Flavour Structure of Baryons and SU(3) Low Energy Constants

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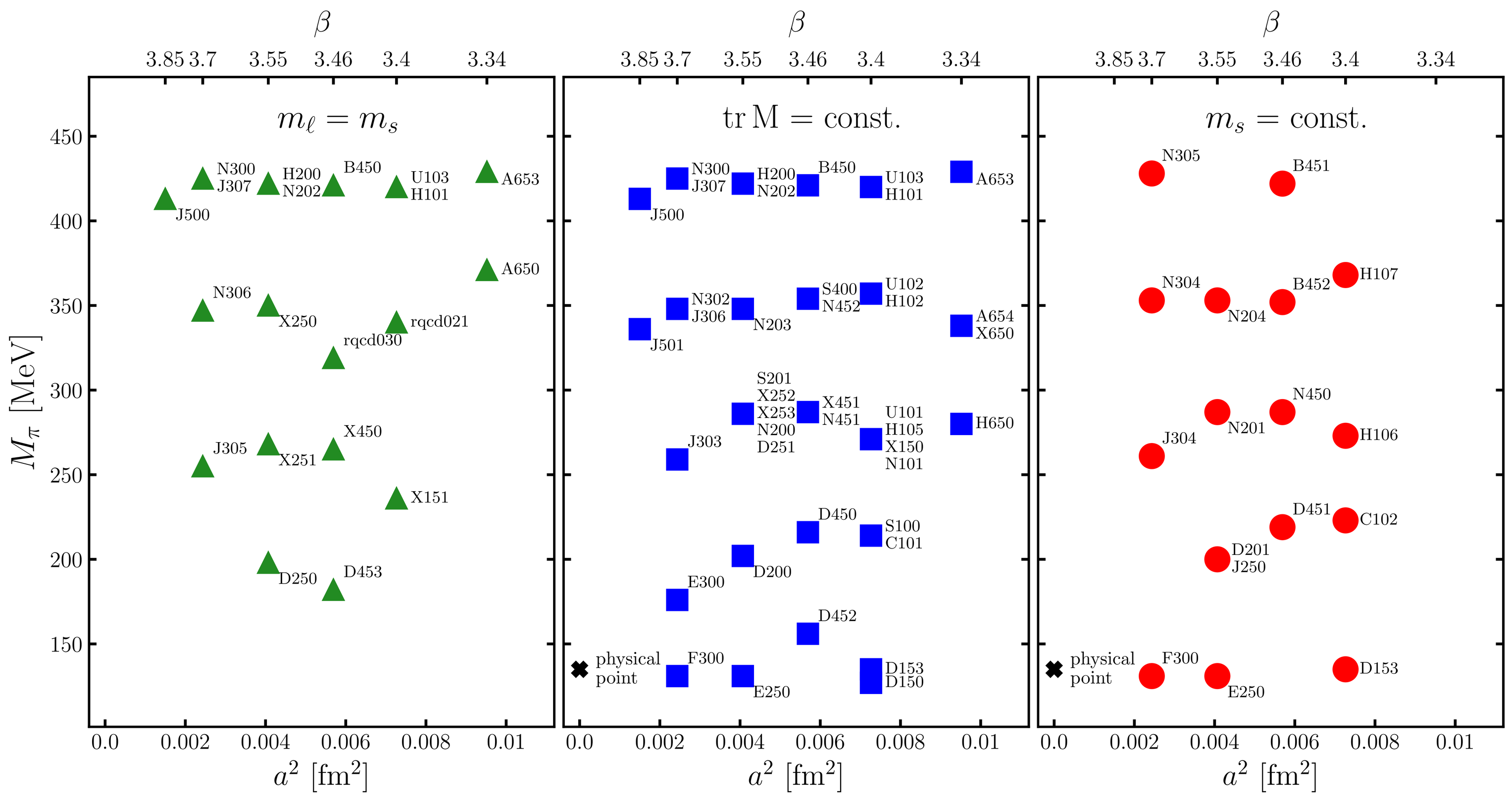
In fixed target and collider experiments, like at CERN, Geneva, or at the upcoming Electron Ion Collider (EIC) at Brookhaven National Laboratory (New York), the structure of the constituents of nuclei, i.e. protons and neutrons (nucleons), is extensively studied. We know that nucleons are made of quarks and gluons, however, less is known about how these building blocks arrange themselves to form a nucleon. While protons and neutrons make up the bulk of everything we see in the universe, the quark masses account only for a small fraction of their total mass. In contrast, the dynamics of the massless gluons gives rise to over 90 per cent of the visible mass of the universe. Insight into this world of the strong force can be obtained from numerical studies within the framework of lattice Quantum Chromo Dynamics (QCD). Here we report on results for the mass spectrum, test the validity of quark flavour symmetry relations and predict low energy constants of SU(3) chiral perturbation theory (ChPT).

