Dynamical QCD+QED from the lattice: π^0–η–η' masses and mixing

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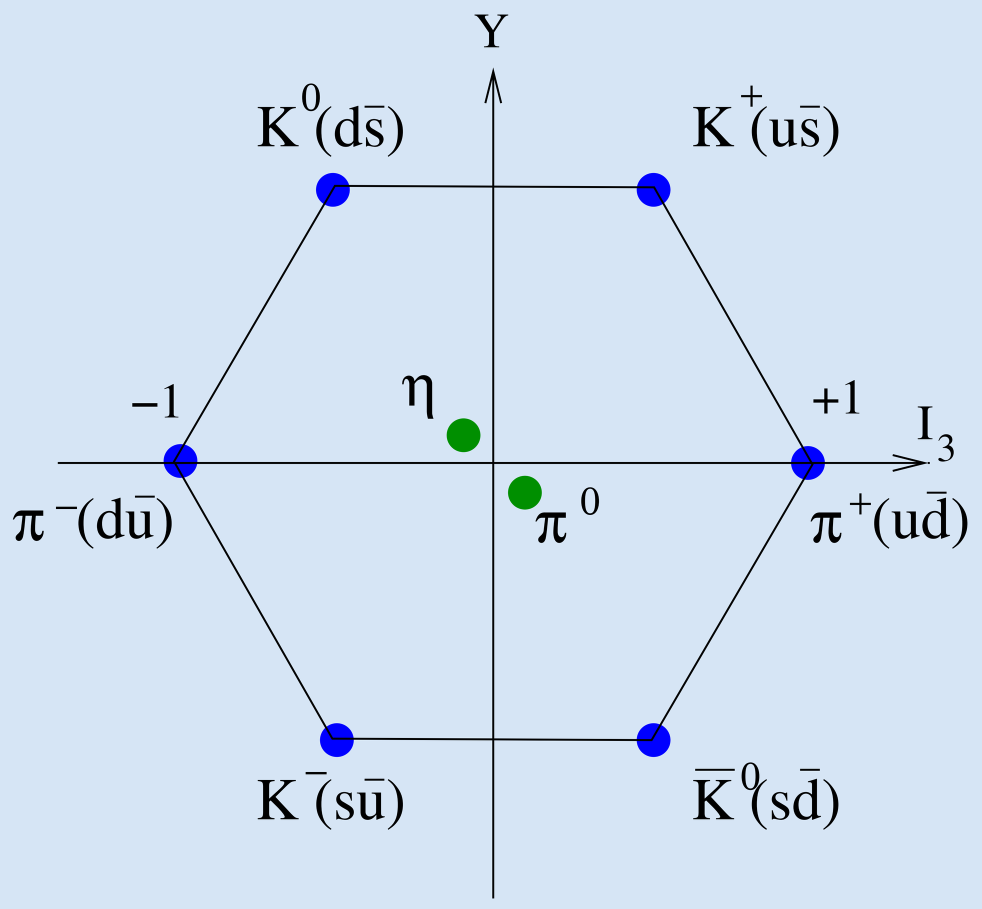
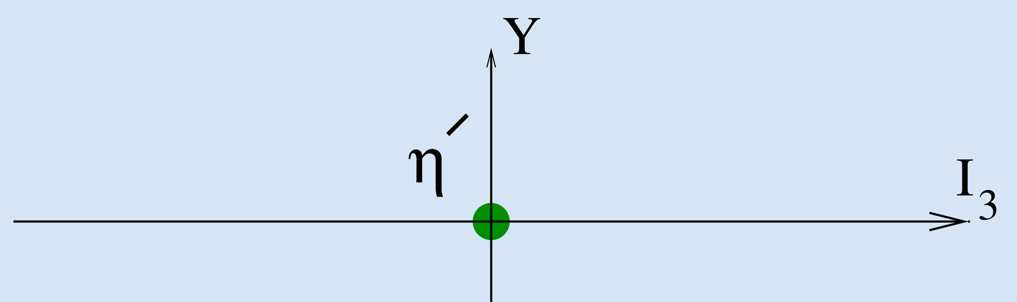
Introduction

This poster presents essential results [1] from supercomputing project CKM: ***Leptonic and Semileptonic Decays from QCD+QED***.





Low lying mesons consist of quark–anti-quark states either up, u, down, d or s strange quark. The $SU(3)$ classification leads to mesons belonging to an octet or singlet as illustrated in the figure.

