Calculating Proton Structure using Lattice QCD

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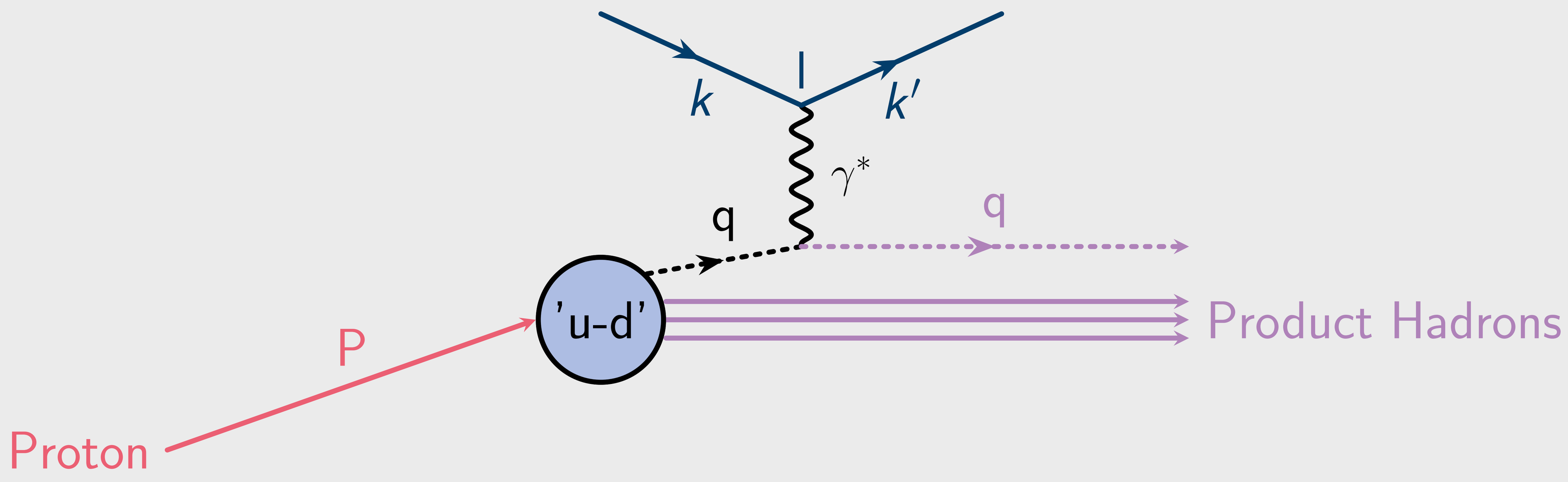
Moments of Parton Distribution Functions

Deep Inelastic Scattering (DIS) provides valuable processes to probe the structure of Hadrons. In DIS a target proton is scattered with an incoming particle beam (e.g. leptons) which has high enough energy to shatter the target. At first order, this process can be understood by a parton (q), e.g. a quark of the target, interacting with the incoming beam (l) via exchange of a virtual photon (γ^*).



DIS defines energy regimes $(P+k)^2 \gg -q^2 \equiv Q^2$ at which QCD factorization allows to split cross-sections into process dependent and perturbative (Wilson) coefficients C_p, and process independent but non-perturbative contributions, Parton Distribution Functions (PDFs) f_p

$$\frac{d\sigma^2}{dx\,dy} \sim \sum_{p=g,u,d,\dots} C_p(x,q)\, f_p(x)\,. \qquad (1)$$

