RUHR-UNIVERSITÄT BOCHUM

Simulations of collisionless plasmas: multiphysics coupling and low rank decomposition

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General Introduction

Kinetic and Fluid models:

s = electrons/ions

- Vlasov/Maxwell

$$\frac{\partial f_s}{\partial t} + \mathbf{v}\cdot\nabla_x f_s + \frac{q_s}{m_s}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_v f_s = 0$$

- 10 moment Fluid/Maxwell

$$\frac{\partial n_s}{\partial t} + \nabla\cdot(n_s\mathbf{u}_s) = 0$$
$$m_s\frac{\partial(n_s\mathbf{u}_s)}{\partial t} - n_s q_s(\mathbf{E}+\mathbf{u}_s\times\mathbf{B}) + \nabla\cdot\mathcal{P}_s = 0$$
$$s\frac{\partial\mathcal{P}_s}{\partial t} - q_s(n_s\mathrm{sym}(\mathbf{u}_s\otimes\mathbf{E}) + \frac{1}{m_s}\mathrm{sym}(\mathcal{P}_s\times\mathbf{B})) + \nabla\cdot\mathcal{Q}_s = 0.$$

- 5 moment Fluid/Maxwell

$$\frac{\partial n_s}{\partial t} + \nabla\cdot(n_s\mathbf{u}_s) = 0$$
$$m_s\frac{\partial(n_s\mathbf{u}_s)}{\partial t} - n_s q_s(\mathbf{E}+\mathbf{u}_s\times\mathbf{B}) + \frac{1}{N}\nabla(2\mathcal{E}_s - m_s n_s\mathbf{u}_s^2) + \nabla\cdot(m_s n_s\mathbf{u}_s\otimes\mathbf{u}_s) = 0$$
$$\frac{\partial\mathcal{E}_s}{\partial t} - q_s n_s\mathbf{u}_s\cdot\mathbf{E} + \frac{1}{N}\nabla\cdot\left(\mathbf{u}_s((N+2)\mathcal{E}_s - m_s n_s u_s^2)\right) = 0$$

- Maxwell

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0},\quad \nabla\cdot\mathbf{B} = 0,\quad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t},\quad \nabla\times\mathbf{B} = \mu_0\mathbf{j} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

- density $n_s = \int f_s\,\mathrm{d}\mathbf{v}$,
  mean velocity $\mathbf{u}_s = \frac{1}{n_s}\int\mathbf{v}\,f_s\,\mathrm{d}\mathbf{v}$,
  charge density $\rho = \sum_s q_s n_s$, current density $\mathbf{j} = \sum_s q_s n_s\mathbf{u}_s$,
  second moment $\mathcal{P}_s = m_s\int\mathbf{v}\otimes\mathbf{v}f_s\mathrm{d}\mathbf{v}$,
  scalar second moment $\mathcal{E}_s = \frac{m_s}{2}\int\mathbf{v}^2 f_s\mathrm{d}\mathbf{v} = \mathrm{tr}(\mathcal{P}_s)/2$,
  third moment $\mathcal{Q}_s = m_s\int\mathbf{v}\otimes\mathbf{v}\otimes\mathbf{v}f_s\mathrm{d}\mathbf{v}$
  dim. of velocity space N