A particularly challenging topic is the (π^0, η, η') mass splittings and mixings. With exact SU(3) symmetry, the decomposition is known in the isospin, (I-spin), flavour operator basis $\mathcal{O}_q = \bar{q}\gamma_5 q$:

$$\pi_3 = \frac{1}{\sqrt{2}}(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$$
$$\eta_8 = \frac{1}{\sqrt{6}}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s)$$
$$\eta_1 = \frac{1}{\sqrt{3}}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s)$$

However, when isospin is broken due to differing quark masses (i.e. $m_u \neq m_d$) and QED effects (i.e. electromagnetically charged quarks) these states mix to form the experimentally observed mass spectrum

$$(\pi_3, \eta_8, \eta_1) \to (\pi^0, \eta, \eta')$$

Previous works have employed isospin-symmetric simulations and hence have focussed on the η–η' mixing. Here, [1], we make use of the natural breaking of isospin that occurs when electromagnetic effects are incorporated into a lattice simulation [2] and perform full $N_f = 1+1+1$ simulations to investigate the mass splittings and mixings of these states.

The strategy

We wish to take $(\delta m_u, \delta m_d, \delta m_s)$ as the distance from the QCD symmetric point where all the quarks are mass degenerate. The inclusion of QED complicates this definition of the flavour symmetric point somewhat and we take it to be the point where the neutral mesons have the same mass, [2].

A systematic expansion to second order for the mass-squared neutral meson matrix has been developed [1]:

$$\mathcal{M}^2 = M_0^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{A}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + 2\alpha \begin{pmatrix} \delta m_u & 0 & 0 \\ 0 & \delta m_d & 0 \\ 0 & 0 & \delta m_s \end{pmatrix} + \ldots$$

along a path where the average quark mass is held constant. In addition $\delta m_u + \delta m_d + \delta m_s$ automatically vanishes. The neglected terms also include terms with the charges e_u, e_d, e_s. The A term represents the annihilation term $\bar{q}q \to \text{glue} \to q\bar{q}$. As an illustration the lowest order eigenvalues are

$$M_{\pi^0}^2 = M_0^2 - \alpha\sqrt{\frac{2}{3}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2)} + \ldots$$
$$M_\eta^2 = M_0^2 + \alpha\sqrt{\frac{2}{3}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2)} + \ldots$$
$$M_{\eta'}^2 = M_0^2 + A + \ldots$$

Configuration generation

In this first study, we have the chosen quark masses to be as close to the QCD flavour symmetric point as possible and taken an artificially large QED coupling, $\alpha = 0.1$, in order to maximise our ability to amplify the QED part of the splitting.

This is most efficiently achieved by choosing a quark mass trajectory that keeps the down quark mass fixed i.e. $\delta m_d = 0$, $\delta m_u = -\delta m_s$.

We have generated 3 data sets on this trajectory using fully dynamical QCD+QED quarks on $24^3 \times 48$ lattices.

We measure the two-point correlation functions

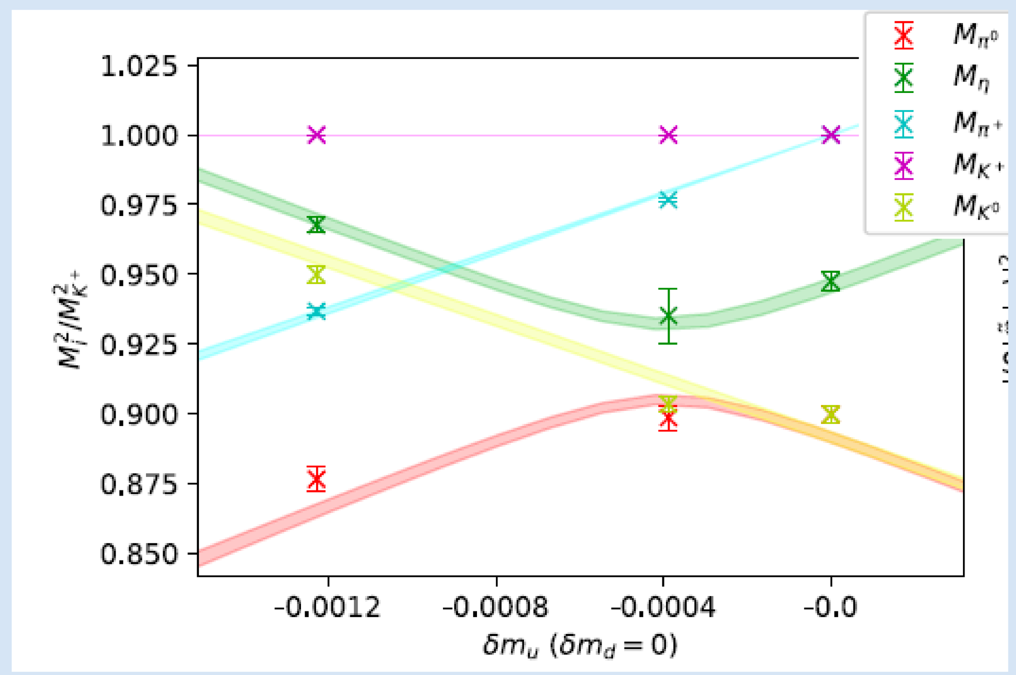
$$C_{q'q} = \sum_{\vec{x},\vec{y}} \langle \mathcal{O}_{q'}(\vec{y}, t)\mathcal{O}_q^\dagger(\vec{x}, 0)\rangle$$

It can be shown that this gives

$$C_{q'q} = \sum_n \langle 0|\mathcal{O}_{q'}|n\rangle\langle 0|\mathcal{O}_q|n\rangle^* e^{-M_n t}$$

where M_n are the lowest lying energies (taken as 6 here). This equation can be regarded as a Generalised EigenValue Problem (GEVP) and which can be solved to yield M_n and the overlaps $\langle 0|\mathcal{O}_q|n\rangle$.

