Non-perturbative studies of Beyond Standard Models

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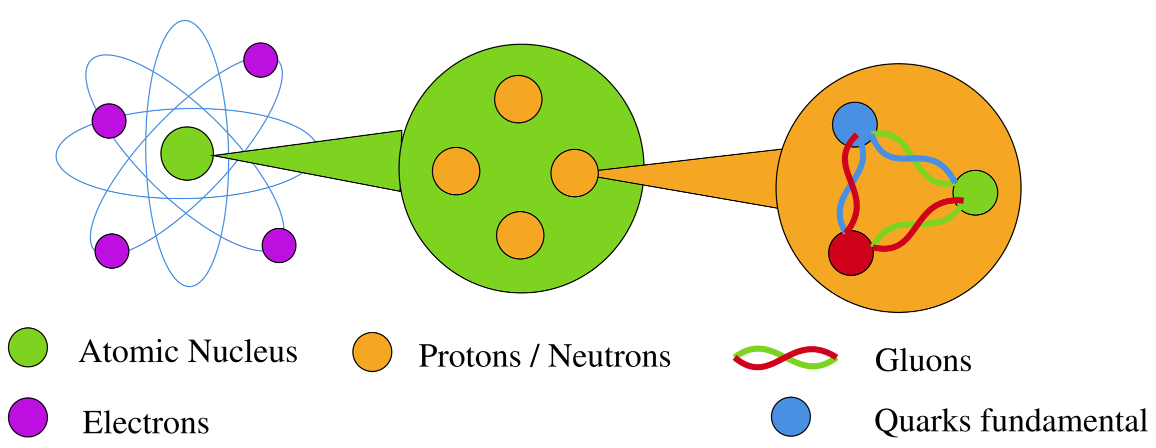
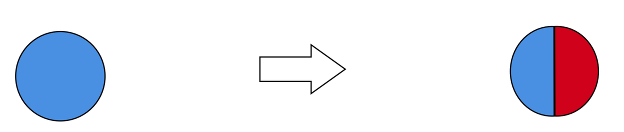
Motivation

Building blocks of strong nuclear interaction (QCD):



Perturbative methods can not be used to investigate the strongly coupled regime. Lattice field theory gives us a numerical first principle tool to study these theories. Interesting theories can be obtained by adding extra degree of symmetry.

Theories with adjoint fermions: Giving a bit of extra color to the fermions we make them more similar to the gluons → Higher degree of symmetry → More tractable



Opens the possibility to study Supersymmetric theories and Adjoint QCD theories. Why study Supersymmetric / Adjoint QCD theories on the lattice?

- Explain beyond Standard Model puzzles: Dark matter, Dark energy ...
- Gain insights into confinement and chiral symmetry breaking, crucial to understand mass generation. These more symmetric theories provide a powerful background to tackle these problems but they need to be complemented and extended by numerical methods.

Theories with Adjoint Fermions

$\mathcal{N}=1$ Supersymmetric (SUSY) Yang-Mills

- SUSY extension of Gauge sector of QCD
- Simplest model with SUSY and local gauge invariance
- Orientifold planar equivalence: SUSY Yang-Mills theory with N_c colors is equivalent to QCD with a single quark flavour, $N_f=1$ QCD, in the limit $N_c\to\infty$ with Quarks in antisymmetric repr. of SU(N_c).
- Continuity to semiclassical regime

Lagrangian:

$$\mathcal{L}=\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\frac{1}{2}\overline{\lambda}^a\gamma_\mu(\mathcal{D}_\mu\lambda)^a+m_{\tilde{g}}\,\overline{\lambda}^a\lambda^a+\frac{1}{2}D^aD^a$$

- Gauge field $A^a_\mu(x)$, $a=1,\dots,N_c^2-1$, "Gluon" Gauge group SU(N_c)
- Majorana-spinor field $\lambda^a(x)$, $\overline{\lambda}=\lambda^T C$, "Gluino"
- Gluino mass term $m_{\tilde{g}}\,\overline{\lambda}^a\lambda^a$ breaks SUSY softly.
- (auxiliary field $D^a(x)$)
- SUSY: (on-shell) $\delta A^a_\mu=-2\,\mathrm{i}\,\overline{\lambda}^a\gamma_\mu\varepsilon$, $\delta\lambda^a=-\sigma_{\mu\nu}F^a_{\mu\nu}\varepsilon$

