Contribution of QCD theta-term to neutron EDM with stabilized Wilson fermions and scalar content of a nucleon on the lattice

Jangho Kim[1], Thomas Luu[1,3], Giovanni Pederiva[2], Andrea Shindler[2]

[1]Institute for Advanced Simulation (IAS-4), Forschungszentrum Jülich, Germany

[2]Facility for Rare Isotope Beams Department of Physics and Astronomy, Michigan State University, USA

[3]JARA-HPC, CJIAS40

Contribution of QCD $\theta - term$ to Neutron Electric Dipole Moment with Stabilized Wilson Fermion

The QCD theta term is a source of CP-violation that could potentially induce a non-vanishing neutron electric dipole moment (nEDM). see the Ref. [1] for the nEDM results on the lattice. We present first results obtained with Stabilized Wilson Fermions [2, 3]. We focus on SU(3) flavor-symmetric OpenLat ensembles at $\sim$ 400 MeV pion mass, and 3 different lattice spacings in the range of 0.065 fm $< a <$ 0.12. We use O(100) stochastic source locations to improve the signal-to-noise ratio.

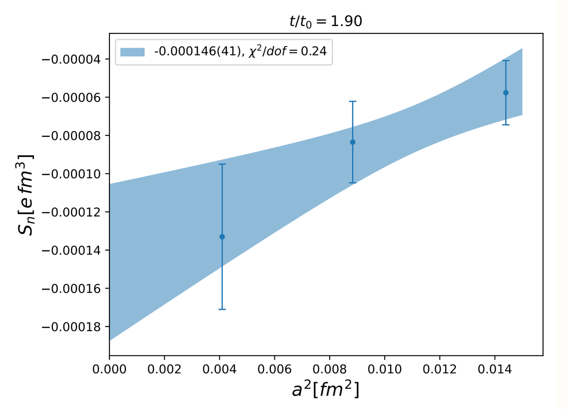
Stabilized Wilson Fermion

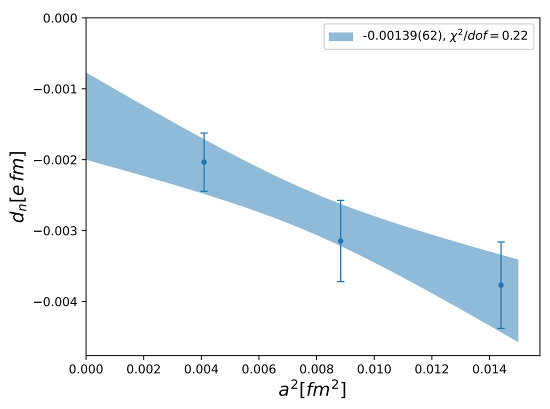
Reaching the physical point at coarse lattice spacings is particularly challenging with Wilson-type fermions. Accidental zero-modes can cause instabilities along the Hybrid Monte Carlo (HMC) and the presence of the clover term worsens these effects. With SWF the local term of the Wilson-clover action is exponentiated, becoming positive definite and providing a bound to the abovementioned instabilities. At the same time $O(a)$ improvement is not spoiled.

	m_π (MeV)	β	a[fm]	κ_ℓ	dimension	# of confs
C_4	410	3.685	0.12	0.1394305	$24^3 \times 96$	1189
F_4	408	3.8	0.094	0.1389630	$32^3 \times 96$	1001
U_4	411	4.0	0.065	0.1382720	$48^3 \times 96$	214

Preliminary Results at 400 MeV pion mass

- Schiff moment by Chiral perturbation theory: $S_n = -1.7(3) \times 10^{-4} \bar{\theta} e fm^3$
- Our lattice calculation: $S_n = -1.46(41) \times 10^{-4} \bar{\theta} e fm^3$
- neutron electric dipole moment at 400 MeV pion mass: $|d_n| = 1.39(62) \times 10^{-3} e fm$





Scalar content of a nucleon

A weakly interacting massive particle (WIMP) is a very popular dark matter (DM) candidate. Various ongoing experiments around the world provide rather severe constraints for the parameters of these DM models.

A possible scenario for the detection of WIMP type of DM particles relies on the idea that the WIMP, due to its assumed large mass, produces a Higgs boson that couples to the various quark flavor scalar density operators taken between nucleon states.

$$\sigma_{\chi N} \sim \left| \sum_f G_f(m_\chi^2) f_{T_f} \right|^2 \quad \text{with} \quad f_{T_f} = \frac{m_f}{m_N} \langle N | \bar{q}_f q_f | N \rangle .$$

The functions G_f depend on several parameters of the particular BSM theory used for the calculation of $\sigma_{\chi N}$, which include the WIMP mass, m_χ. The dimensionless and renormalization group invariant (RGI) coupling f_{T_f} depends on the mass m_f of the quark of flavor f and the nucleon mass, m_N. As evident from Eq. (1), the cross section, $\sigma_{\chi N}$ depends quadratically on f_{T_f}, and is therefore very sensitive to the size of the scalar content contributions of different flavours. A O(10%) variation of f_{T_f}, for example, can lead to significant changes in $\sigma_{\chi N}$. It is therefore necessary to compute accurately and with controlled error the hadronic matrix elements $\langle N|\bar{q}_f q_f|N\rangle$. This approach is numerically very challenging and to date have not provided statistically relevant results. Therefore, to make any radical progress in this area, a novel approach that uses the gradient flow is required.

