Beyond the Standard Model Matrix Elements from lattice QCD: renormalization

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JARA-HPC, Project: `CJIAS40`

Introduction

The observed baryon asymmetry in the universe cannot be reconciled with the current form of the Standard Model (SM) of particle physics. The Standard Model breaks charge conjugation parity (CP) symmetry, but not in a sufficient amount to explain the observed matter-antimatter asymmetry. Historically one of the first systems to be studied in the search of symmetry breaking within the Standard Model is the electric dipole moment (EDM) of the neutron. The contribution to the neutron EDM coming from the SM is several order of magnitudes smaller than the current experimental bound, thus providing a unique, background-free window for potential discovery of physics Beyond the Standard Model (BSM). CP-violating effective operators describing, at energies below the electro-weak scale, the effects of BSM physics can contribute to the neutron EDM. To constrain all these contributions to the neutron EDM we need to precisely determine the hadronic matrix elements of the corresponding renormalized operators.

In this work we show recent results obtained with the gradient flow, that, for the first time, establish a strategy for the non-perturbative renormalization of higher dimensional CP-violating operators. This paves the way to the determination of the BSM contributions to the neutron EDM and the determination of other phenomenologically relevant quantities such as the CP-conserving long-distance contributions to $K^0 - \overline{K}^0$ mixing, direct CP-violation in hyperon decays, ϵ'/ϵ and the $\Delta I = 1/2$, $K \to \pi\pi$ transition, or the CP-violating part of the $K \to 3\pi$ decay.

CP-violating operators

Below the energy scale $\Lambda_{\rm BSM}$ beyond the Standard Model (BSM) effects are described by local effective operators (of dimension D) made of the fundamental degrees of freedom of the SM and suppressed by powers of $1/\Lambda_{\rm BSM}^{D-4}$. Among the resulting operators one of the dominant contribution to the EDM is given by the $D=5$ quark-chromo EDM (qCEDM) operator

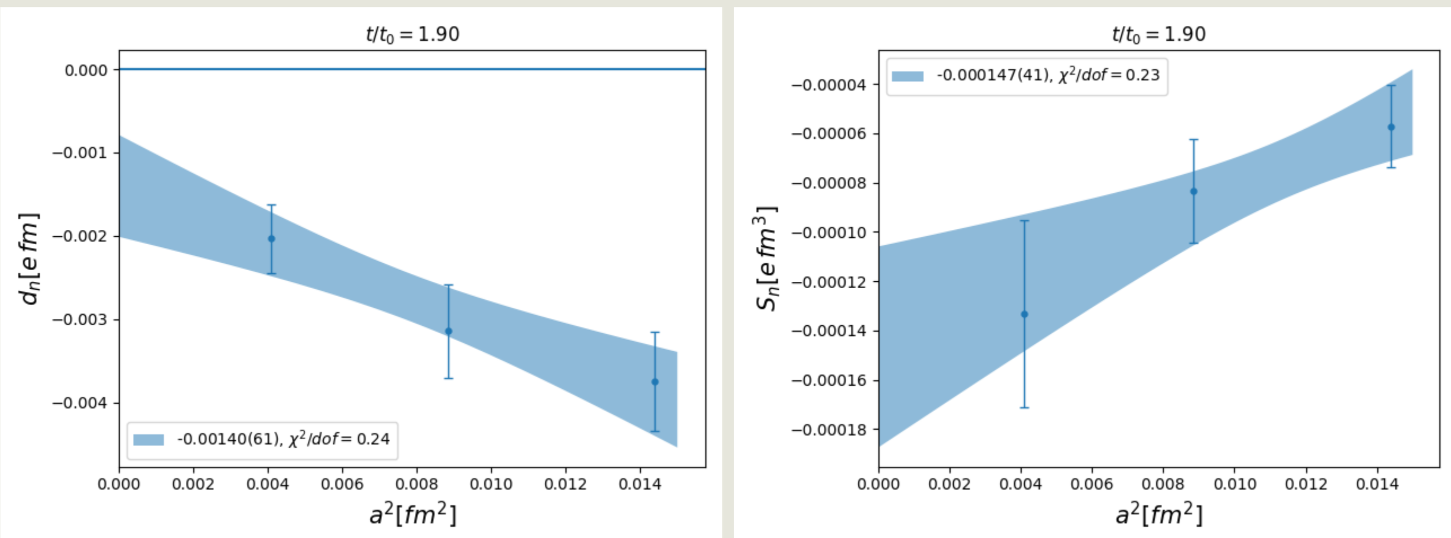
$$\mathcal{O}_{\rm CE}(x) = \overline{\psi}(x)\gamma_5\sigma^{\mu\nu}G_{\mu\nu}\psi(x)\,. \tag{1}$$

The goal of lattice QCD (LQCD) calculations is to provide the renormalized hadronic matrix elements of the CP-violating operators. Those matrix elements represent the key input to constrain and disentangle the different contributions to any future experimental measurement of a non-zero EDM.

