Comparing QCD Thermodynamics in the DWF and HISQ Discretization Schemes

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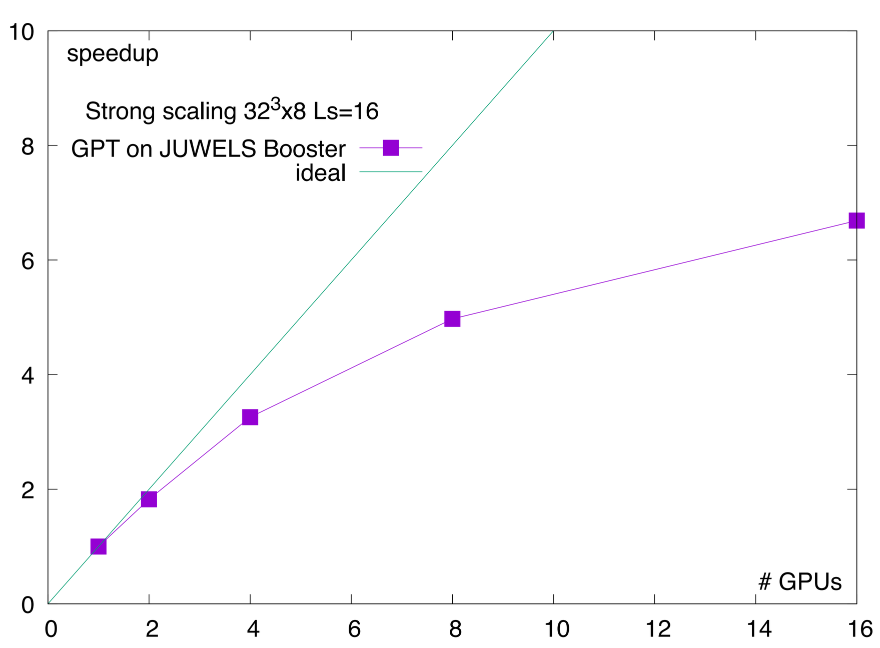
INTRODUCTION

- Understanding the chiral transition in the QCD phase diagram with 2 light quark flavors and its impact on strongly interacting matter is a key challenge in QCD thermodynamics.
- It is an ongoing effort to determine if the anomalously broken $\mathbb{U}_A(1)$ part of chiral symmetry is effectively restored at the same crossover transition temperature, T_c, as the nonsinglet part, and what is the symmetry group near T_c, which defines the universality class of the chiral transition.
- It is important to study chiral symmetry on the lattice using discretizations where the chiral and continuum limits are disentangled.
- The QCD phase transition is mainly studied using **staggered fermions**.
  - These are computationally cheaper than chiral fermion discretizations.
  - However, they preserve only an remnant $\mathbb{U}(1) \times \mathbb{U}(1)$ part of chiral symmetry on the lattice, introducing additional cutoff effects, so-called **taste violations**.
  - Large lattices are required for observables sensitive to the light scalar and pseudo-scalar sectors.
- In this work we used the **Möbius domain-wall fermions (MDWF)** discretization, which preserves chiral symmetry on the lattice to a good extent, to study the thermodynamics of the chiral transition in QCD.
- Observables studied, sensitive to different subgroups of chiral symmetry:
  - Chiral condensate: $\langle \bar{\psi}\,\psi \rangle_f = \frac{1}{N_\sigma^3 N_\tau} \langle \mathrm{tr}[M_f^{-1}\,\partial_{m_f}\, M_f] \rangle$
  - Subtracted condensate: $\Delta_{ls} = (m_s\, \langle \bar{\psi}\psi \rangle_\ell - m_\ell \langle \bar{\psi}\psi \rangle_s)/T^4$
  - χ_{disc} is the disconnected part of the mass derivative of the chiral condensate.
  - Topological susceptibility: $\chi_{\mathrm{top}} = \frac{T}{V}\, \langle Q^2 \rangle$.
- $m_\ell^2\, \chi_{\mathrm{disc}} - \chi_{\mathrm{top}}$ measures breaking of nonsinglet chiral symmetry.
- Esimate $\mathbb{U}_A(1)$ breaking from the temperature dependence of χ_{top}.