Coordinated Lattice Simulations (CLS) gauge ensembles

Large scale effort to generate gauge field configurations (in a Markov chain using the hybrid Monte Carlo (HMC) algorithm) to perform a well controlled continuum limit



- $N_f = 2+1$ flavours of nonperturbatively $\mathcal{O}(a)$ improved Wilson fermions with tree-level Symanzik improved gauge action
- Open boundary conditions in time for small lattice spacings to avoid freezing of the topological charge
- Three different trajectories: Symmetric line ($m_\ell = m_s$), constant average quark mass ($\mathrm{Tr}\, M = 2m_\ell + m_s = \mathrm{const.}$) and constant physical strange quark mass ($m_s = \mathrm{const.}$)
- Six different lattice spacings with 0.039 fm $\lesssim a \lesssim$ 0.098 fm, pion masses from 430 MeV down to 130 MeV and volumes between $3.3 \leq LM_\pi \leq 6.4$
- High statistics (typically between 1000 and 2000 configs each)
- Ensemble generation still ongoing to further improve control over systematic effects

Chiral and continuum extrapolation of baryon masses

Chiral extrapolation including finite volume effects by NNLO BChPT

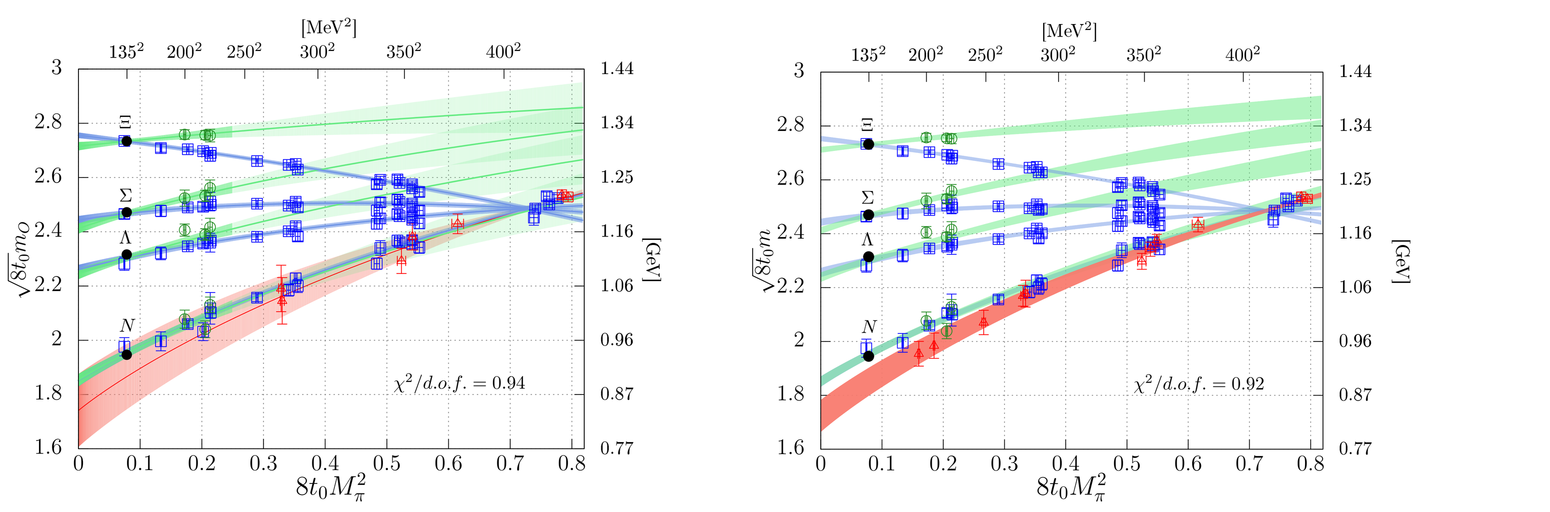
- Parameterized by only 6 LECs: m_0, $\bar{b}$, δb_O (combinations of b_0, b_F, b_D), F and D

$$m_0 + \bar{b}\bar{M}^2 + \delta b_O \delta M^2 + \frac{m_0^3}{(4\pi F_0^2)}\left[g_{O,\pi} f_O\left(\frac{M_\pi}{m_0}\right) + g_{O,K} f_O\left(\frac{M_K}{m_0}\right) + g_{O,\eta_8} f_O\left(\frac{M_{\eta_8}}{m_0}\right)\right]$$