Calculation of hadronic matrix elements that couple to DM probes

This calculation is based on the possibility to relate the matrix element of a scalar density $\bar{q}_r(t,x)q_s(t,x)$ at non-vanishing flow-time between nucleon states with the one at vanishing flow-time [4]. This can be done considering the small flow-time expansion of the scalar density

$$S^{rs}(t,x) = \bar{q}_r(t,x) q_s(t,x) .$$

To compute the matrix element we are interested in we need to compute

$$\frac{G_\pi}{G_{\pi,t}} \cdot \left[\langle \mathcal{N} S^{rs}(t) \mathcal{N} \rangle - \langle S^{rs}(t) \rangle \langle \mathcal{N}\mathcal{N} \rangle \right] .$$

We observe that the renormalization factor of $S^{rs}(t)$ simplifies with the one of $G_{\pi,t}$, thus the only renormalization factor needed is the one for G_π, i.e. Z_P.

Method

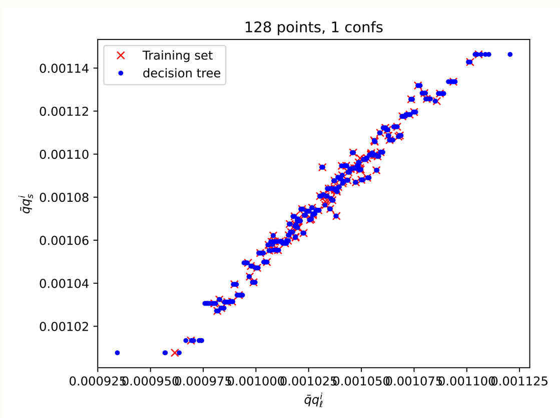
We compute the quark disconnected diagrams using the Hutchinson trace method and gradient flow method.

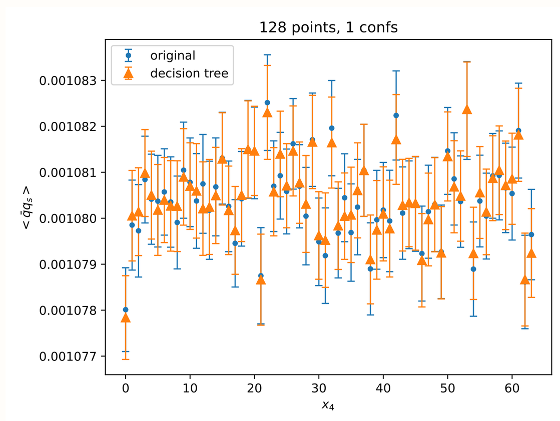
$$\mathrm{tr}[A] \approx \frac{1}{N_\xi} \sum_k^{N_\xi} z_k^\dagger A z_k$$

Here, k is the index of noise source, z_k is one of $\{1, -1, i, -i\}$ as in a Z_4 random noise source. We apply the gradient flow and we define $A = K(t)SK^\dagger$, where $K^\dagger$ is the adjoint kernel which flows the source of the propagator S, while K flows the propagator sink. Hence, $KSK^\dagger = S(t,t)$. We use the light and strange quark propagators in this calculation.

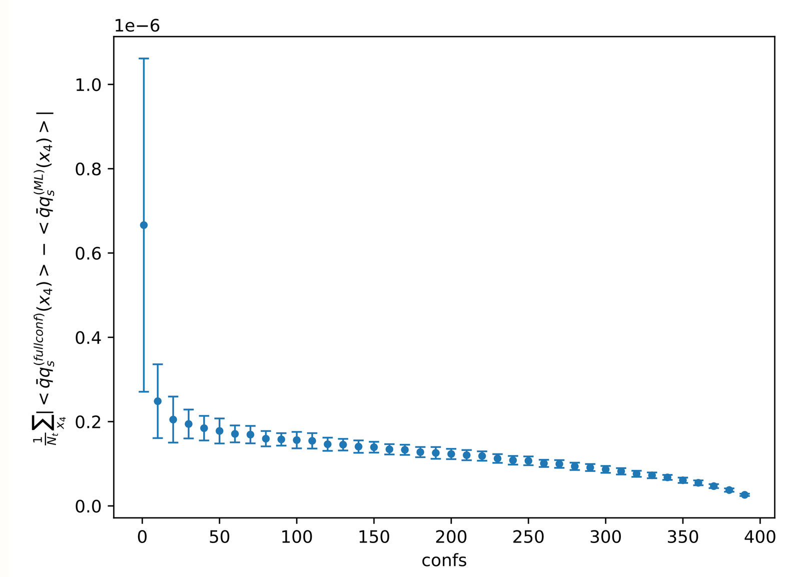
Machine learning method to reduce the amount of computing

We test the machine learning method that trains the correlation between $\bar{q}q_\ell$ and $\bar{q}q_s$ on the same gauge field. First, the input set is $\bar{q}q_\ell$ on 399 configurations with 2 random noise sources at flow-time $t = 1$. The training set is $\bar{q}q_\ell$ with the part of 399 configurations at the same random sources and flow-time.