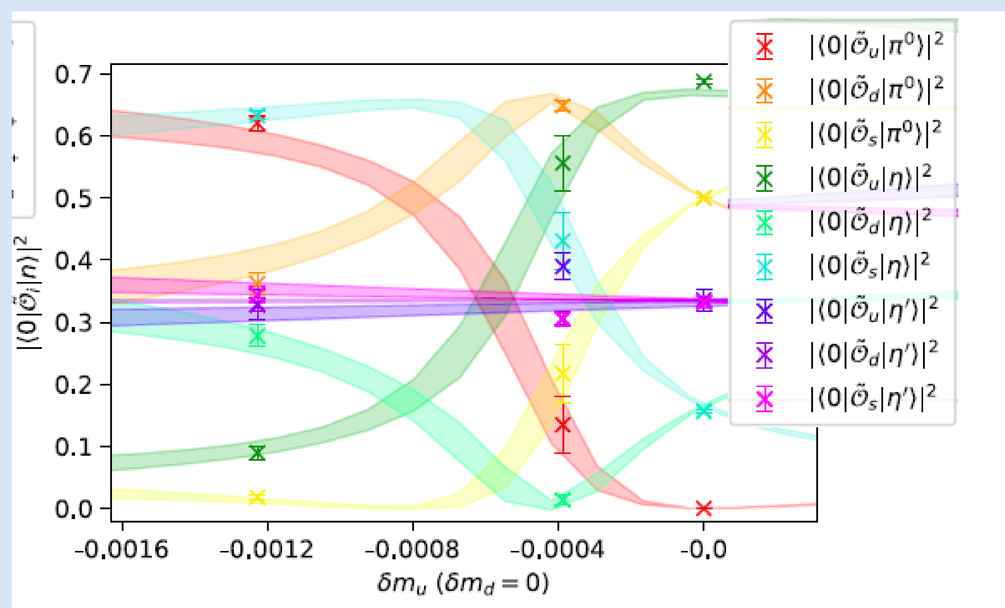
Masses



$(\text{Masses})^2$ for the flavour-diagonal pseudoscalar mesons normalised by $M_{K^+}^2$ along a trajectory holding the down quark mass fixed at $\delta m_d = 0$.

The avoided level crossing at $\delta m_u \approx -0.0004$ shows the dynamical interaction between the two light pseudoscalars, π^0 (red) and η (green).

Overlaps



The evolution of the flavour composition of the flavour-neutral states along this trajectory.

We highlight the evolution of the π^0 state, which is one of an exact U-spin, when $m_d = m_s$, $\pi_U = (\bar{d}\gamma_5 d - \bar{s}\gamma_5 s)/\sqrt{2}$ at the point $\delta m_u = 0$.

Moving to the left, this state takes the form of a V-spin state $\pi_V^0 = (\bar{u}\gamma_5 u + \bar{s}\gamma_5 s - 2\bar{d}\gamma_5 d)/\sqrt{6}$ near our middle mass, then in the vicinity of $\delta m_u \approx -0.0008$, this state takes the form of a conventional isospin $\pi^0 = (\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)/\sqrt{2}$.

By convention, we choose the π^0 state to denote the lightest state, and η the second lightest state irrespective of the quark content.

Conclusions

- Extrapolating to the physical quark masses and charges ($\alpha = 1/137$) we find

$$|\pi^0\rangle \approx 0.85|\pi_3\rangle - 0.27|\eta_8\rangle + 0.29|\eta_1\rangle$$
$$|\eta\rangle \approx -0.07|\pi_3\rangle + 0.76|\eta_8\rangle + 0.56|\eta_1\rangle$$
$$|\eta'\rangle \approx -0.005|\pi_3\rangle - 0.26|\eta_8\rangle + 0.96|\eta_1\rangle$$

- With present levels of accuracy can only determine the η–η' mixing angle

$$\theta_{\eta'\eta} = \sin^{-1}(-0.26 \pm 0.10) = -15.1^{+5.9\,\circ}_{-6}$$

- Consistent with phenomenology

References

[1] Z. R. Kordov *et al.* [CSSM/QCDSF/UKQCD], Phys. Rev. D **104** (2021) 114514, [arXiv:2110.11533 [hep-lat]].

[2] R. Horsley, *et al.* [QCDSF-UKQCD], JHEP **04** (2016) 093, [arXiv:1509.00799 [hep-lat]].