Gradient flow

No lattice QCD calculations exist in the literature of the neutron EDM stemming from CP-violating operators like the qCEDM (1). The main obstacle is represented by the very complicated renormalization pattern of such operators. The mixing with lower dimensional operators and the high number of other CP-odd operators contributing in the renormalization procedure has rendered these type of calculations practically unfeasible. We have proposed [1, 2, 3] to use the gradient flow [4, 5] to resolve this challenge.

The gradient flow (GF) is a differential operator that modifies the fields at short distances with a renormalizable smoothing procedure. The new flowed gauge and fermion fields are a complicated non-linear function of the original degrees of freedom which are the initial conditions of the GF evolution. The smoothing procedure is governed by a new scale, the flow-time t of dimension $D=-2$. The GF has many applications such as allowing a theoretically sound and numerically robust definition of the topological charge [4] and the neutron EDM induced by the θ term [6].



Here we show our preliminary continuum limit for the neutron EDM, d_n, and the neutron Schiff moment, S_n, stemming from the θ term, obtained with Stabilized Wilson fermions [7] at 3 different lattice spacings and at the SU(3) isosymmetric point with $m_\pi \simeq 400$ MeV. Flowed local fields have also a very simple renormalization pattern. As a consequence any flowed fermionic operator renormalizes multiplicatively with the same factor depending only on the fermion content of the operator [5]. Obviously flowed local operator are not physical. To determine the physical renormalized matrix elements is necessary to find a strategy to connect the flowed renormalized operators with the physical ones.

There are few ways to achieve this like the use of Ward identities [8] or the use of an operator product expansion around $t \sim 0$, also called short flow time expansion (SFTX)

$$\mathcal{O}_i^{\rm R}(x,t) \sim \sum_j c_{ij}(\mu,t)\mathcal{O}_j^{\rm R}(x,\mu)\,, \tag{2}$$

where μ represents the renormalization scale and $c_{ij}(\mu,t)$ are the Wilson, or matching coefficients relating the renormalized flowed operators with the physical ones $\mathcal{O}_j^{\rm R}(x,\mu)$. The list of operators contributing to the SFTX in Eq. (2) can be obtained directly in the continuum and each operator can be probed perturbatively and non-perturbatively in an independent manner, since the SFTX is an operator relation.

Our strategy is to define and calculate matrix elements of the flowed CP-odd operator like the qCEDM (l.h.s of Eq. (2)), and then calculate the matching coefficients. Inverting Eq. (2) it is possible to determine the physical matrix elements, without directly calculating them. The problem of the renormalization is shifted into the determination of the matching coefficients. The big advantages of this method are:1) the matching coefficients can be determined in the continuum limit keeping the flow time t fixed in physical units; 2) the SFTX is an operator relation, thus it can be probed in the most advantageous way to determine the matching coefficients; 3) it allows a perturbative and non-perturbative determination of the matching coefficients.

Power divergences

The dominant contribution to the SFTX in Eq. (2) of the qCEDM comes from the pseudoscalar density $P(x) = \overline{\psi}(x)\gamma_5\psi(x)$ ($D=3$ operator). This implies that the corresponding matching coefficient is power divergent when $t \to 0$

$$\mathcal{O}_{\rm CE}(x,t) \sim \frac{c_{\rm P}}{t}P_{\rm R}(x) + \cdots\,. \tag{3}$$

To avoid uncontrolled systematics power divergences need to be subtracted non-perturbatively [9]. In Ref. [10] we have defined a non-perturbative scheme to define the matching coefficient using the dimensionless ratio