$N_f=1$ Adjoint (Adj) QCD
Connected to $\mathcal{N}=1$ and $\mathcal{N}=2$ SYM

$$\mathcal{N}=2\text{ SYM}\xrightarrow{m_s\to\infty}N_f=1\text{ Adj QCD}\xrightarrow{m_{f1}\to\infty}\mathcal{N}=1\text{ SYM}$$

Two Majorana "half" fermions "$N_f=2\times\frac{1}{2}=1$" gives rise to a non-trivial chiral symmetry $U(1)_A\otimes SU(2)$ with many possible interesting phenomena

- Chiral symmetry breaking
- Pions as massless Goldstone bosons
- Rich phase structure and connection to confinement

A proper study needs to capture chiral symmetry on the lattice → Overlap fermions

Supersymmetric QCD

- Additional quarks ψ and squarks Φ_i in fundamental representation
- Covariant derivatives, mass terms for (ψ,Φ_i)
- Yukawa interactions and scalar potential

$$i\sqrt{2}g\bar{\lambda}^a\left(\Phi_1^\dagger T^a P_+ + \Phi_2 T^a P_-\right)\psi$$
$$-\,i\sqrt{2}g\bar{\psi}\left(P_- T^a\Phi_1 + P_+ T^a\Phi_2^\dagger\right)\lambda^a$$
$$\frac{g^2}{2}\left(\Phi_1^\dagger T^a\Phi_1-\Phi_2^\dagger T^a\Phi_2\right)^2.$$

Non-perturbative Problems

SUSY Models: The lattice breaks SUSY but it can be restored in the continuum limit by fine tuning of parameters. How do we tune the parameters?

- Gain information from perturbation theory
- Determination of the renormalization factors for $\mathcal{N}=1$ SYM

$N_f=1$ Adj QCD: Still not well investigated. Is the theory IR Conformal? [1]. This means for the IR effective theory:

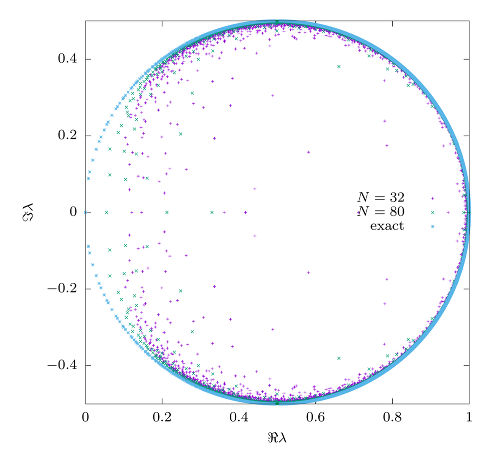
- Scale invariance, states become massless
- Absence of confinement and chiral symmetry breaking
- Infra-red fix point present

$N_f=1$ Adj QCD with Overlap fermions

Chiral Symmetry

Approximate symmetry of nature that explains the small masses of up and down quarks and pions. Overlap fermions implement chiral symmetry exactly on the lattice but are challenging to simulate. RHMC + Overlap leads to a stable polynomial approximation of sign-function to order N:

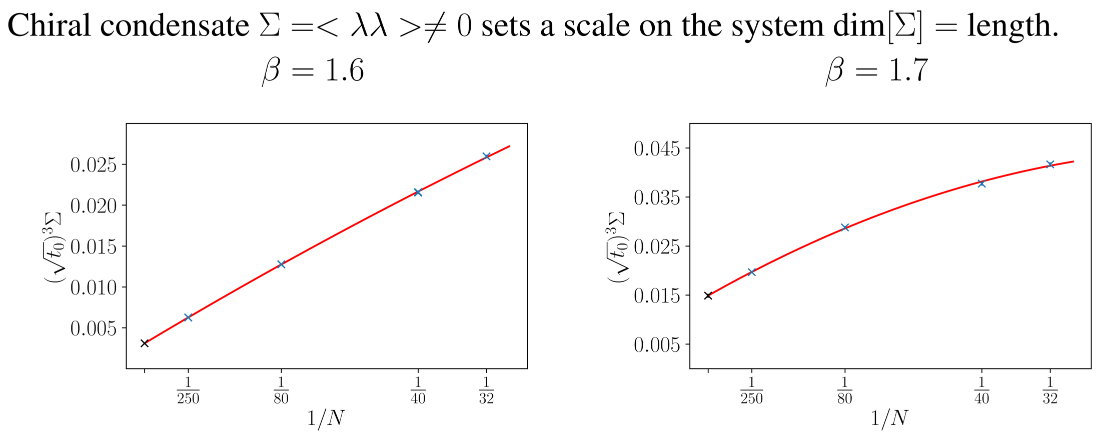
$$D_{\text{ov}}=\frac{1}{2}+\frac{1}{2}\gamma_5\text{sign}(\gamma_5 D_W),\qquad \text{sign}(\gamma_5 D_W)\approx P_N(\gamma_5 D_{\text{ov}})$$