COMPUTER RESOURCES & ALGORITHMS

- We use code written by us using the `Grid Python Toolkit (GPT)` [2], which is based on the `Grid` [3] library.
- Runs were performed on the **JUWELS Booster** on single nodes parallelized over 4 GPUs.
- We generated $32^3 \times 8$ lattices with $L_s = 16$ for 7 temperatures.
- New optimizations for the HMC algorithm to achieve $\geq 20\%$ faster generation of gauge configurations compared to Ref. [4].
  - 6 steps of Hasenbusch mass preconditioning with evenly-spaced ratios, instead of 5 steps as used in [4].
  - 8 fermion integrator steps, instead of 12 as used in [4], keeping 4 steps for the gauge integrator.
  - 24% faster generation at $T = 159\,\mathrm{MeV}$, and over 30% faster generation at higher temperatures, with an acceptance rate of ≥ 0.8.



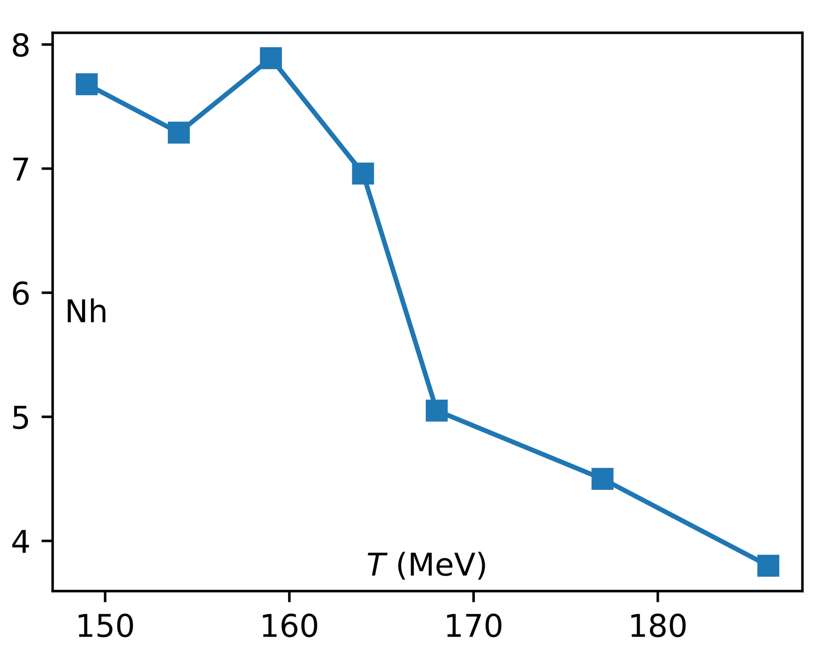
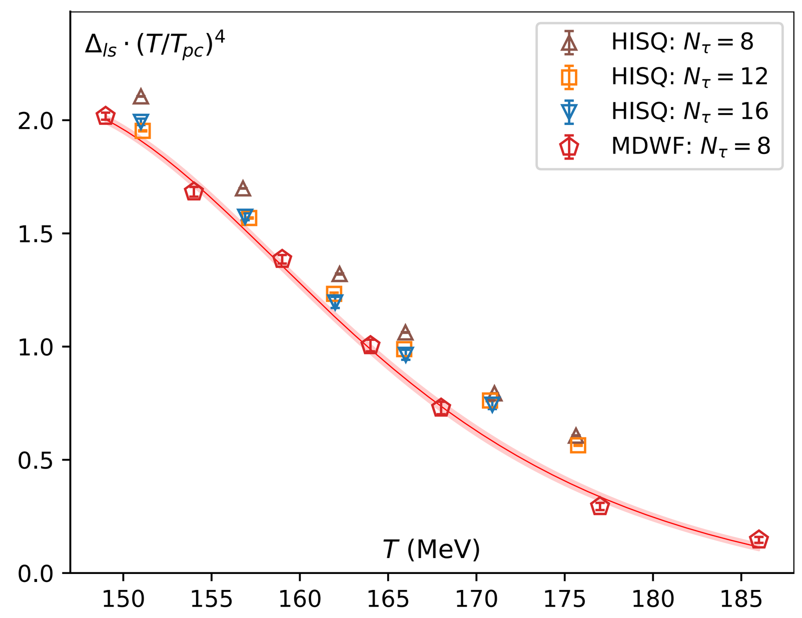
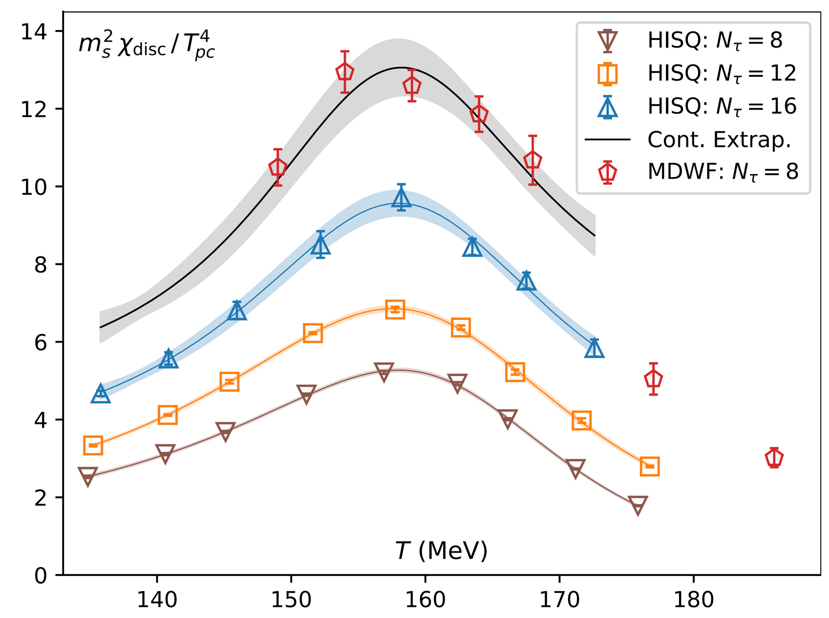


