The Pion's Quark and Gluon Momentum Fractions

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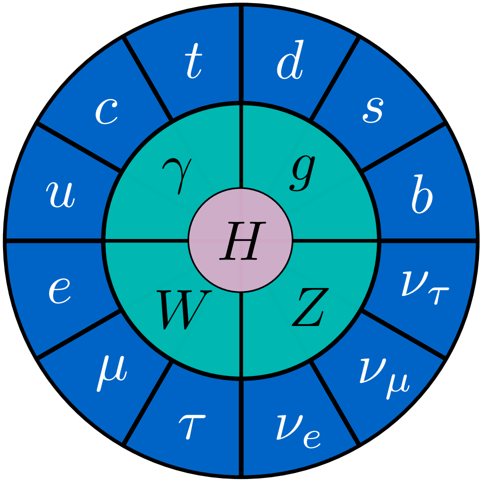
What is a Pion?



- Standard Model of Particle Physics
- combines the
  - electroweak
  - strong

  forces
- particle content

$\Rightarrow$ force carriers and matter fields

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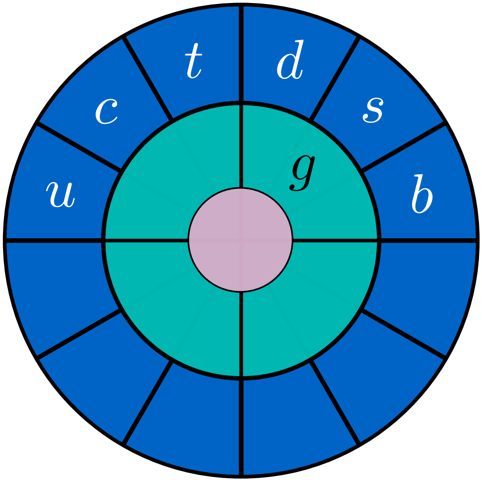

- Standard Model of Particle Physics
- combines the
  - electroweak
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  forces
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$\Rightarrow$ force carriers and matter fields

- here: focus on strong force only

$\Rightarrow$ Quantum Chromodynamics (QCD)

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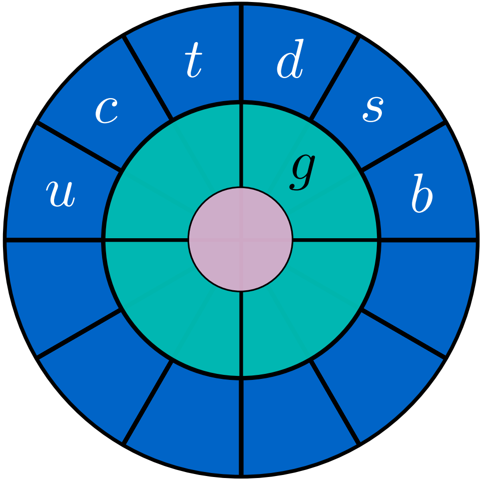


- QCD particle content
  - quarks (u, c, t, d, s, b)
  - gluons (g)
- these particles can bind

$\Rightarrow$ hadrons
  - mesons (quark-antiquark)
  - baryons (three quarks)
- the pion is the lightest hadron in the spectrum