- Overlap operator eigenvalue spectrum lies on a circle
- Gap on the spectrum at finite N stabilizes the RHMC algorithm
- No need of fine tunning. Chiral (massless) limit reached in the $N\to\infty$ limit

Chiral condensate

Chiral condensate $\Sigma=<\lambda\lambda>\neq 0$ sets a scale on the system dim$[\Sigma]$ = length.



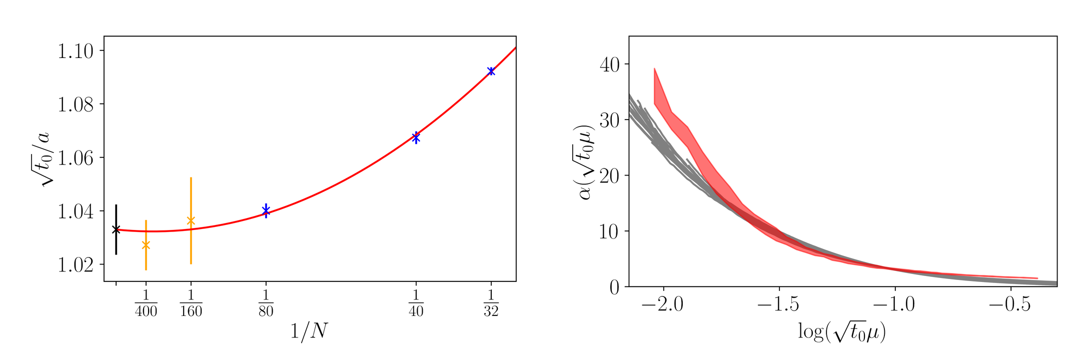
Wilson Flow

Flow the lattice fields following the steepest descent direction of the action

$$\dot{V}_t(x,\mu)+-g_0^2\{\partial_{x,\mu}S_W(V_t)\}V_t(x,\mu),\quad V_t(x,\mu)|_{t=0}=U(x,\mu)$$

It can be used to define an energy scale t_0 and to obtained the running of the coupling constant g_{GF} to check for fix points.

$$t_0^2\langle E\rangle=0.3,\quad g^2_{\text{GF}}(\mu)=\frac{16\pi^2}{3(N^2-1)\tau^2\langle E(\tau)\rangle}\bigg|_{\tau^2=1/8\mu}.$$



$\mathcal{N}=1$ SYM on the lattice

Lattice breaking of SUSY

Local lattice theory breaks SUSY unavoidably at any finite lattice spacing. Approach for SUSY Yang-Mills theory (Curci, Veneziano) [2]

1. Wilson action:

$$S=-\frac{\beta}{N_c}\sum_p\text{Re Tr }U_p$$

$$+\frac{1}{2}\sum_x\left\{\overline{\lambda}^a_x\lambda^a_x-K\sum_{\mu=1}^{4}\left[\overline{\lambda}^a_{x+\hat{\mu}}V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x+\overline{\lambda}^a_x V^t_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat{\mu}}\right]\right\}$$

$\beta=\frac{2N_c}{g^2}$, $K=\frac{1}{2m_0+8}$ hopping parameter, m_0 : bare gluino mass

$V_{ab,x\mu}=2\,\text{Tr}\,(U^\dagger_{x\mu}T_aU_{x\mu}T_b)$, adjoint link variables

2. Tuning towards the chiral supersymmetric continuum limit:
   - Wilson term breaks chiral symmetry and SUSY → both recovered in the continuum limit
   - Degenerate mass spectrum (SUSY partners) found in the continuum limit [3, 4]

Challenging extension towards supersymmetric QCD

- Yukawa couplings and scalar potential need to be fine tuned ~ 10 parameters
- reduced tuning for chiral symmetric formulations (overlap fermions)

Ward Identities

Supersymmetry leads to conserved quantity $S_\mu=\sigma_{\nu\rho}\gamma_\mu\text{Tr}_c(F_{\nu\rho}\lambda)$ and a SUSY WI $Z_S\langle\nabla_\mu\langle S_\mu(x)\mathcal{O}(y)\rangle=0$. However the lattice discretization breaks SUSY adding extra terms and obstructing its simulation.