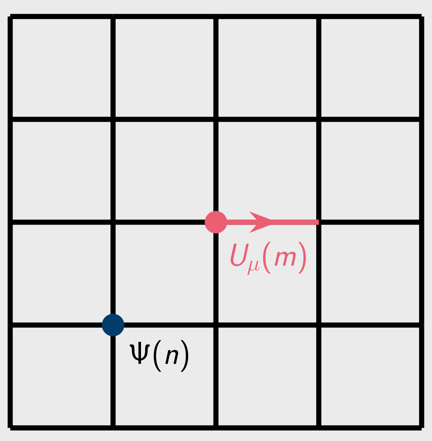
Further, PDFs can be related to forward ($k = k'$) matrix elements of local leading twist operators [1, 2]

$$\mathcal{O}_n^X = \bar{\Psi}\Gamma^X_{\{\alpha}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n\}}\Psi \equiv \mathcal{O}^X_{\{\alpha,\mu_1,\cdots,\mu_n\}} \qquad (2)$$

via the n[th] Mellin moment of $f_p(x)$, i.e. $\langle x^n\rangle = \int_{-1}^{1} x^n f_p(x)\,\mathrm{d}x$ computed by

$$\boxed{\langle P|\mathcal{O}_n^X|P\rangle = \langle x^n\rangle\, \bar{\Psi}\Gamma^X_{\{\alpha} ip_{\mu_1}\cdots ip_{\mu_n\}}\Psi} \qquad (3)$$

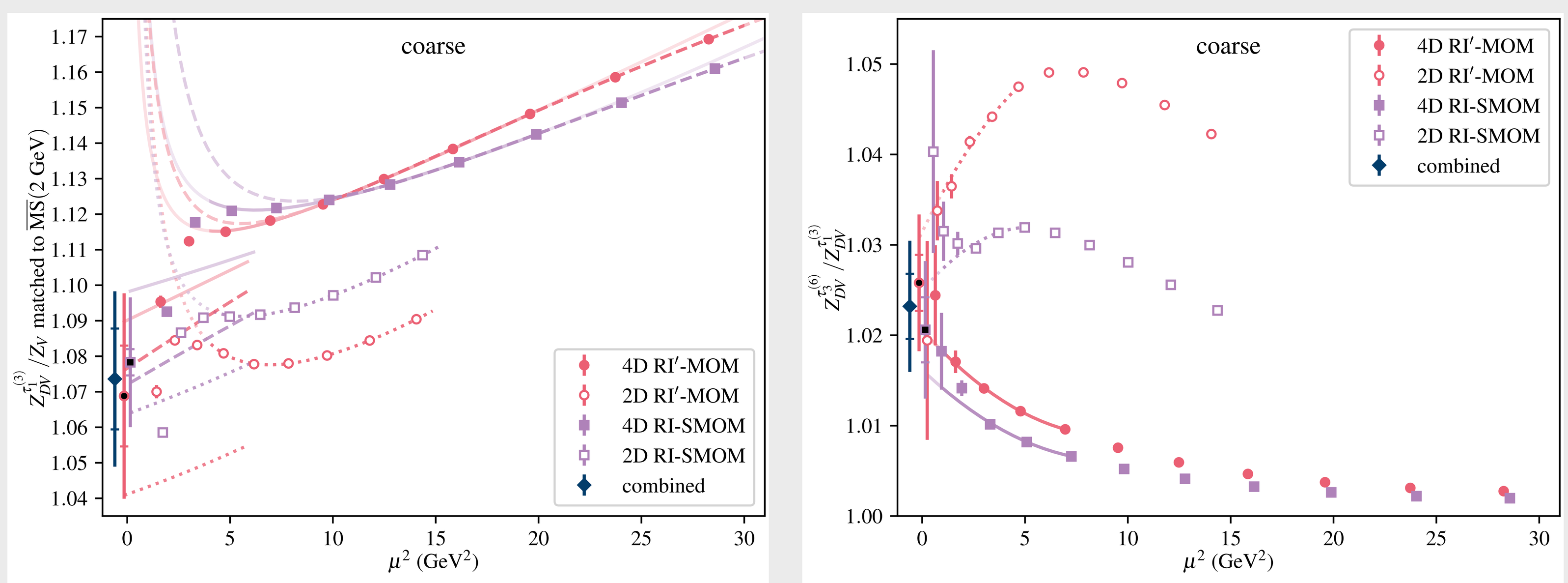
By discretizing (Euclidean) space-time onto a Hypercubic lattice, placing the fermion field onto the lattice sites $\Psi(x) \to \Psi(n)$ and the gluon field on the link between two sites $A_\mu(x) \to U_\mu(m) = \exp(iaA_\mu(m))$, it is possible to reformulate observables in terms of a Markov process. Each Markov element is then generated by the Hybrid/Hamilton Monte Carlo algorithm.