Numerical Setup

- Scaling on JUWELS Booster

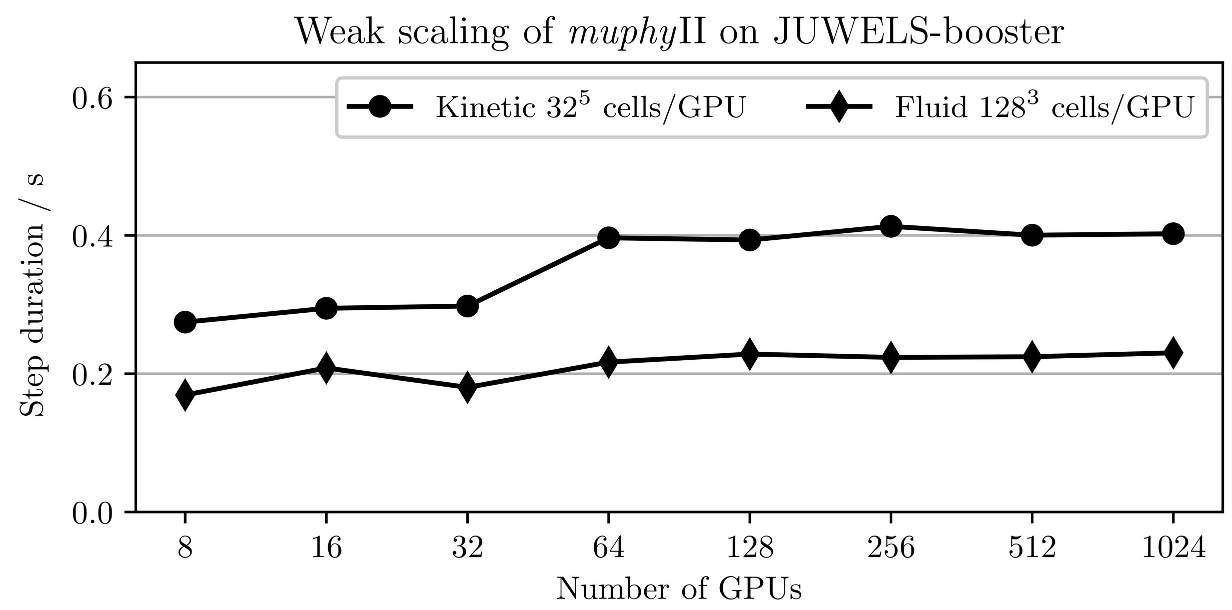

Figure 1: Computing time scaling of *muphy*II while increasing resolution and number of GPUs on JUWELS Booster at Jülich Supercomputing Centre.

- From fluid to kinetics

Scheme	Description	Criterion
VeViM	Vlasov electrons, Vlasov ions	$j > 1.25\ n_0 v_{A,0}\ \vee\ u_e > 4.0\ v_{A,0}$
F10eViM	10 moment electrons, Vlasov ions	$j > 0.75\ n_0 v_{A,0}\ \vee\ u_e > 2.0\ v_{A,0}$
F10eF10iM	10 moment electrons, 10 moment ions	$j > 0.30\ n_0 v_{A,0}\ \vee\ u_e > 1.0\ v_{A,0}$
F5eF10iM	5 moment electrons, 10 moment ions	$j > 0.10\ n_0 v_{A,0}\ \vee\ u_e > 0.5\ v_{A,0}$
F5eF5iM	5 moment electrons , 5 moment ions	else.

Table 1: Plasma models and criteria used in the coupled simulation of foreshock reconnection.

- Hierarchical Tucker

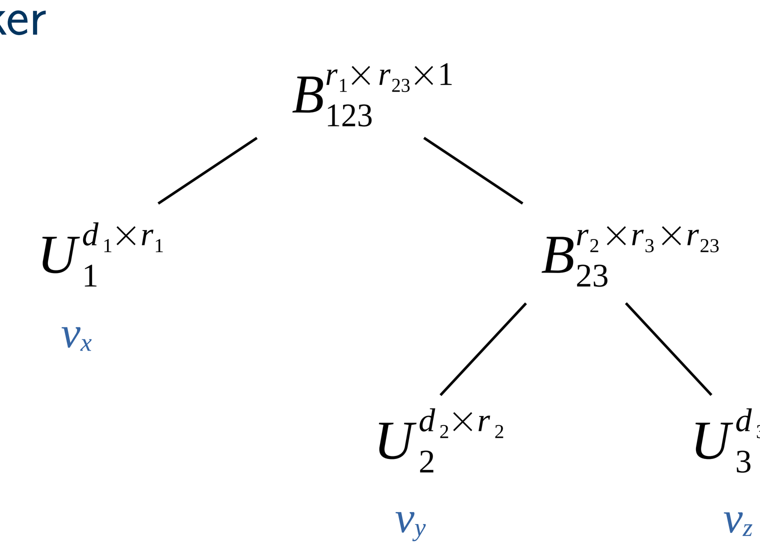

Figure 2: At each coordinate (x, y, z) the velocity distribution $f(v_x, v_y, v_z)$ is decomposed according to the shown hierarchical Tucker tree. The coordinates which the respective factor matrices belong to are highlighted in blue.