$\Rightarrow$ π meson of light u and d (anti)quarks

Quantum Chromodynamics

QCD

Asymptotic Freedom
at high energies

- quarks and gluons become free
- interaction becomes weak

Quantum Chromodynamics

QCD

Confinement

at low energies

- quarks and gluons not observable
- only colour neutral hadrons observable

Asymptotic Freedom

at high energies

- quarks and gluons become free
- interaction becomes weak

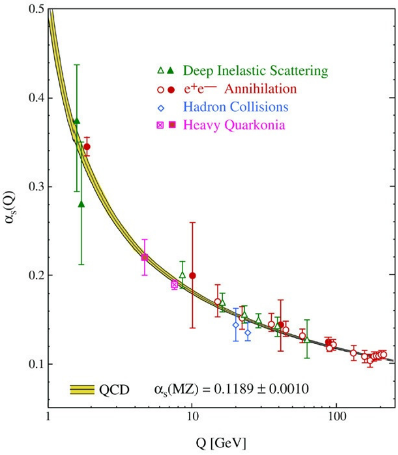
Quantum Chromodynamics

- QCD has a running coupling $\alpha(Q)$
- α large at small Q

$\Rightarrow$ QCD becomes non-perturbative

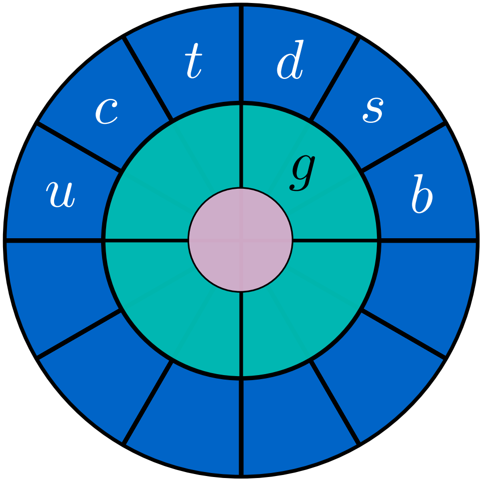
- bound states are low energy properties $Q \ll 1$
- need a non-perturbative method to study their structure

$\Rightarrow$ cannot expand in α!



[Bethke, 2006]

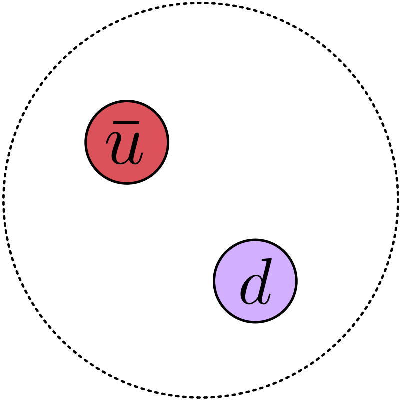
Quantum Chromodynamics



- all of QCD described by an astonishingly simple action

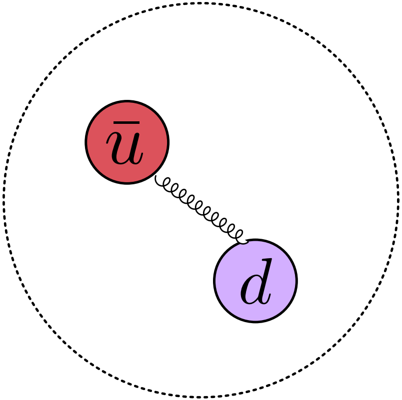
$$
\begin{aligned}
S[A_\mu, \bar{\psi}, \psi] \\
&= \int d^4x \left\{ \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}_q \left(\gamma_\mu D_\mu + m_q \right) \psi_q \right\} \\
&= S_{\mathsf{G}} + S_f
\end{aligned}
$$

What is a Pion's content?



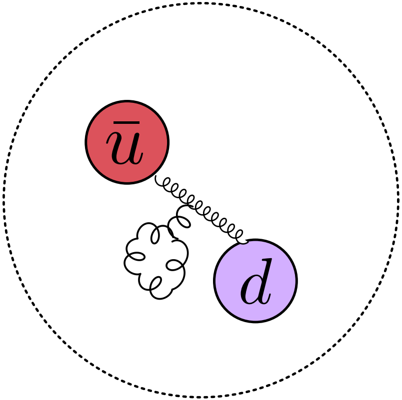
- simplest model:
  two quarks in a bag

What is a Pion's content?