Setup

$N_t \times L^3$	β	a[fm]	m_π[MeV]	N_{cfg}
48^4	3.31	0.1163(4)	137(2)	212

The ensemble was generated close to the physical point using a tree-level Symanzik improved gauge action with 2+1 HEX smeared Wilson-clover flavors [3]. Measurements of two-point correlators and three-point correlators with eight source-sink separations $T/a \in \{3,4,5,6,7,8,10,12\}$ are performed. The shown matrix elements are renormalized in both RI-SMOM [4] and RI'-MOM [5] scheme, and converted to the $\overline{\text{MS}}$ scheme at $\mu = 2$GeV[6].



Analysis

The forward matrix elements of (3) can be extracted from ratios of two-point $C_{2pt}(\tau) = \int e^{-i\vec{p}\vec{x}}\,\mathrm{Tr}\left\{\Gamma_{pol}\left\langle \chi(\tau,\vec{x})\bar{\chi}(0,\vec{0})\right\rangle\right\}\mathrm{d}^3x$ and three-point $C_{3pt}(\tau,T) = \int e^{-i\vec{p}'\vec{x}}e^{-i(\vec{p}'-\vec{p})\vec{y}}\,\mathrm{Tr}\left\{\Gamma_{pol}\left\langle \chi(T,\vec{x})\mathcal{O}_n^X(\tau,\vec{y})\bar{\chi}(0,\vec{0})\right\rangle\right\}\mathrm{d}^3x\,\mathrm{d}^3y$ correlators

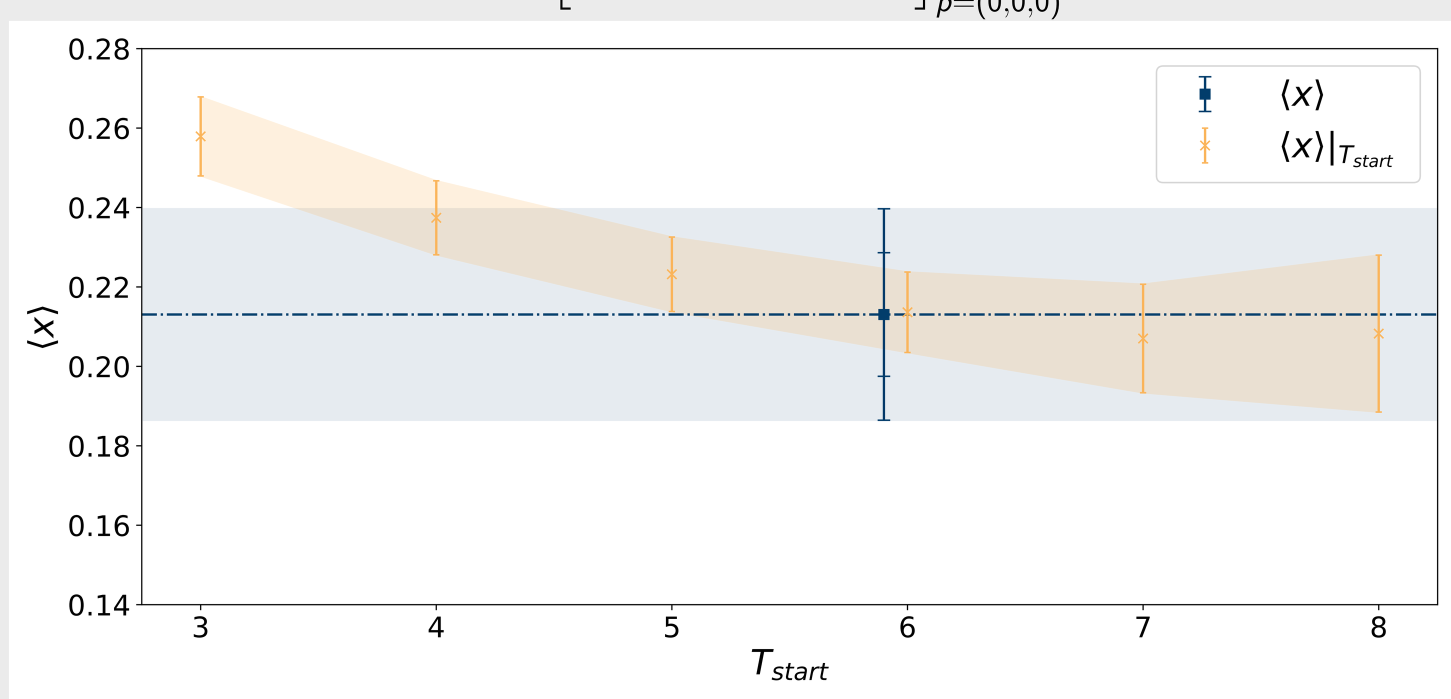
$$\langle P|\mathcal{O}_n^X|P\rangle = \lim_{\tau-T,\tau\to\infty} R(\tau,T) = \lim_{\tau-T,\tau\to\infty} \frac{C_{3pt}(\tau,T)}{C_{2pt}(T)} \qquad (4)$$

