RUHR-UNIVERSITÄT BOCHUM

A numerical study of precession driven flow using the pseudo-spectral approach combined with an immersed boundary method

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General Introduction

Goal:

- Numerical study of precession driven flow in a cylindrical cavity spinning around two axes → development of C++ code *SpedDyn*
- Focus on flows suitable for magnetic dynamo action in the *DRESDYN* dynamo experiment
- Special attention on baffles with adjustable penetration depth at the cylinder end caps with the aim to make the precession forcing more efficient

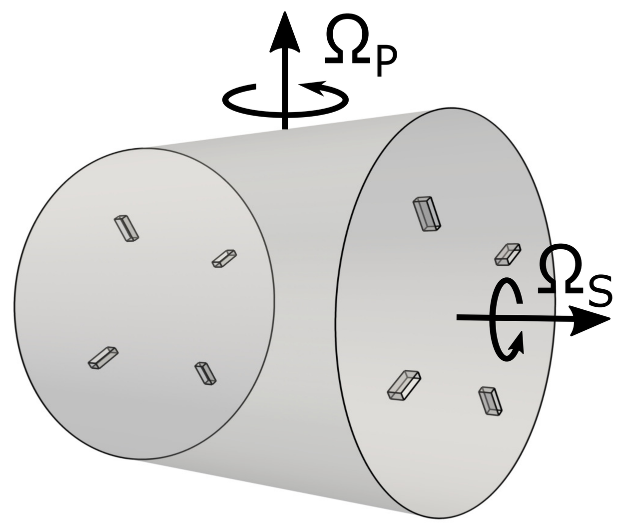


Figure 1: Schematic drawing of the precessing cylinder of radius R with (fully extended) baffles at the end caps. It spins around the symmetry axis with the angular velocity $\boldsymbol{\Omega}_S$ and is simultaneously in orthogonal precession with angular velocity $\boldsymbol{\Omega}_P$. The maximum penetration depth of the baffles is $d_0 = 0.065\,R$. They have a width of $w = 0.04\,R$ and their inner and outer radius is $r_i = 0.49\,R$ and $r_o = 0.645\,R$.

Problem:

$$\partial_t \mathbf{u} = \underbrace{\mathbf{u}\times(\nabla\times\mathbf{u})}_{\text{self advection}} + \underbrace{\frac{1}{Re}\Delta\mathbf{u}}_{\text{diffusion}} - \underbrace{\nabla p}_{\text{modified pressure}} + \underbrace{2\,\mathbf{u}\times\boldsymbol{\Omega}}_{\text{Coriolis force}} + \underbrace{\mathbf{r}\times\dot{\boldsymbol{\Omega}}}_{\text{Poincaré force}} \qquad (1)$$

$$\nabla\cdot\mathbf{u} = 0 \qquad (2) \qquad\qquad \mathbf{u} = 0 \text{ (at the boundaries)} \qquad (3)$$

Analytical predictions:

- Inviscid fluid in steadily and rapidly rotating cylinder → Kelvin modes

$$\mathbf{u}(r,\phi,z,t) = A_{mnk}\,\mathbf{R}_{mnk}(r)\,exp[i\,(m\,\phi + n\,\pi\,z + \omega_{mnk}\,t)] \qquad (4)$$

- Weak precession: Resonance of Poincaré force with mode $(m,n,k) = (1,1,1)$
- Higher precession makes forced mode break down due to non-linear interactions
- Experimental finding of excitation of double roll structure $(0,2,k)$ around $Po \sim 0.095$
- Kinematic dynamo simulations at *HZDR* show critical role of double roll for the onset of dynamo action.

Parallelization

- FFT is a global operation.
- Computational domain is split into pencils → one dimension is completely available on each process
- Processor grid is transposed using *MPI_Datatypes* and the API function *MPI_Alltoallw*