- Analysis of octet and decuplet (not shown here) baryons including all systematic errors
- Continuum limit extrapolation (additional 6 parameters): $1 + a^2\left(c + \bar{c}\bar{M}^2 + \delta c_O \delta M^2\right)$

$\Rightarrow$ achieved sub-percent level accuracy of masses and Wilson flow parameter $\sqrt{t_{0,\mathrm{ph}}}$ [1]

$\Rightarrow$ ongoing effort to further improve on the error on $\sqrt{t_{0,\mathrm{ph}}}$ $\leftarrow$ important for scale setting



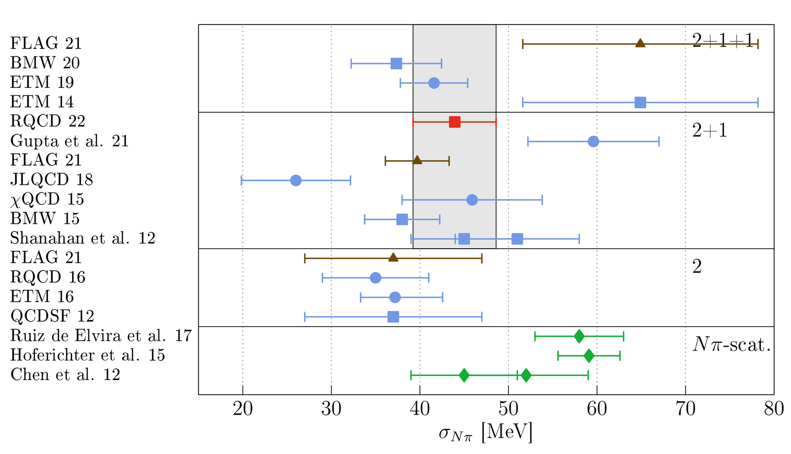
RQCD (2023, Ref. [1]) RQCD (2025, preliminary)

Sigma terms of the baryon octet

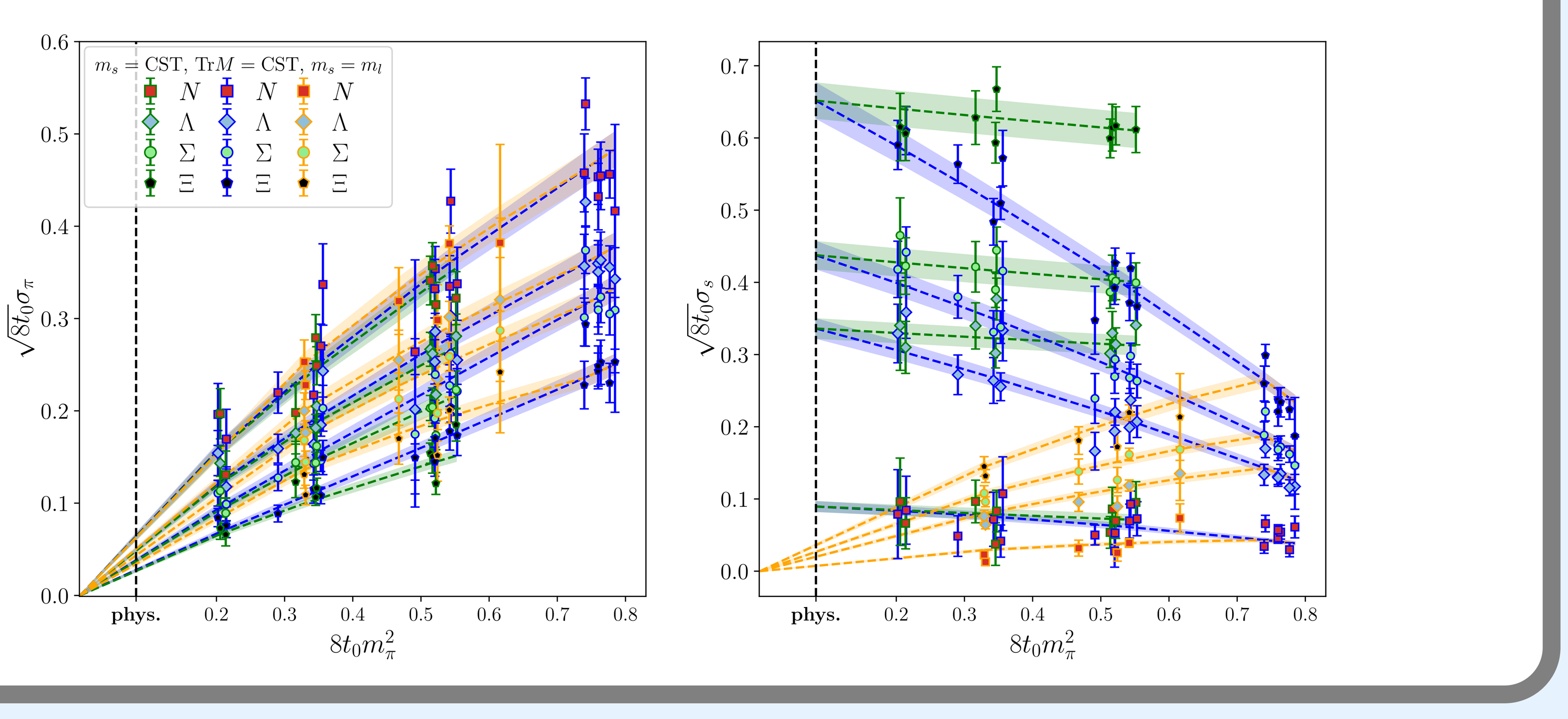
Defined by scalar singlet matrix elements