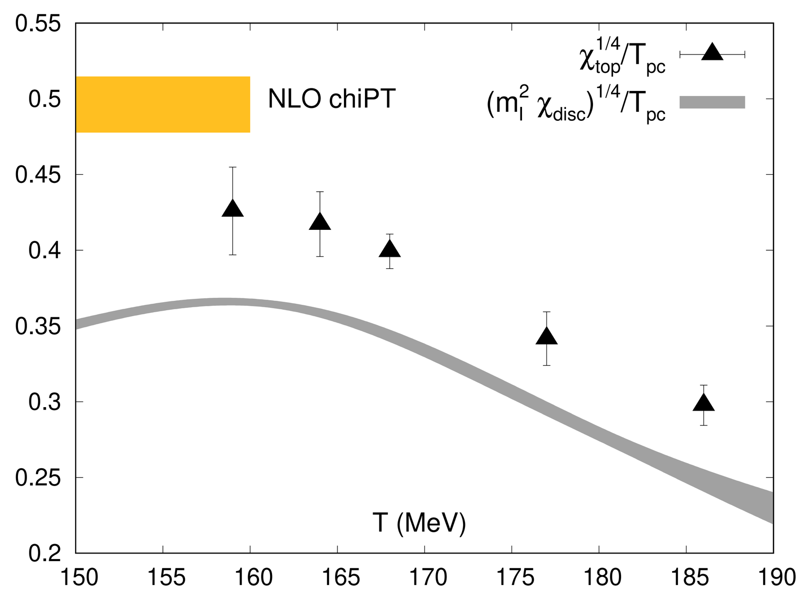
Figure 1: Generation of gauge configurations with dynamical MDWF for $N_\sigma = 32$, $N_\tau = 8$ and $L_s = 16$, using the `Grid` library on the JUWELS Booster. **Left:** Strong scaling plot. **Right:** Node hours for 5 HMC steps of unit length.