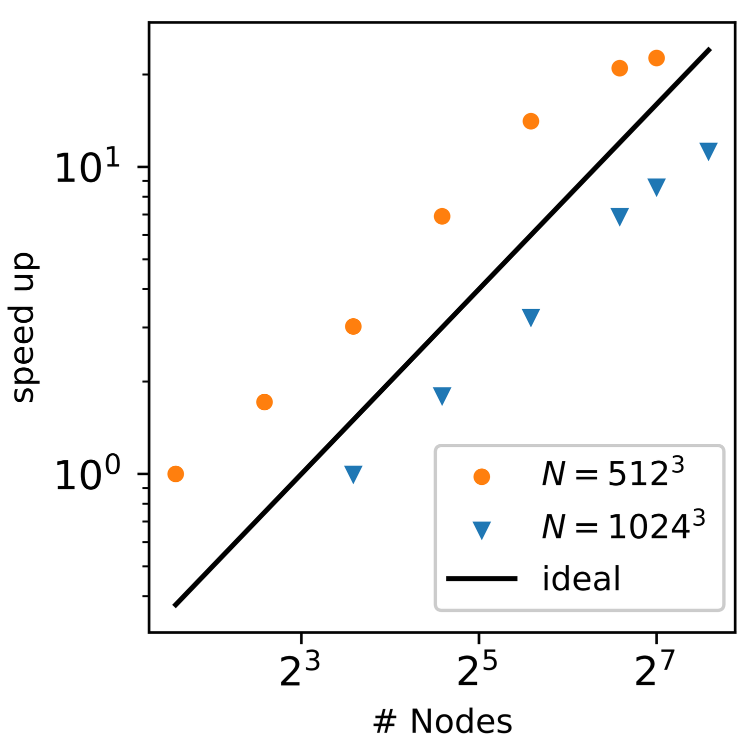


Figure 2: Scaling of *SpecDyn* for 100 time steps on a grid of size 512^3 and 1024^3. The computation was performed on the *JUWELS* CPU cluster with 48 CPUs per node. The speed up is normalized to 3 nodes for $N = 512^3$ and to 12 nodes for $N = 1024^3$.

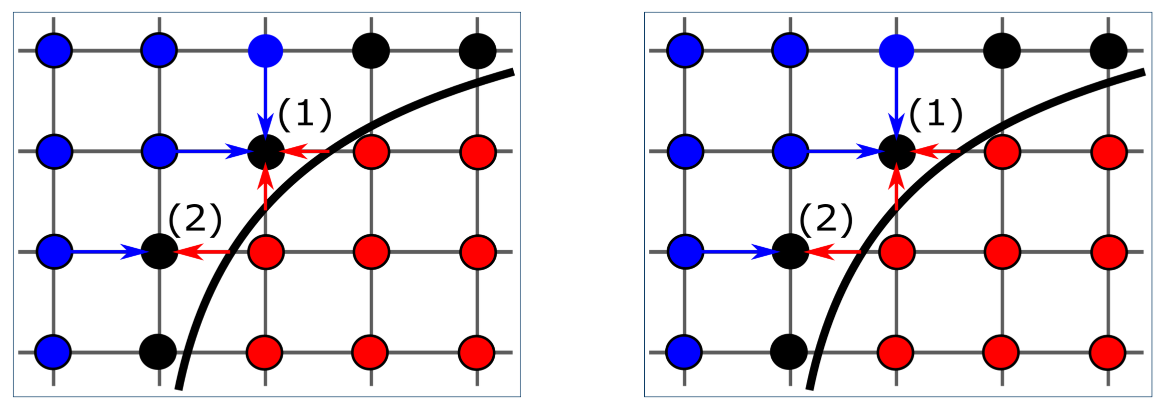
Numerical Methods

Pseudo-Spectral Method:

- Compute spatial derivatives in Fourier and multiplications in real space
- Speed advantage due to efficiency of the FFT
- Pressure Poisson equation can be directly solved in Fourier space
- Implies periodic boundaries

Immersed Boundary Method

- Boundary represented by triangular mesh
- No-slip boundary condition is enforced by linear interpolation between mesh and adjacent fluid nodes on Cartesian grid → $\mathcal{O}([\Delta x]^2)$
- Does not commute with pressure calculation → pressure corrector method ($\epsilon_u \sim \mathcal{O}(\Delta t^2)$, $\epsilon_p \sim \mathcal{O}(\Delta t^{3/2})$)



(a) Cconvex surface (b) Concave corner

Figure 3: Schematic drawing presenting the linear interpolation of the immersed boundary to the IB nodes in two dimensions for a convex surface (a) and a concave corner (b). Shown are the three different types of nodes: fluid (blue), solid (red) and IB (black). The boundary is represented by a thick black line.

Results

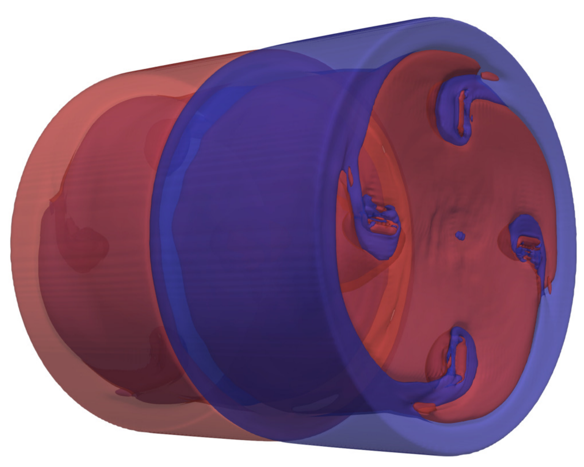


Figure 4: Contour plot of the time averaged axial velocity V_z in the mantle frame of the cylinder. The presented flow is dominated by the double roll structure.