$$\sigma_{qB} = m_q \langle B|\bar{q}q|B\rangle = m_q \frac{\partial m_B}{\partial m_q}$$

$$\sigma_{\pi B} = \sigma_{uB} + \sigma_{dB}$$

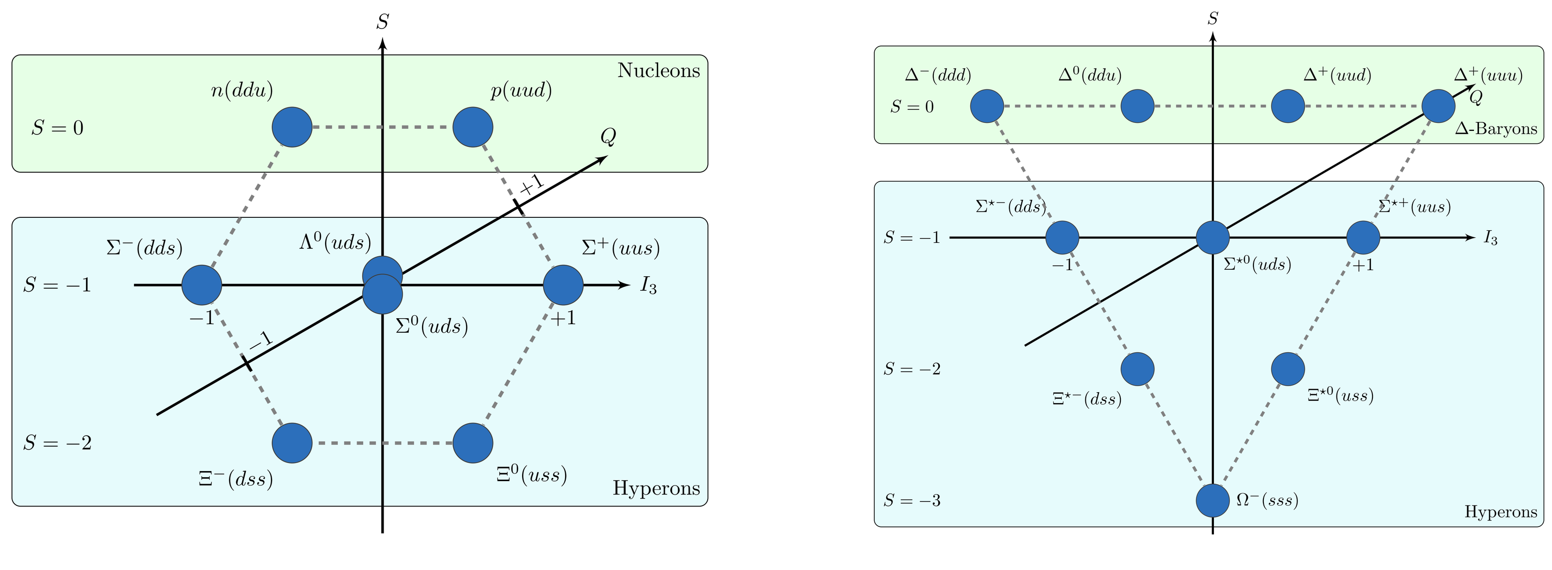


- Direct determination in progress (prelim. results [2] as data points in lower plots)
- Quark mass dependence of $\sigma_{\pi B}$ and σ_{sB} via the Feynman-Hellmann theorem and NNLO BChPT [1] consistent with direct determination (error bands lower plots) and determinations from literature (upper plot)