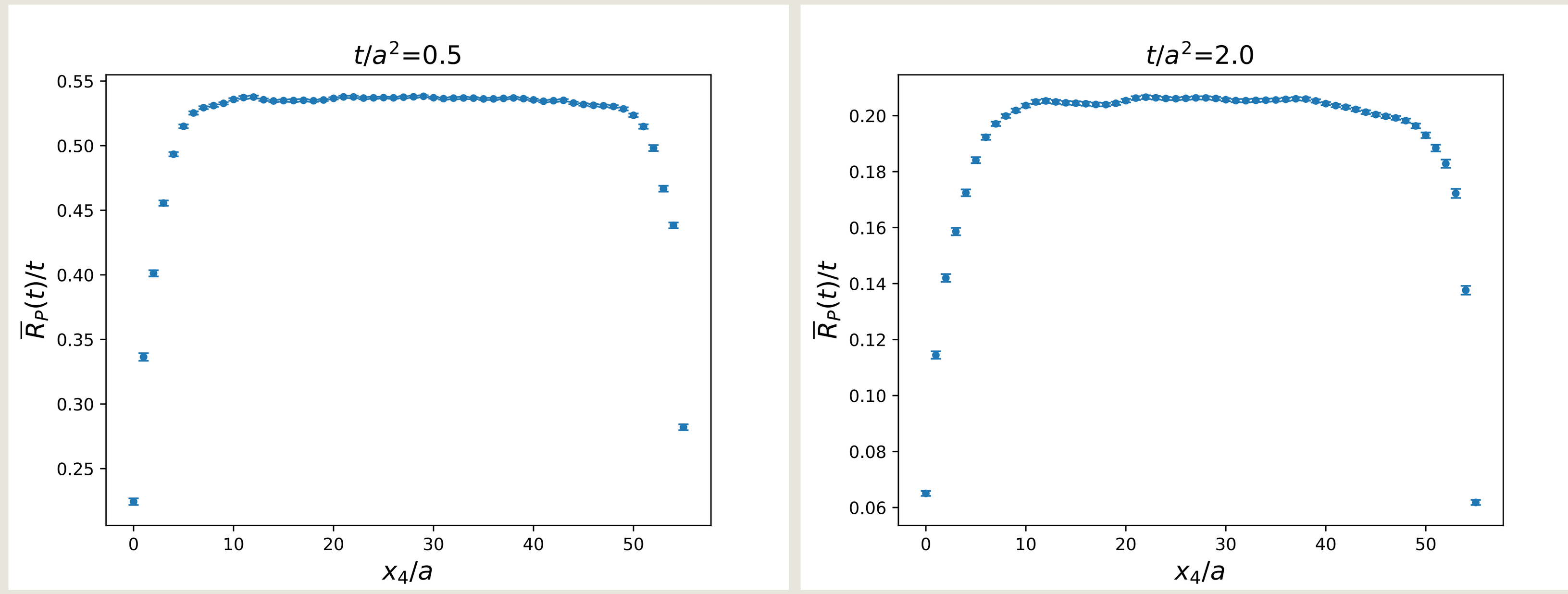
$$\overline{R}_{\rm P}(x_4,t) = t\frac{a^3\sum_{\mathbf{x}}\langle\mathcal{O}_{\rm CE}(x,t)P(0)\rangle}{a^3\sum_{\mathbf{x}}\langle P(x,t)P(0)\rangle}\,. \tag{4}$$

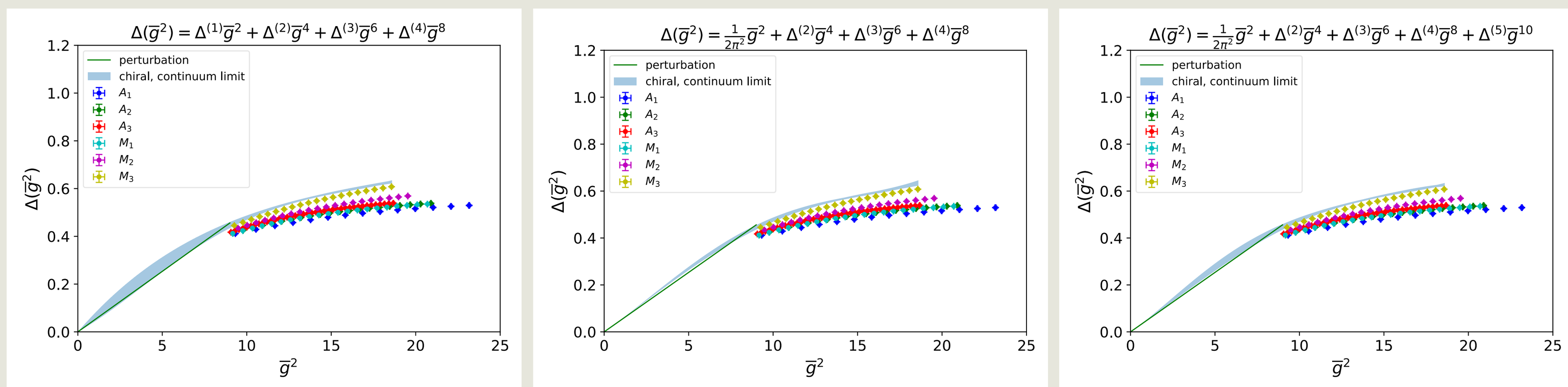
We have tested our strategy on PACS-CS ensembles from Ref. [11] and the lattice parameters are summarized in this table.