Results

- Coupled simulations

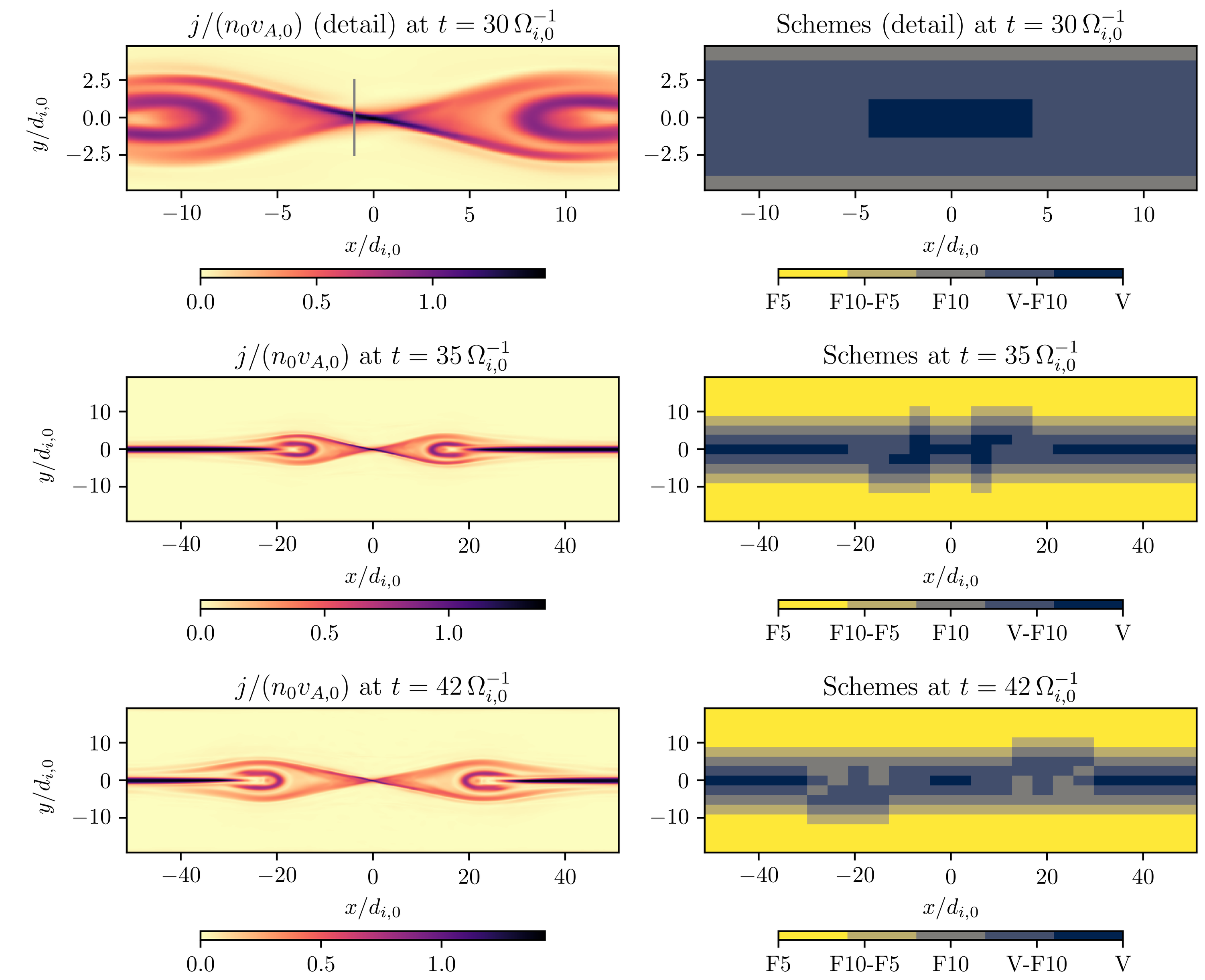

Figure 3: Current density $|\mathbf{j}|$ (left) next to the utilized plasma schemes (right) at different times in the foreshock reconnection simulation.