SU(3) baryon flavour structure

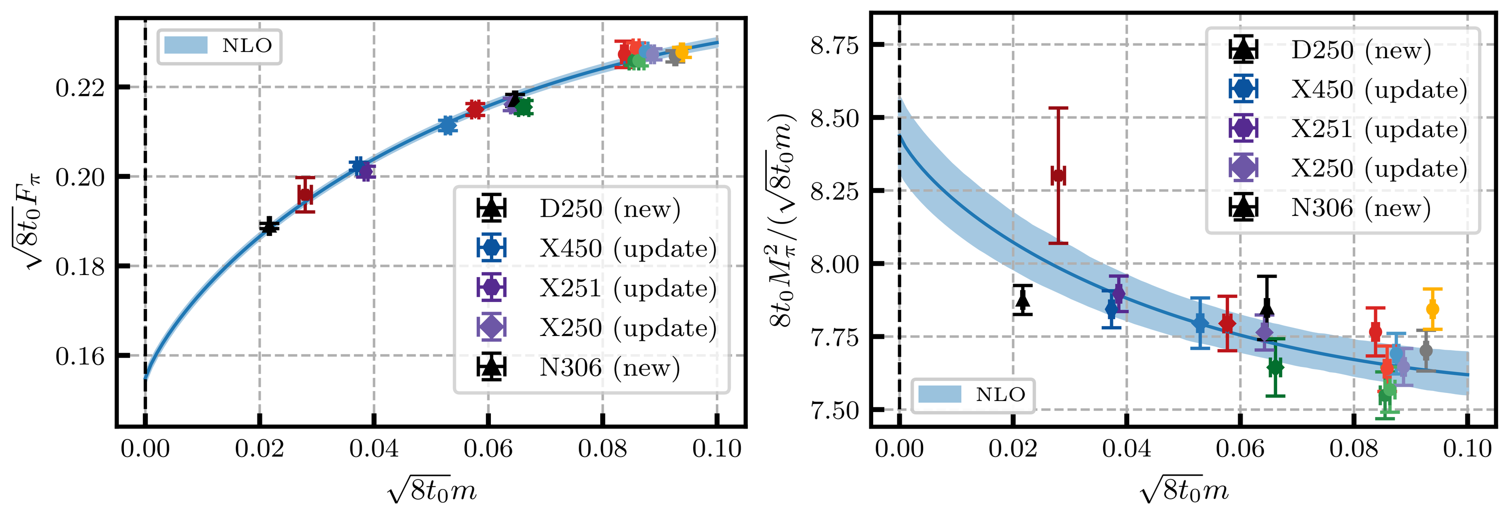
Including hyperons (i.e. baryons containing at least one valence strange quark) provides a wealth of additional information and allows to extract SU(3) low energy constants (LECs)



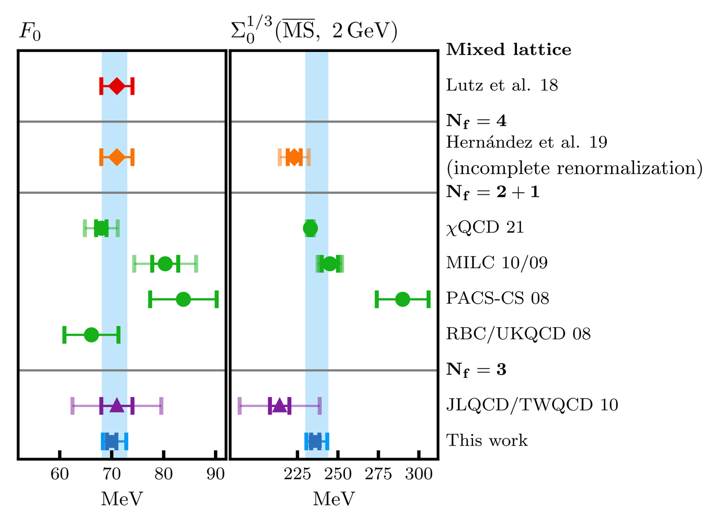
Spin $\frac{1}{2}$ baryon octet Spin $\frac{3}{2}$ baryon decuplet