In order to verify the proper number of configurations for the training set, we compute the difference between the full configuration and machine learning results.



The formula for the disconnected bubble for light and strange quark is

$$\bar{q}q_f(t) = \sum_{x \in V_s} \sum_{s,c,k} z_k^\dagger K(t) S K^\dagger(t) z_k - c_{fl} \sum_{x \in V_s} \sum_{s,c,k} z_k^\dagger K(t) K^\dagger(t) z_k$$

where $c_{fl} = 0.5734$, S is light or strange quark propagator, s and c are spin and color indices. To compute the sigma term σ_f, we combine this $\bar{q}q$ with nucleon-nucleon 2pt function.

$$\begin{aligned}
&\langle N(y_4,0) | \bar{q}q_f(x_4,t) | \bar{N}(0,0) \rangle \\
&= \frac{1}{N_{conf}} \sum_i^{N_{conf}} \bar{q}q_f^i N^i(y_4,0) \bar{N}^i(0,0) \qquad (1) \\
&\langle \bar{q}q_f(x_4,t) \rangle \langle N(y_4,0) \bar{N}(0,0) \rangle \\
&= \left(\frac{1}{N_{conf}} \sum_i^{N_{conf}} \bar{q}q_f^i \right) \left(\frac{1}{N_{conf}} \sum_j^{N_{conf}} N^j(y_4,0) \bar{N}^j(0,0) \right)
\end{aligned}$$

Here y_4 is the nucleon source and sink separation and x_4 is the time slice of the disconnected piece.

We show the results of

$$R(y_4, x_4, t) \equiv \frac{\langle N(y_4,0) | \bar{q}q_f(x_4,t) | \bar{N}(0,0) \rangle}{\langle N(y_4,0) \bar{N}(0,0) \rangle} - \frac{\langle \bar{q}q_f(x_4,t) \rangle \langle N(y_4,0) \bar{N}(0,0) \rangle}{\langle N(y_4,0) \bar{N}(0,0) \rangle}$$

The σ_f term is defined as

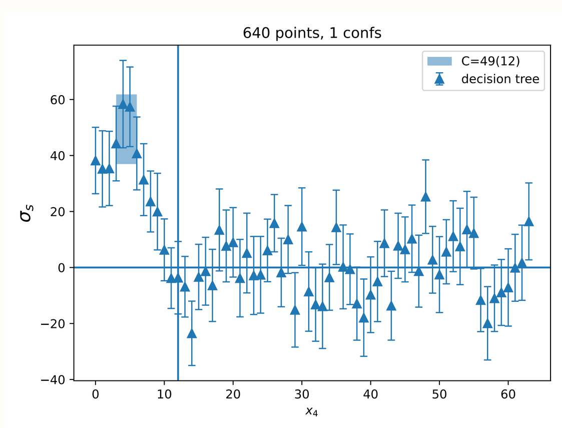
$$\sigma_f = m_f \langle N | \bar{q}^f q^f | N \rangle = m_f g_S^f$$

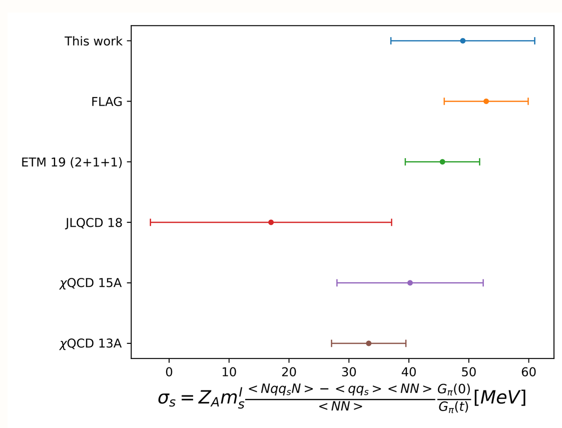
or the mass fraction is defined

$$f_{T_f} = \sigma_f / M_N$$

We compute the disconnected loop with light quark mass $\langle \bar{q}q_\ell \rangle$ on M_1 ensemble [5] at flowtime $t = 1$ with 10 noise sources. For the $\langle \bar{q}q_s \rangle$, the calculation is done for only one configuration with 10 same noise sources and the machine learning method is applied. The $\sigma_{\pi N}$ requires the calculation of both connected and disconnected diagrams, so here we only present the preliminary result of σ_s which needs only disconnected diagram and compare with other group's results [6].

	m_π (MeV)	β	a[fm]	κ_ℓ	dimension	# of confs
M_1	700	1.9	0.0907	0.13700	$32^3 \times 64$	399





Our preliminary result of the mass fraction is

$$f_{T_{ud}} = 0.045(10) , \quad f_{T_s} = 0.0315(79)$$

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