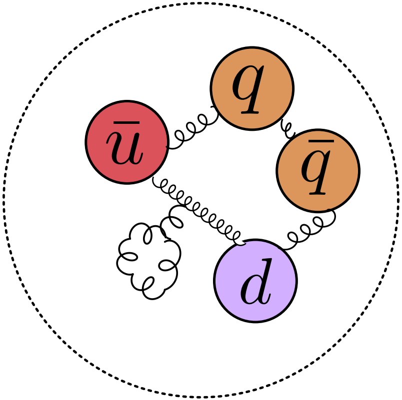
- simplest model:
  two quarks in a bag
- there are also gluons

What is a Pion's content?



- simplest model:
  two quarks in a bag
- there are also gluons
- gluons can self interact

What is a Pion's content?



- simplest model:
  two quarks in a bag
- there are also gluons
- gluons can self interact
- and there are $\bar{q}q$ pairs
  with $q = u, c, t, d, s, b$

$\Rightarrow$ with t, b too heavy to play a role

Expectation Values in the Path Integral Formulation

- expectation value of operator $\mathcal{O}$, action S

$$\langle\mathcal{O}\rangle \;=\; \int \mathcal{D}A\,\mathcal{D}\bar\psi\,\mathcal{D}\psi\,\mathcal{O}\;\exp(\,iS[\bar\psi,\psi,A]\,)$$

- Ken Wilson noticed: rotate $t \;\to\; i\tau$ to Euclidean space-time

$$\langle\mathcal{O}\rangle \;=\; \int \mathcal{D}A\,\mathcal{D}\bar\psi\,\mathcal{D}\psi\,\mathcal{O}\;\exp(\,-S_{\mathsf{E}}[\bar\psi,\psi,A]\,)$$

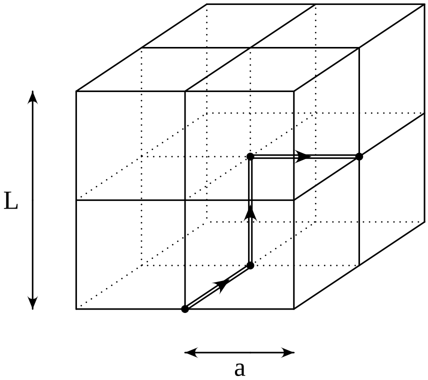
- allows to apply Monte-Carlo methods
  interpreting $\exp(-S_{\mathsf{E}})$ as a probability density

- can still obtain Minkowski space quantities

  [Osterwalder and Schrader (1973,1975)]

Lattice QCD Regularisation

- work in Euclidean space-time
- lattice regularisation: discretise space-time
  - hyper-cubic $L^3 \times T$-lattice with lattice spacing a
  - $\Rightarrow$ momentum cut-off: $k_{\mathrm{max}} \propto 1/a$
  - derivatives $\Rightarrow$ finite differences
  - integrals $\Rightarrow$ sums
  - gauge potentials A_μ in $G_{\mu\nu}$ $\Rightarrow$ link matrices U_μ ('')



- use large scale Monte Carlo methods to solve QCD

Hadron Structure

distribution of quarks and gluons in hadrons described by **parton distribution functions (PDFs)**

PDFs are functions of x, the fraction of momentum carried by a parton in a hadron

$\Rightarrow$ **important to our understanding of QCD**
Fundamental properties of QCD states

$\Rightarrow$ **important input to experiments**
needed to understand and analyse the data (together with a perturbative part)

$\Rightarrow$ **interesting to compare Theory $\leftrightarrow$ Experiment**
a discrepancy might point towards new phenomena

Pion Structure

Pion

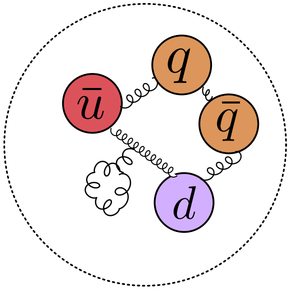
- lightest state in QCD spectrum
- Goldstone boson of spontaneously broken chiral symmetry

it turns out

- there is surprisingly little known about pion PDFs

since of course

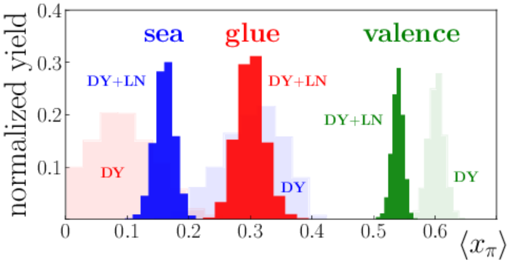
- there is no meson target for experiments!