Designation	β	κ_l	κ_s	L/a	T/a	$c_{\rm SW}$	N_G	a [fm]	m_π [MeV]	m_N [GeV]	Z_P
M$_1$	1.90	0.13700	0.1364	32	64	1.715	399	0.0907(13)	699.0(3)	1.585(2)	0.49605
M$_2$	1.90	0.13727	0.1364	32	64	1.715	400	0.0907(13)	567.6(3)	1.415(3)	0.49605
M$_3$	1.90	0.13754	0.1364	32	64	1.715	450	0.0907(13)	409.7(7)	1.219(4)	0.49605
A$_1$	1.83	0.13825	0.1371	16	32	1.761	800	0.1095(25)	710(1)	1.65(1)	0.44601
A$_2$	1.90	0.13700	0.1364	20	40	1.715	790	0.0936(33)	676.3(7)	1.549(6)	0.49605
A$_3$	2.05	0.13560	0.1351	28	56	1.628	650	0.0684(41)	660.4(7)	1.492(5)	0.51155

Comparison with perturbation theory

Performing the spectral decomposition, the ratio $\overline{R}$ is independent on x_4 for $x_4 \gg \sqrt{8t}$, where the ground state dominates. From the plateau (see following figure), for each value of the renormalized coupling $\overline{g}$ evaluated at flow time t, one reads off the value of a function $\Delta(\overline{g}^2)$, that, while is still not the coefficient of the power divergence, it has the same behavior at leading order in perturbation theory.





In this figure we compare $\Delta(\overline{g}^2)$ with our leading order perturbative result [2] (green line). This result is very encouraging and in the next computer time allocation we plan to extend this calculation using Stabilized Wilson fermions (SWF) [7], and determine the normalization of the flowed fermion fields. This will allow a direct determination of the coefficient $c_{\rm P}(\overline{g})$ and a robust continuum limit.

Conclusions and outlook

We have analyzed the quark chromo-EDM operator, for the first time using the gradient flow method to provide control on the power divergences that occur due to mixing during renormalization when using a discrete spacetime regulator. In essence, our gradient flow analysis trades induced power divergences with cutoff dependencies, the latter being much more amenable to a continuum limit extrapolation.

Our most important result is the non-perturbative determination of the finite renormalization connecting, for a wide range of renormalized coupling values, the qCEDM operator with the pseudoscalar density at finite flow time. It shows the feasibility of our method and the successful matching with perturbation theory. The calculation of this finite renormalization reduces the power divergence problem to the determination of the nonperturbative evolution of the pseudoscalar density at finite flow time. In the next step of the calculation we will determine the correction factor to replace the flowed pseudoscalar density with the unflowed and renormalized one. We have recently determined, perturbatively and in the continuum, also the logarithmic corrections stemming from $D=5$ operators [3]. Combining these results with the nonperturbative determination of the power-divergent matching coefficient of the qCEDM, we have all the elements for a first determination of the quark-chromo EDM contribution to the neutron.

Acknowledgements and References

M.D.R. and A.S. acknowledge funding support under the National Science Foundation grants PHY-1913287 and PHY-2209185. We acknowledge the computing time granted through JARA-HPC on the supercomputer JURECA at Forschungszentrum Jülich. We also would like to thank the Center for Scientific Computing, University of Frankfurt for making their High Performance Computing facilities available.

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