Animations: foreshock reconnection   thin current sheet

- Hierarchical Tucker

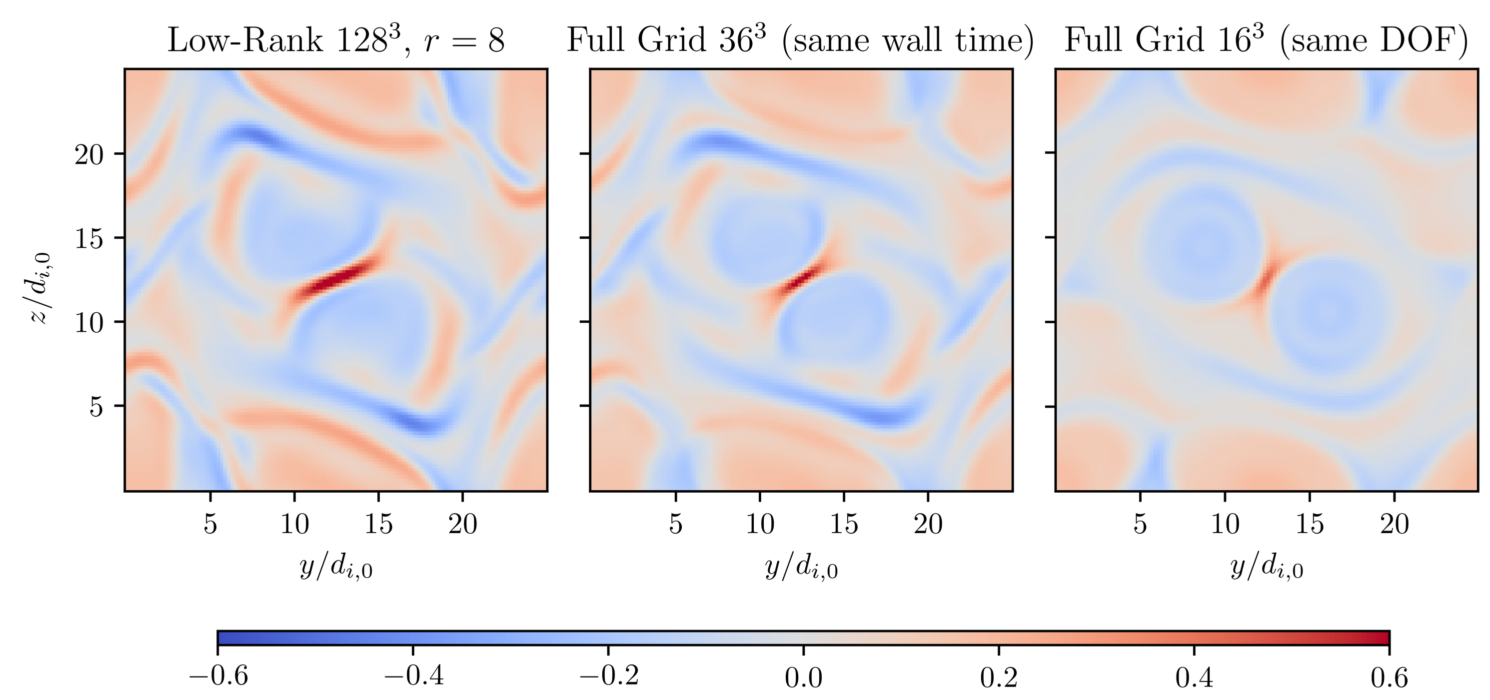

Figure 4: Out-of-plane current density at $t = 62.83\,\Omega_{i,0}$ in the low-rank simulation and comparable full grid simulations.

Summary

Results achieved so far:

- dynamic coupling of
  VeViM $\longleftrightarrow$ F10eViM $\longleftrightarrow$ F10eF10iM $\longleftrightarrow$ F5eF10iM $\longleftrightarrow$ F5eF5iM
- hierachical Tucker: speedup 70
- step to global simulations

What's next?

- coupling to MHD

Publications

F. Allmann-Rahn, R. Grauer, K. Kormann
JCP 469 (2022) 111562

F. Allmann-Rahn, S. Lautenbach, R. Grauer
JGR - Space Physics 127 (2022) 29976

F. Allmann-Rahn, S. Lautenbach, R. Grauer, R. D. Sydora
JPP 87 (2021) 905870115

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Frontiers in Physics 6 (2018) 113

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