Status Phenomenology: Strong Dependence on Data Set!

- recent Monte Carlo global QCD analysis of pion momentum fraction

  [Barry et al., PRL, 121, 152001 (2019)]
- Drell-Yan (DY) and Leading Neutron Electroproduction (LN) data
- DY: light shaded
  DY + LN: dark shaded



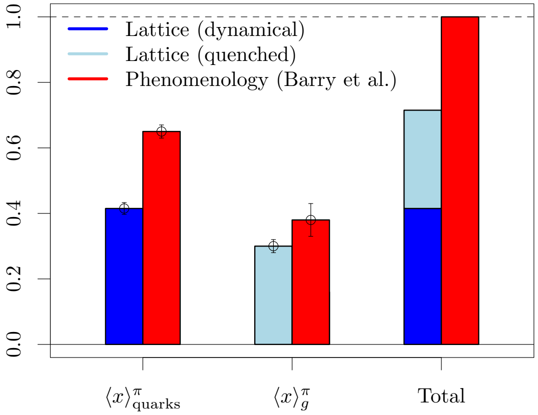
- analysis based on DY alone and DY + LN lead to significantly different pion momentum fractions
- more data and first principles theoretical analysis needed!

Status Theory (was)

- no full calculation was available!
- $\langle x \rangle_g$ only quenched

  [Meyer, Negele, Phys.Rev. D77 (2008)]
- $\langle x \rangle_{\text{quarks}}$ missing contributions

  [Oehm, CU et al., Phys.Rev. D99 (2019)]
- could not even check sum rule!
  (enforced by phenomenology)



Opportunities in Upcoming Experiments

Pion PDFs will be further constrained by upcoming experiments

- COMPASS will provide new πA DY data

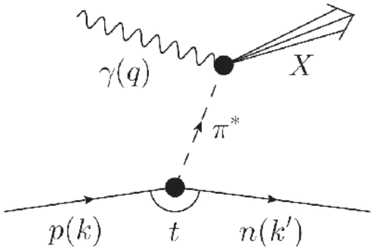
  wwwcompas.cern.ch/compass/future_physics/drellyan/

- Tagged DIS (TDIS) at JLAB
  pion structure through $ed \to eppX$

  [J. Annand et al., JLab experiment E12-15-006]

- a future electron-ion collider (eRHIC, JLEIC, LHCeC, ...)

Sullivan Process relevant for PDFs:



[A. C. Aguilar et al., arXiv:1907.08218]

Parton Distribution Functions from Lattice QCD

Nowadays two complementary approaches:

- computing moments of PDFs

  [G. Martinelli and C. T. Sachrajda, Phys.Lett. B196, 184 (1987)]

  - relate moments of PDFs to matrix elements of local operators
  - only lowest few moments accessible

- compute the x-dependence via quasi/pseudo/...-PDFs

  [X. Ji, Phys. Rev. Lett. 110, 262002 (2013), ...]