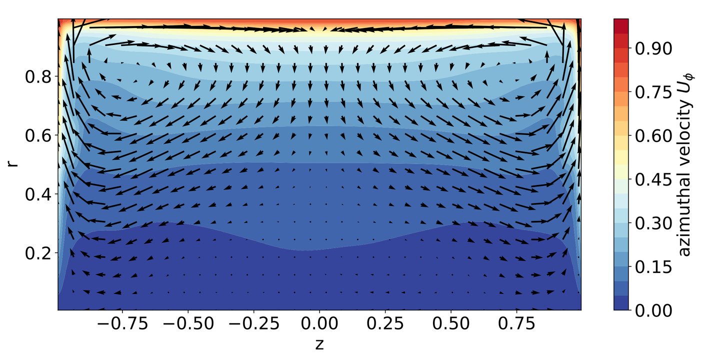


Figure 6: Azimuthal and temporal averaged velocity field in the turntable frame for extended baffles.

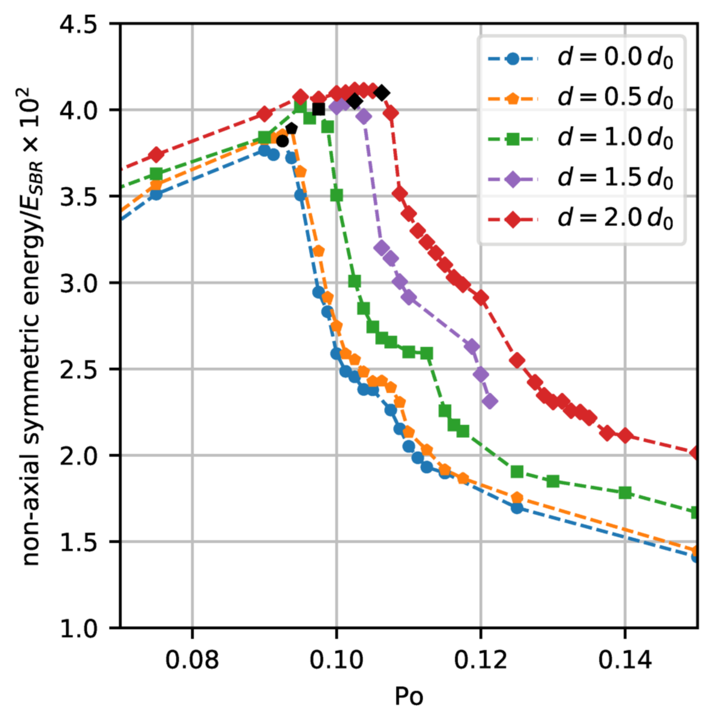


Figure 5: The non-axialsymmetric energy as a function of Po in the vicinity of the critical Poincaré number Po_E^c for different baffle lengths d.

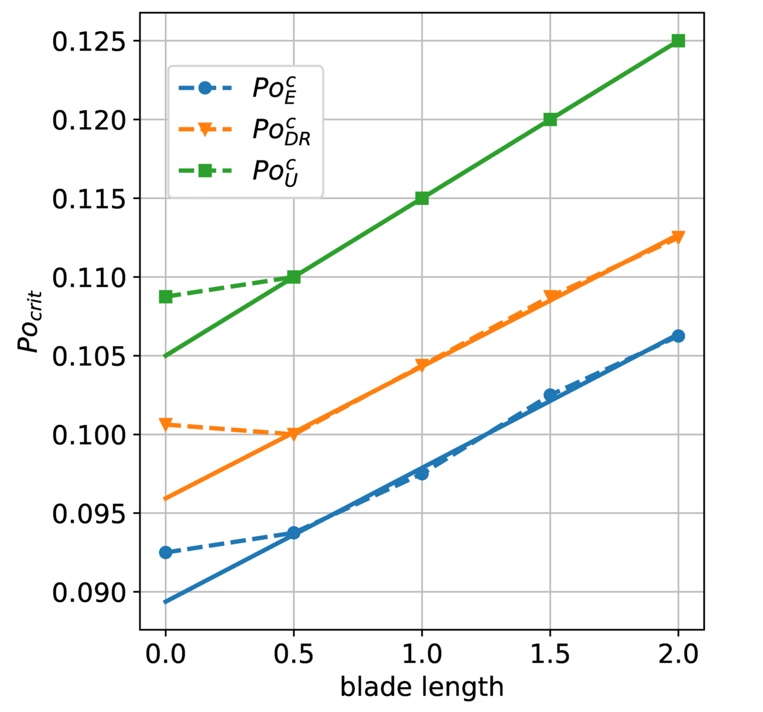


Figure 7: Critical Poincaré numbers of the non-axialsymmetric energy (Po_E^c), the double roll structure (Po_{DE}^c) and the turbulent energy (Po_U^c) as functions of the baffle length.

Outlook

- Dynamical MHD simulations already in progress
- Determination of critical magnetic Reynolds numbers for different Poincaré numbers and of the general magnetic field structure.

Publications

[1] M. Wilbert, A. Giesecke, and R. Grauer.
Numerical investigation of the flow inside a precession driven cylindrical cavity with additional baffles using an immersed boundary method.
POF, in press, 2022.

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