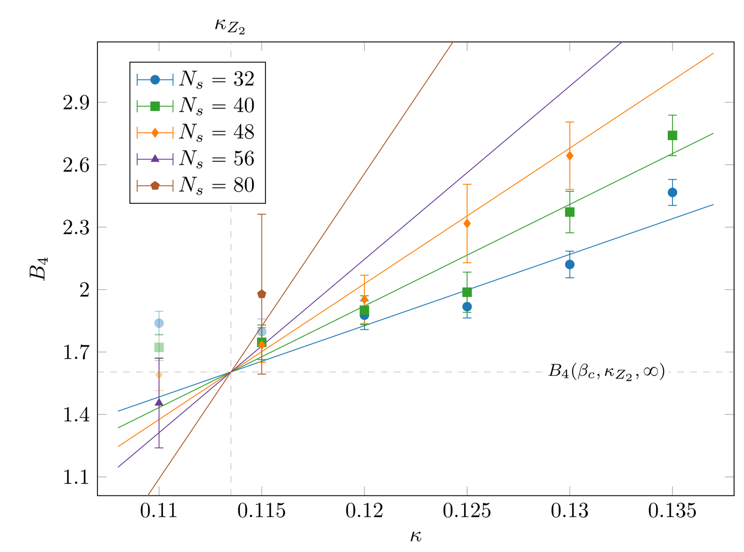
- on finite N_σ expand B_4 about the critical point

$$B_4(\beta_{pc}, am, N_\tau, N_\sigma) = (1.604 + Bx + \dots)\left(1 + CN_\sigma^{y_t - y_h} + \dots\right)$$

with the scaling variable $x(am, N_\sigma) \propto N_\sigma^{1/\nu}$ and Ising 3D critical exponents $y_t = 1/\nu, y_h$



(a) $B_3(|L|)$ **and** $B_4(|L|)$ **of different Markov chains (left) and of merged raw and reweighted data (right) (Wilson fermions,** $\kappa = 0.11$, $N_\tau \times N_\sigma^3 = 6 \times 36^3$**)**



(b) $B_4(\beta_{pc})$ **for** $N_\tau = 8$ **Wilson fermions and a common fit for all volumes according to the kurtosis scaling formula with** $C = 0$