where the limit projects onto the ground state that we are interested in. To remove disconnected diagrams we only considere the iso-vector combination ($p = u - d$). Higher state contribution can be reduced by considering sum of ratios

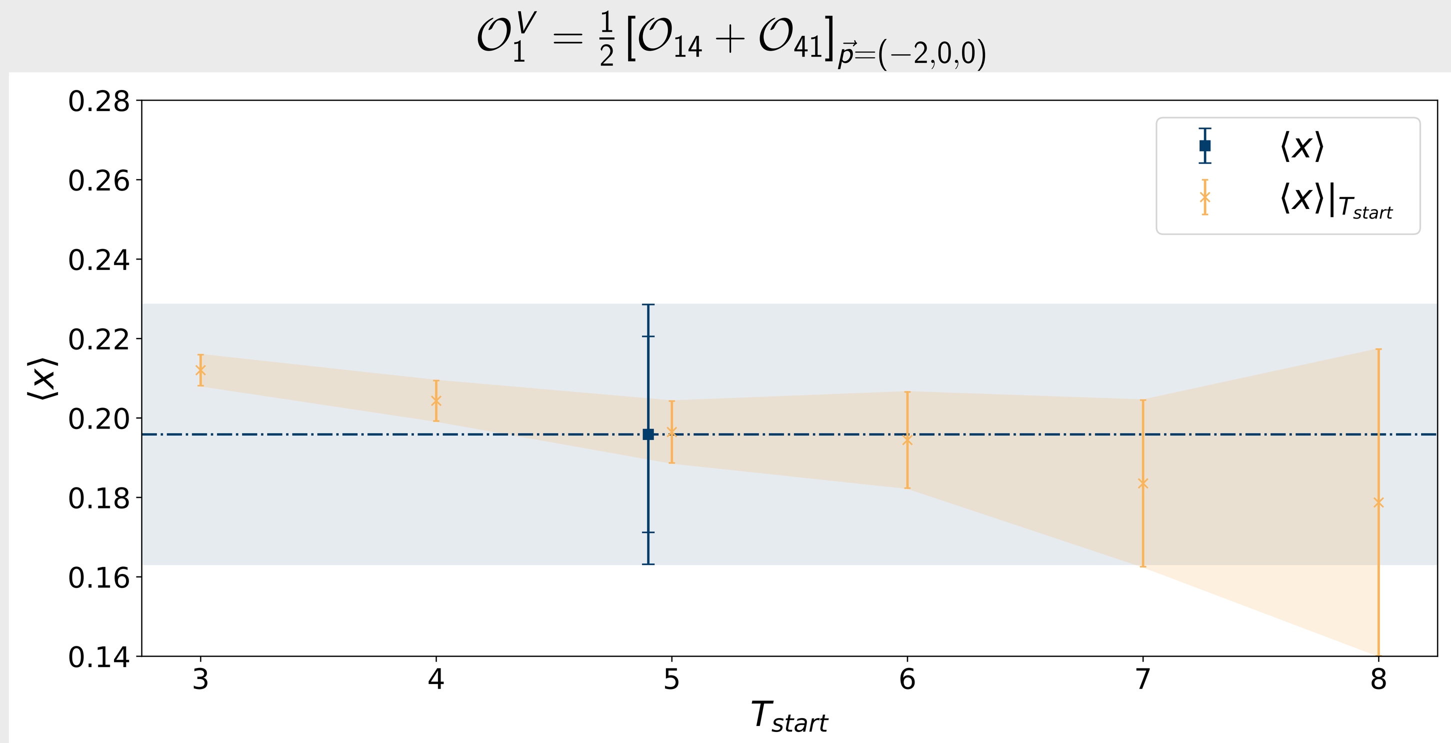
$$S(\tau_{skip},T) = \sum_{\tau=\tau_{skip}}^{T} R(\tau,T) \xrightarrow{T\to\infty} \langle P|\mathcal{O}_n^X|P\rangle\,(T-\tau_{skip}) + c. \qquad (5)$$