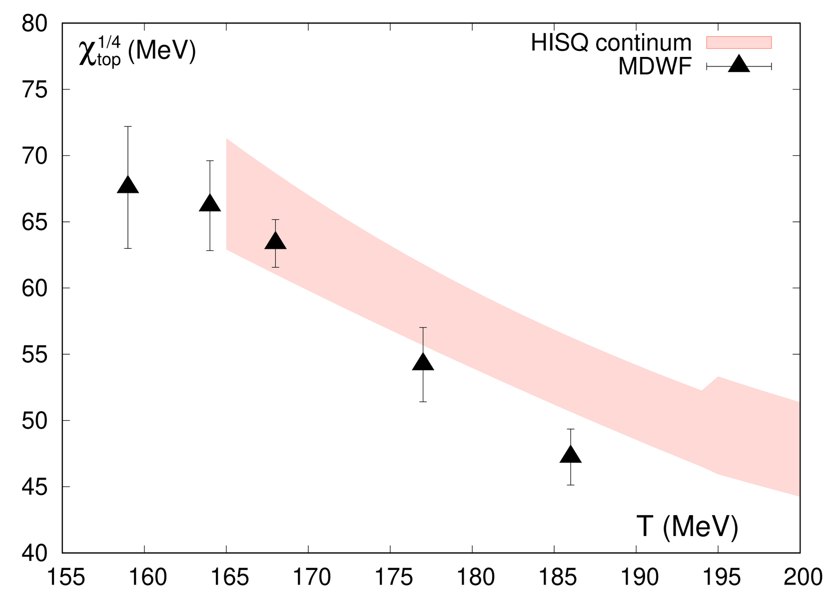
RESULTS





- Peak location of MDWF $m_s \chi_{\mathrm{disc}}/T_{\mathrm{pc}}$ defines a pseudo-critical temperature:
$$T_{\mathrm{pc},N_\tau=8} = 158.7^{+2.6}_{-2.3}\,\mathrm{MeV}.$$
  - This is consistent with continuum extrapolated HISQ and stout results [5].
- Comparison of $\chi_{\mathrm{disc}}(T)$ for HISQ $N_\tau = 8, 12, 16$ and MDWF $N_\tau = 8$
  - HISQ continuum estimate $T_{\mathrm{pc}}^{\mathrm{HISQ}} = 157.4(7)\,\mathrm{MeV}$.
  - MDWF results agree well with HISQ continuum estimate even at $N_\tau = 8$.
  - Highlights importance of using lattice actions with disentangled chiral and continuum limits.





- **Left Figure:** Computed χ_{top} as the variance of the number of zero modes of the overlap Dirac operator.
  - The χ_{top} deviates from $\mathrm{NLO}\chi\mathrm{PT}$ prediction and at $T > T_{\mathrm{pc}}$ approaches $m_\ell^2\,\chi_{\mathrm{disc}}$. ⇒ Effective resotarion of nonsinglet chiral symmetry.
  - At $T > 160\,\mathrm{MeV}$ we find the temperature dependence of $\chi_{\mathrm{top}}^{1/4}/T_{\mathrm{pc}}$ to be $(T/T_{\mathrm{pc}})^{-\alpha}$ with $\alpha = 2.8(1)$.
  - $\alpha > 2$ implies a nontrivial $\mathbb{U}_A(1)$ breaking up to $T = 186\,\mathrm{MeV}$, which is *not* compatible with a dilute instanton gas approximation (DIGA).
  - The phase transition of (2+1)-favor QCD in the massless limit is likely of 2nd order with $\mathbb{O}(4)$ critical exponents.
- **Right Figure:** Compared $\chi_{\mathrm{top}}^{1/4}$ on MDWF $N_\tau = 8$ against HISQ continuum extrapolated results, obtained using improved clover definition, after performing Symanzik flow.
  - Even on a finite $N_\tau = 8$ lattice, the MDWF results are consistent with the continuum extrapolated HISQ results.
  - Chiral symmetry on the lattice plays signficant role for topological sampling.
- **The singlet $\mathbb{U}_A(1)$ and nonsinglet parts of chiral symmetry are not restored at the same temperature.**

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