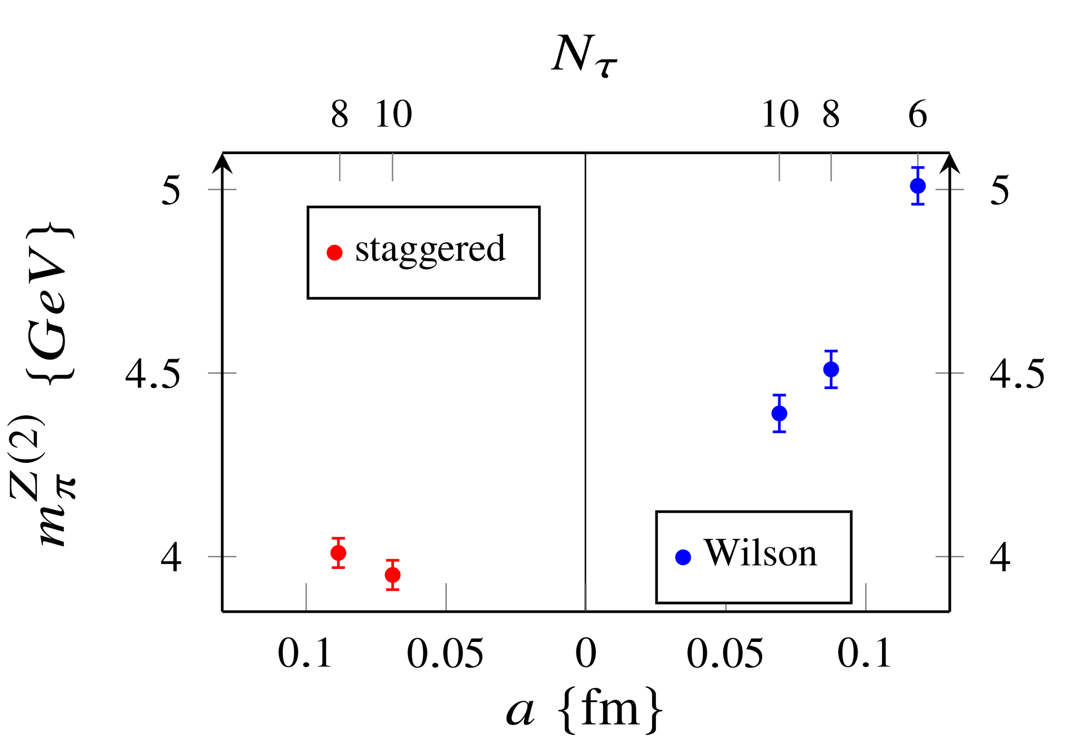
The Heavy Mass Corner

- motivation to study the heavy quark mass regime:
  - understand the **interplay of the chiral symmetry restoration and deconfinement** at the physical point
  - $Z(2)$-critical boundary as **first-principles benchmark** for effective theories
- simulations are extremely expensive due to the requirement of large and fine lattices
- order parameter associated with spontaneous breaking of global $Z(3)$ center symmetry is absolute value of **Polyakov loop**

$$L = \frac{1}{N_\sigma^3}\sum_{\boldsymbol{n}}\frac{1}{3}\,\mathrm{Tr}\left[\prod_{n_4=0}^{N_\tau-1} U_4(\boldsymbol{n}, n_4)\right].$$

- simulations for N_f=2 Wilson [8] and staggered fermions [9]
- scaling variables for Wilson and staggered fermions are

$$x_W(\kappa, N_\sigma) = (\kappa - \kappa_c)\cdot N_\sigma^{1/\nu}$$
$$x_S(am, N_\sigma) = \left(\frac{1}{am} - \frac{1}{am_c}\right)\cdot N_\sigma^{1/\nu}$$



Critical pion masses for staggered and Wilson fermions

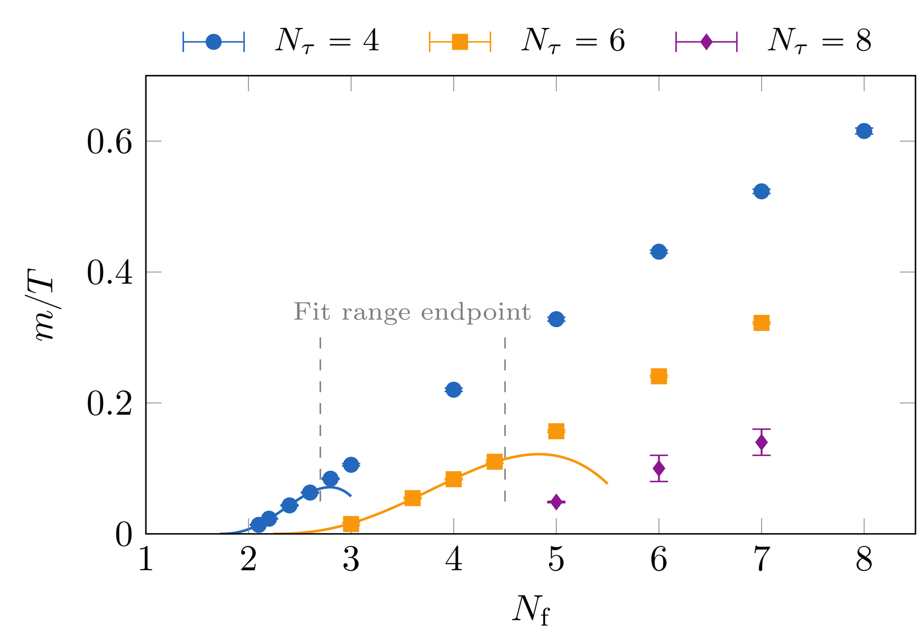
- critical phase boundary expressed by **pseudo-scalar meson masses**
- Wilson fermions: results for three lattice spacings N_τ=6, 8, 10
- running simulations on finer lattices for continuum limit

The Chiral Transition

A novel way of analyzing the chiral limit [10]

- reduces systematic errors of required extrapolations significantly
- **non-integer** N_f are simulated employing the staggered fermion action for degenerate quarks [11]
- order parameter: **chiral condensate** $\langle\bar\psi\psi\rangle$, scaling variable for analysis: $x = (am - am_c)N_\sigma^{1/\nu}$
- change from first order to second order chiral phase transition (as in second-order scenario Columbia plot) requires a **tricritical** N_f^{tric}
- functional form of $Z(2)$-critical surface entering the tricritical point is governed by **tricritical scaling**