Combined analysis of baryonic and mesonic SU(3) LECs

Octet baryon masses and the axial charges for the nucleon and the Σ baryon are analyzed together in order to determine the leading order baryonic SU(3) LECs



- Pilot study for $N_f = 3$ on the $m_\ell = m_s$ trajectory [3] (including mesonic LECs F_0 and B_0)
- Consistent description of the pion mass and volume dependence of the axial charges and the octet baryon mass was possible with the same set of LECs
- Extend analysis to the $N_f = 2+1$ case in order to further improve the accuracy, to test the applicability range of SU(3) ChPT and also to determine higher order LECs



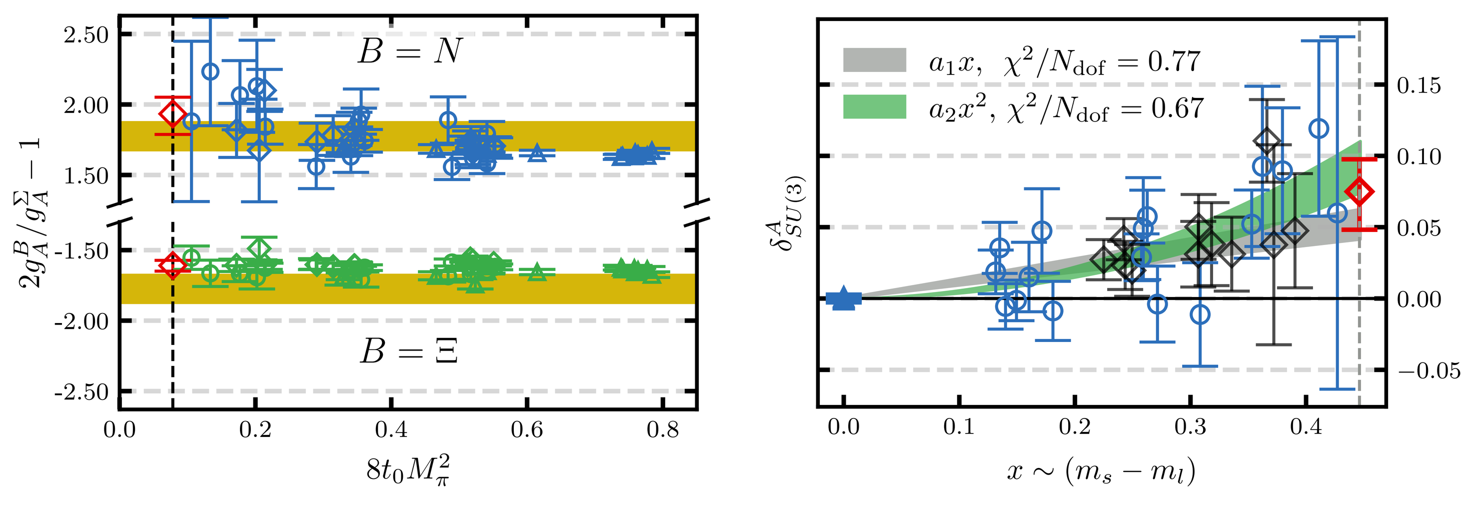
Hyperon charges and SU(3) symmetry breaking

Hyperon charges are experimentally hard to access but determinations from lattice QCD via computation of matrix elements $g_J^B = \langle B|O(J)|B\rangle$ with $J \in \{A, S, T\}$ are possible

- Decomposition of the axial charges in LECs: $g_A^N = F + D$, $g_A^\Sigma = 2F$, $g_A^\Xi = F - D$
- Investigation of SU(3) flavour symmetry breaking, publication in preparation [4]

$\Rightarrow$ preliminary results, e.g. axial charges (at physical point, shown by red diamonds):

$$\delta^A_{\mathrm{SU}(3)} = \frac{g_A^\Xi + g_A^N - g_A^\Sigma}{g_A^\Xi + g_A^N + g_A^\Sigma} = 0.075^{(23)}_{(27)},\ g_A^N = 1.284^{(28)}_{(27)},\ g_A^\Sigma = 0.875^{(30)}_{(39)},\ g_A^\Xi = -0.267^{(13)}_{(12)}$$



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References

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