  - direct access to PDFs
  - several extrapolations and systematics need to be understood

here we focus on the moments approach

Moments of PDFs from Lattice QCD

- Mellin moments of PDFs

$$\langle x^n \rangle_q = \int_0^1 x^n \left(f_q(x) + (-1)^{n+1} f_{\bar{q}}(x)\right) dx$$

- through operator product expansion at leading twist related to matrix elements of twist two operators $\mathcal{O}_q^{\mu\nu\ldots\rho}$

$$\langle H | \bar{q} \gamma_{\{\mu} \overleftrightarrow{D}_\nu \cdots \overleftrightarrow{D}_{\rho\}}\, q | H \rangle - \text{traces}\,.$$

- note that lattice has access to
  - valence + sea contribution for n odd (e.g. $\langle x \rangle$)
  - valence only contribution for n even (e.g. $\langle x^2 \rangle$)

Average Momentum Fraction

- the Mellin moment $\langle x \rangle$ corresponds to the average momentum fraction
- e.g. for quark $\langle x \rangle_{\text{quarks}}$ a possible operator reads

$$\mathcal{O}_{44} = \bar{q}\left[\gamma_0 \overleftrightarrow{D}_0 - \frac{1}{3}\sum \gamma_k \overleftrightarrow{D}_k\right] q$$

$\Rightarrow$ single gauge covariant derivative

- similar for the gluon fraction $\langle x \rangle_g$
- sum rule:
  all contributions need to sum to one!

Different Contributions

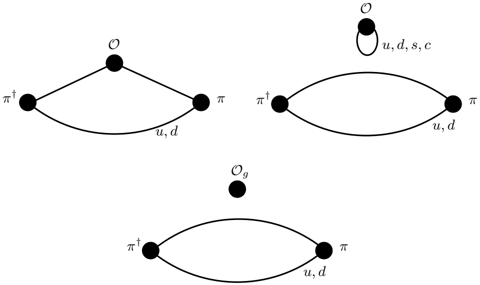
central object

$$R(t, t_f, t_i; \mathbf{p}) = \frac{\langle \pi(t_f, \vec{p})\, \mathcal{O}(t)\, \pi(t_i, \mathbf{p}) \rangle}{\langle \pi(t_f, \mathbf{p})\, \pi(t_i, \mathbf{p}) \rangle}$$

on the computational level

- quark and gluon contributions
- connected and disconnected
- **Challenge**
  - disconnected

  $\Rightarrow$ very noisy



Extracting $\langle x \rangle$

- for $t_f - t$, $t - t_i$ (and thus $t_f - t_i$) large enough

$$R(t, t_f, t_i; \mathbf{p}) \quad \to \quad \frac{1}{2E_\pi} \frac{\langle \pi(\mathbf{p}) | \mathcal{O} | \pi(\mathbf{p}) \rangle}{1 + \exp(-E_\pi (T - 2(t_f - t_i)))}$$

  with E_π the pion energy at given momentum

- at the same time need time extend $T \gtrsim 2(t_f - t_i)$ to avoid large finite size effects
- R depends on $t_f - t_i$ and $t - t_i$ only!
- then

$$\langle \pi(\mathbf{p}) | \mathcal{O} | \pi(\mathbf{p}) \rangle \quad \propto \quad \langle x \rangle$$

  up to kinematic factors

Results

- results on one ensemble
  - one discretisation length $a \approx 0.08$ fm
  - one box size $L \approx 5.1$ fm

- leading discretisation effects are of $O(a^2\Lambda^2_{\mathsf{QCD}})$

$\Rightarrow$ could be expected to be of order $\sim 2.6\%$

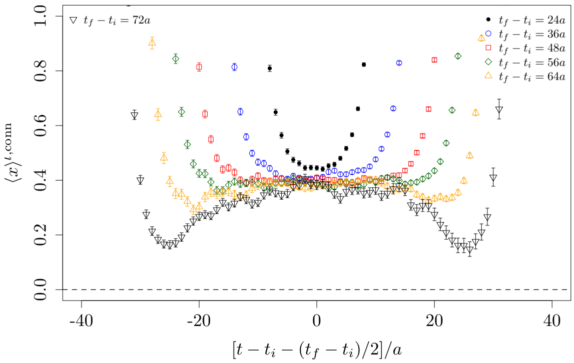
- all quark mass values tuned to their physical values