The chiral critical surface

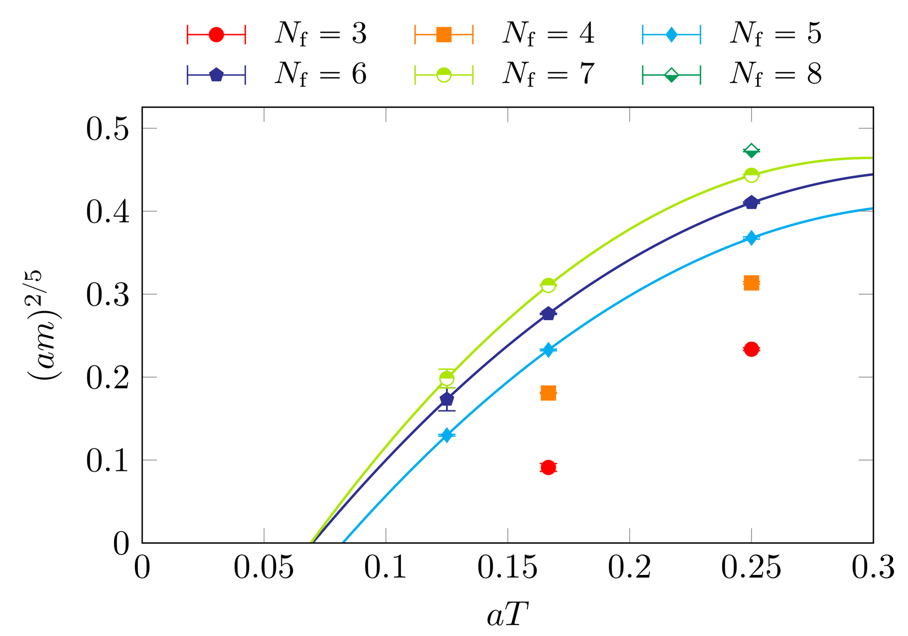


Chiral critical line for three different lattice spacings in $((m/T), N_f)$**-plane and leading order (LO) + next-to-leading order (NLO) fits**

- critical lines in $((m/T), N_f)$-plane separate crossover (above) from first order transition (below)
- first order region grows with increasing N_f
- fit to LO + NLO **tricritical scaling relation** gives $N_f^{tric}(N_\tau)$

$$am_c(N_f, N_\tau) = D(N_\tau)\left(N_f - N_f^{tric}(N_\tau)\right)^{\frac{5}{2}} + E(N_\tau)\left(N_f - N_f^{tric}(N_\tau)\right)^{\frac{7}{2}} + \dots$$

- tricritical $N_f^{tric}(N_\tau)$ grows for increasing N_τ (decreasing a)

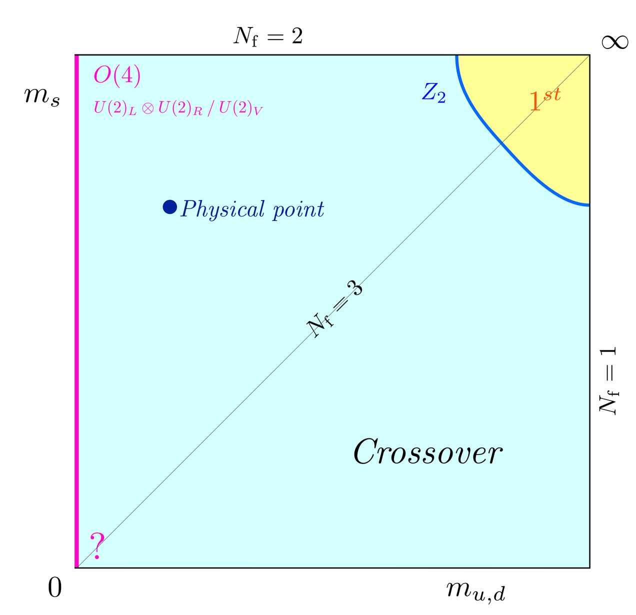


Chiral critical line for several N_f **in** $((am)^{2/5}, N_f)$**-plane and NLO fits**

- up to $N_f = 7$, lines are compatible with the existence of a finite N_f^{tric}
- first order region is not continuously connected to continuum limit
- **continuum chiral phase transition** must be of **second order**

Concluding Remarks

- convincing evidence for the qualitative form of the Columbia plot
- **chiral transition is of second order** for all values of m_s
- open question: **size of scaling regime**, important for physical point
- for a complete understanding of cut-off effects: study critical lines also for larger lattice spacings [12]



The Columbia plot according to our simulation results

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