We restrict ourself to the first moments and the vector case and perform a linear fit to (5). Higher state contribution can further be reduced by fitting only $T > T_{start}$ until a plateau is reached. The lower end is kept fixed at $\tau_{skip} = 1$.

$\mathcal{O}_1^V = \frac{1}{2}\left[-\mathcal{O}_{44} + \frac{1}{3}\sum_{i=1}^{3}\mathcal{O}_{ii}\right]_{\vec{p}=(0,0,0)}$



$\mathcal{O}_1^V = \frac{1}{2}\left[\mathcal{O}_{14} + \mathcal{O}_{41}\right]_{\vec{p}=(-2,0,0)}$



Besides the linear fit, other methods can be applied, e.g. finite difference. All these have different higher order contributions hence can be used to estimate a systematic error as the standard deviation of these methods. For this deviation we consider $\tau_{skip} \in \{0,1\}$, $T_{start} \in \{3,4,5,6,7,8\}$ in both a linear fit and forward, backward as well as central finite difference. Averaging over all possible operators yields the first renormalized mellin moment

$$\boxed{\langle x\rangle = 0.20(1)(2)} \qquad (6)$$

Future Work

For the near future we want to advance this work further. First, higher moments ($n = 2,3$) are in preparation, but they tend to become more noisy as n increases. Second, a finer lattice spacing is on the way to address the continuum limit. And last we want to considering helicity and transversity (X=A,T) moments of PDFs.

[1] P. Hägler. In: *Physics reports* (2010). DOI: 10.1016/j.physrep.2009.12.008.
[2] M. Göckeler et al. In: *Physical Review D* (1996). DOI: 10.1103/PhysRevD.54.5705.
[3] S. Durr et al. In: *Physics Letters B* (2011). DOI: 10.1016/j.physletb.2011.05.053.
[4] C. Sturm et al. In: *Physical Review D* (2009). DOI: 10.1103/PhysRevD.80.014501.
[5] G. Martinelli et al. In: *Nuclear Physics B* (1995). DOI: 10.1016/0550-3213(95)00126-D.
[6] N. Hasan et al. In: *Physical Review D* (2019). DOI: 10.1103/PhysRevD.99.114505.