$\Rightarrow$ no need to extrapolate in the masses

- renormalised results at $\mu = 2$ GeV in the $\overline{\mathsf{MS}}$-scheme

Example: Light Connected Contributions

- extract $\langle x \rangle$ at $(t_f - t_i)/2$
- away from the center: excited state contributions
- how to combine the different $t_f - t_i$ values
- define a plateau symmetrical around $(t_f - t_i)/2$ with length t_p



Combining Different Fits

- results as function of t_f and plateau length t_p

- assign weight

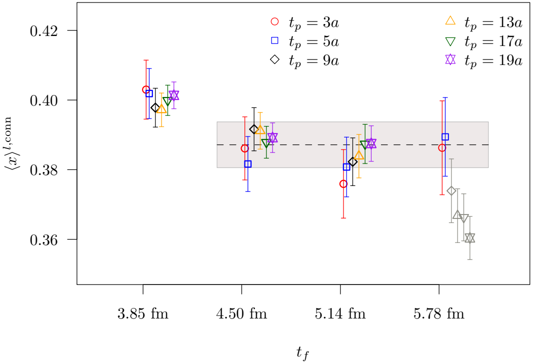
$$w = \exp\left(-\frac{1}{2}[\chi^2 - 2\text{dof}]\right)$$

  to every fit

- take weighted average

  [Borsanyi et al., Nature 593 (2021)]

- repeat on all bootstrap samples

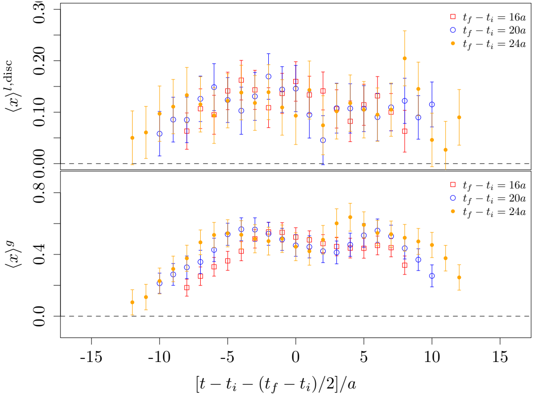


- comparison of weighted average with single fit results

Example for Disconnected Contributions: light and gluon

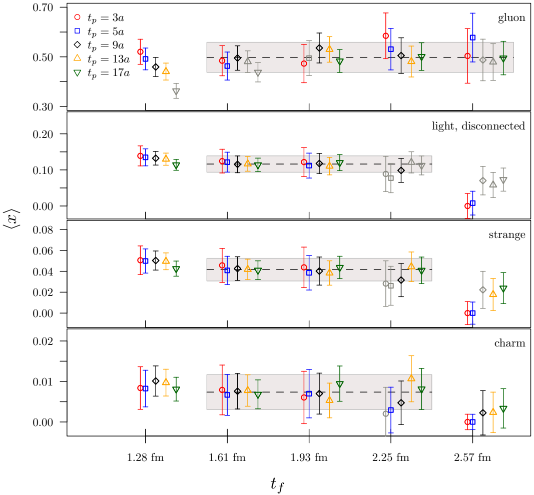
- little dependence on $t_f - t_i$ values
- little uncertainty from excited states
- significantly larger uncertainties compared to connected contribtutions

  despite significantly larger amout of resources used
- the gluon contribution is large



Gluon, Light, Strange and Charm Contributions

- the weighting procedure works well
- we can resolve all contributions
- gluon relatively largest
- charm small, but significantly non zero



Combining the Various Contributions: Renormalisation

- the result depends on the energy scale μ where this is probed
- here, there is even mixing under renormalisation

$$\begin{pmatrix} \sum_f \langle x \rangle_f^{\mathsf{R}} \\ \langle x \rangle_g^{\mathsf{R}} \end{pmatrix} = \begin{pmatrix} Z_{qq}^s & Z_{qg} \\ Z_{gq} & Z_{gg} \end{pmatrix} \begin{pmatrix} \sum_f \langle x \rangle_f \\ \langle x \rangle_g \end{pmatrix}$$

- have determined the diagonal components non-perturbatively

  [ETMC, C. Alexandrou, et al., PRD **101**, 094513 (2020)]

- off-diagonal, mixing contributions perturbatively

  [ETMC, C. Alexandrou, et al., PRD **101**, 094513 (2020)]

- mixing currently not significant!

