# QCD thermal phase transition and chiral fermions

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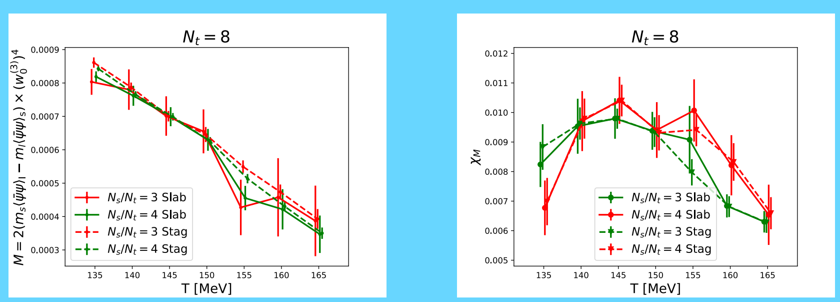
## Introduction

Simulation in lattice QCD with staggered fermions: QCD thermal phase transition is a crossover [1]. But staggered fermions do not have the full chiral symmetry at finite lattice spacing. Chiral fermions are needed!

This project: QCD thermal phase transition with overlap fermions: chiral symmetry at finite lattice spacing. Detailed setup in [2].

Simulations are done in the fixed topology $Q = \mathrm{const}$, we determine the topological susceptibility and sum contributions of different topological sectors.

## Chiral condensate and susceptibility

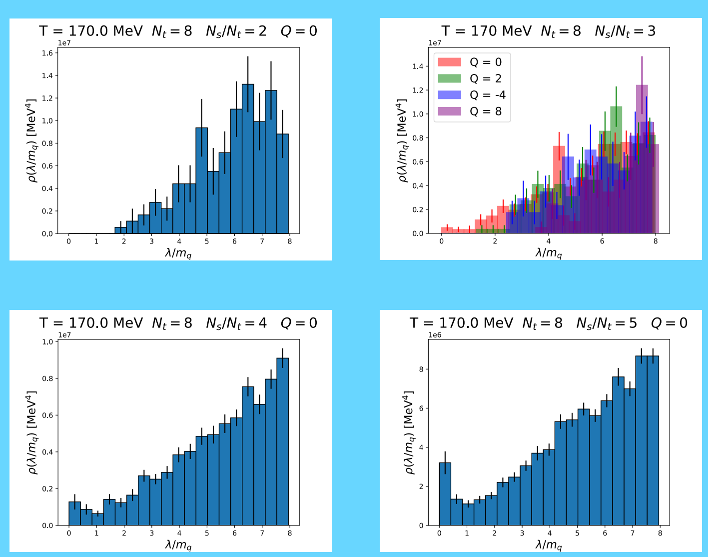



We sum the results in different sectors $Q$ using the topological susceptibility $\chi_Q$ from:

Stag: staggered based data [4]

Slab: overlap results at fixed $Q$

## Spectrum of the Dirac operator




$$D^\dagger_{\mathrm{ov}} D_{\mathrm{ov}} |e_i\rangle = \lambda_i^2 |e_i\rangle$$

Chiral symmetry (Banks-Casher relation):

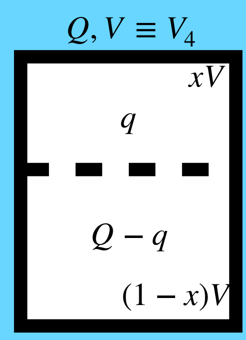
$$\bar\psi\psi \propto \int \frac{m}{\lambda^2 + m^2}\rho(\lambda) \xrightarrow[m\to 0]{} \rho(\lambda = 0)$$

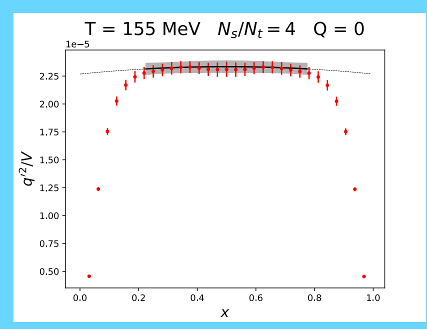
Axial symmetry: $\chi_A = \chi_\pi - \chi_\delta \propto \int d\lambda \frac{m^2}{(m^2+\lambda^2)^2}\rho(\lambda)$

Peak in the spectral density $\rho(\lambda \to 0)$, seen for large aspect ratios $\frac{N_s}{N_t} \gtrsim 4$

Possible explanation: gas of free instantons [5]

## Topological susceptibility from fixed $Q$ simulations (slab method) [3]



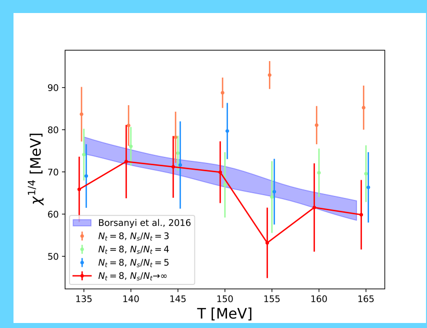

Probability of topological charge $q$ in the subvolume $xV$:

$$p(q, Q-q) \propto p_1(q)p_2(Q-q) \propto e^{-\frac{q^2}{2\chi V x}} e^{-\frac{(Q-q)^2}{2\chi V(1-x)}} \propto e^{-\frac{1}{2\chi V}\frac{q'^2}{x(1-x)}}$$

$$q' = q - xQ$$

$$\langle q'^2 \rangle \propto \chi V x(1-x)$$

FV corrections: $V \to \infty$




Results for topological susceptibility $\chi_Q$: agreement with staggered based data from [4]

## Summary and further steps

- Thermal phase transition with chiral fermions at $N_t = 8$
- Topological susceptibility $\chi_Q$ from simulations at fixed $Q = \mathrm{const}$
- Chiral observables summed over different topological sectors using $\chi_Q$ determined from overlap simulations: purely overlap result!
- Our results suggest that the transition is a crossover
- Peak in the spectral density $\rho(\lambda \to 0)$
- Next steps: continuum limit

## References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4. Sz. Borsanyi et al. Nature 539 (2016) 7627, 69-71.
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