$$Z_S\langle\nabla_\mu S_\mu(x)\mathcal{O}(y)\rangle+Z_T\langle\nabla_\mu T_\mu(x)\mathcal{O}(y)\rangle=m_S\langle(\chi(x))\mathcal{O}(y)\rangle+\mathcal{O}(a),$$

Renormalization coefficients Z_S, Z_T Renormalized mass m_S

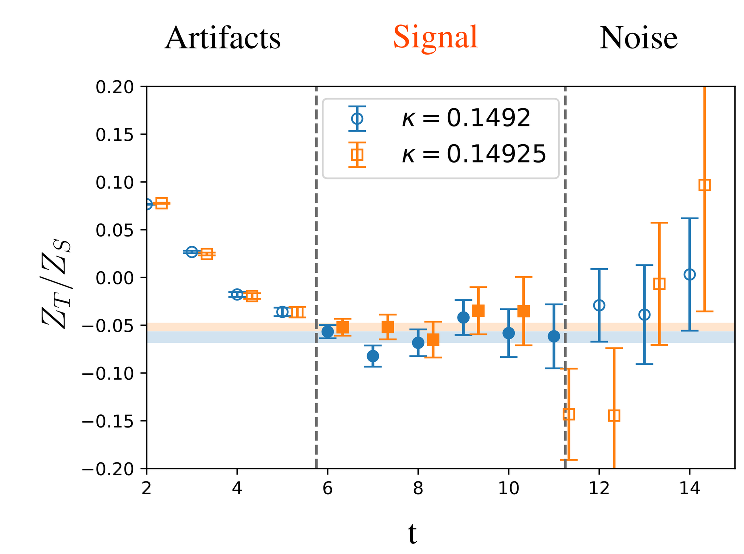
Can we recover the SUSY WI in the continuum limit? Yes, by tuning renormalized mass to zero [5]. If we have extra parameters (SQCD) to which value do we tune them? Maybe we can use their perturbative value. First try: are the perturbative and non-perturbative values of Z_S, Z_T comparable?

Supercurrent Renormalization

GIRS Scheme: Renormalization scheme with only gauge independent / physical observables quantities. Valid both perturbatively and non-perturbatively

$$Z_X^{\text{B,GIRS}}Z_Y^{\text{B,GIRS}}\langle\mathcal{O}_X^{\text{B}}(x)\mathcal{O}_Y^{\text{B}}(y)\rangle|_{x-y=\bar{z}}=\langle\mathcal{O}_X(x)\mathcal{O}_Y(y)\rangle^{\text{tree}}|_{x-y=\bar{z}}$$

Get rid of GIRS scale z dependence ⟶ Translate to $\overline{\text{MS}}$ using conversion factors



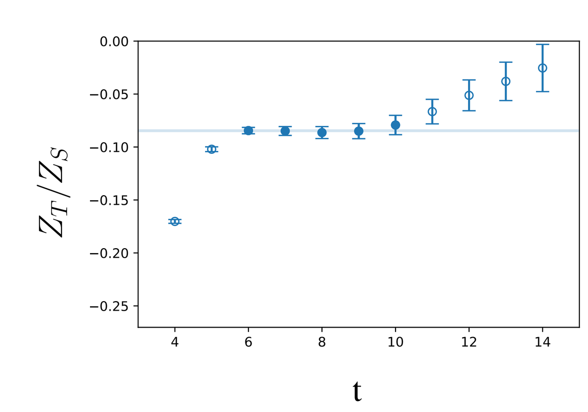
Non-perturbative

$$\frac{Z_T}{Z_S}=-0.0517(84),\ \kappa=0.14920$$
$$\frac{Z_T}{Z_S}=-0.0418(84),\ \kappa=0.14925$$

Perturbative

$$\frac{Z_T}{Z_S}=0.1008,\ \kappa=0.1250$$

- Results are in high tension → Simulating closer to the continuum
- Signal is very noisy → Smearing helps, needs to be included non-perturbatively



Summary

Main results

- Shown hints of non-conformality of $N_f=1$ Adj QCD [6]
- Shown stability of RHMC + Overlap fermions [6, 7]
- First application GIRS scheme to supercurrent renormalization both perturbatively [8] and non-perturbatively [9]

Outlook

- $N_f=1$ Adj QCD chirally broken → Study phase diagram, pions...
- Tuning of SUSY requires further investigation: working on possible approaches (finer lattice, smearing, $\mathcal{O}(a)$ improvement)
- Towards simulation of SQCD [10]

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