Distinguish the Renormalised Contributions

- want to compare separate contributions to phenomenology
- for this we need to separate the contributions again
- define $\delta Z_{qq} = Z^s_{qq} - Z_{qq}$

$$\langle x\rangle_f^{\mathsf{R}} = Z_{qq}\langle x\rangle_f + \frac{\delta Z_{qq}}{N_f}\sum_{f'}\langle x\rangle_{f'} + \frac{Z_{qg}}{N_f}\langle x\rangle_g\,,$$

$$\langle x\rangle_g^{\mathsf{R}} = Z_{gg}\langle x\rangle_g + Z_{gq}\sum_{f'}\langle x\rangle_{f'}\,.$$

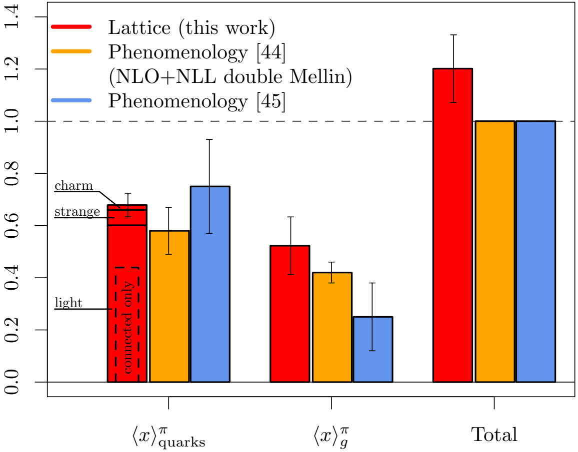
- again, δZ_{qq} zero within errors

Main Result

- first time: computation of all components
- including mixing (perturbatively)
- sum rule fulfilled within errors
- large gluon contribution

Compare with

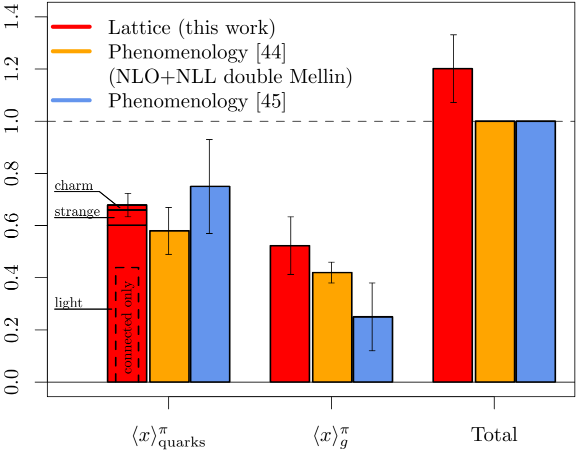
- [[44] Barry et al., PRL 127, (2021)]
- [[45] Novikov et al., PRD 102 (2020)]



Discussion

- statistical errors need to be reduced
- mixing needs to be studied non-perturbatively
- why is the gluon contribution so large?
- only a single lattice spacing value

$\Rightarrow$ need to take continuum limit



Summary

- computed all relevant quark and gluon momentum fractions to the pion
- sum rule fulfilled within error bars
- fair agreement with phenomenology

Outlook

- continuum limit / non-perturbative mixing
- compute higher moments $\langle x^n \rangle$

Thanks to ...!

- the lattice QCD group in Bonn:
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- the ETM collaboration
  inparticular: D. Alexandrou, U. Wenger

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- ... **and for your attention!**