Quantum Sampling on the Quantum Annealer for an Effective Theory of Lattice QCD

UNIVERSITÄT BIELEFELD
Faculty of Physics

NIC Symposium 2025

Jangho Kim, Wolfgang Unger - Bielefeld University
Thomas Luu - IAS-4 & JARA-HPC, FZ Jülich

CRC-TR 211
Strong-interaction matter under extreme conditions

JÜLICH Forschungszentrum | JUNIQ

Motivation

Why strong coupling LQCD on a quantum annealer?

- ***Lattice gauge theory*** is the only non-perturbative, gauge-invariant method to study QCD
- The ***dual representation*** for staggered fermions with U(3) gauge group [1] is well suited for quantum computing as it has a discrete state space

$$Z = \sum_{\{d,m\}} \prod_{b=(n,\hat{\nu})} \frac{(N_c - d_\nu(n))!}{N_c! d_\nu(n)!} \gamma^{2 d_\nu(n) \delta_{\hat{0},\hat{\nu}}} \prod_n \frac{N_c!}{m(n)!} (2am_q)^{m(n)}$$

- Has been studied classically via the Worm algorithm
- Becomes rather ***expensive at low temperatures*** $aT = \frac{\gamma^2}{N_t}$
- Dual variables d, m can be mapped on binary vector.

Annealing and the QUBO Formalism

- Quantum annealers are promising to study aspects of LGT [2]
- The array of qubits can be modelled as an Ising spin glass:

$$H_{Ising} = -\sum_{i<j} K_{ij} \sigma_z^i \sigma_z^j + \sum_i h_i \sigma_z^i \,,$$

- A transverse field is applied with time-dependent coefficients $A(t)$, $B(t)$

$$H(s) = -A(t) \sum_i \sigma_x^i + B(t) H_{Ising} \,.$$

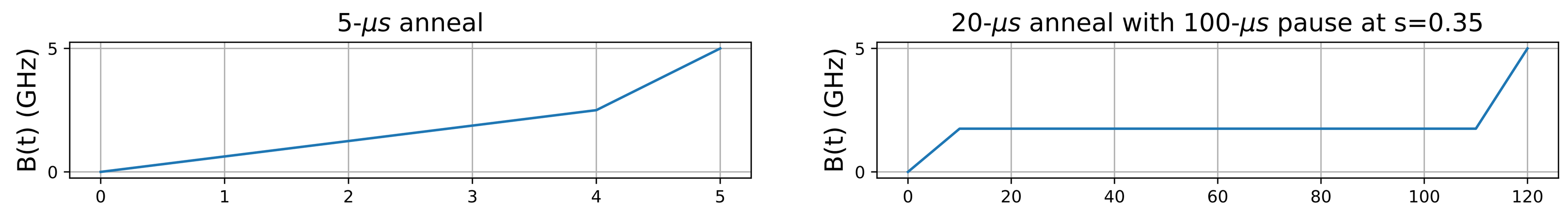

Figure 1: Our choice of annealing profiles. The behavior of $A(t)$ is roughly inversely proportional to $B(t)$. The x-axis represents time in μ-secs.

- During the annealing process, the target Hamiltonian is minimized wih respect to a binary vector x, based on a quadratic unconstrained binary optimization (QUBO) matrix Q [2]:

$$\chi^2 = x^T Q x \,, \qquad Q = W + p(A^T A + \mathrm{diag}(2b^T A)).$$

- We use the Pegasus topology: 5760 qubits

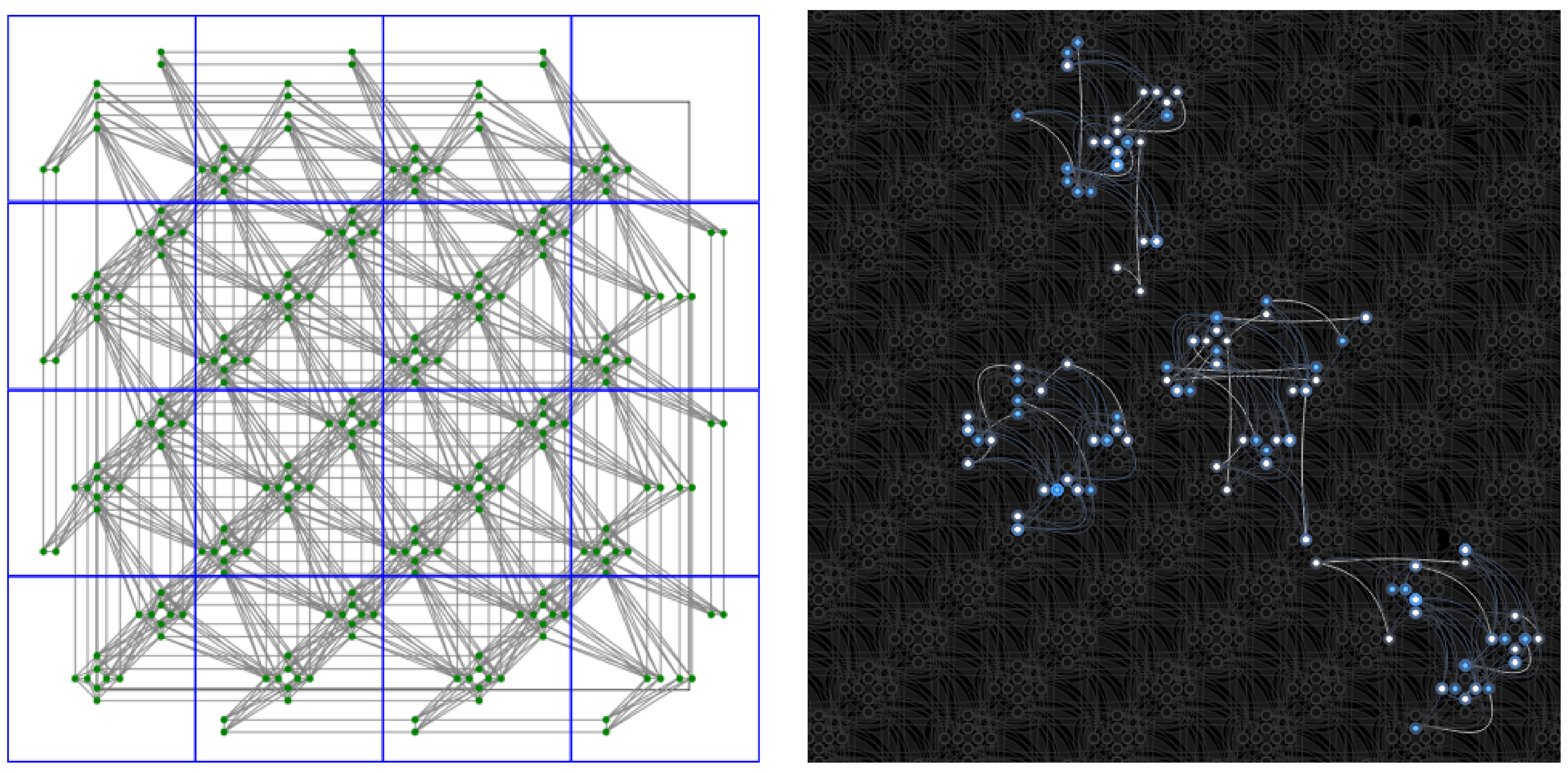

Figure 2: *Left:* connectivity between qubits of the Pegasus topology, showing a part of the full 5760 qubits. *Right:* embedding of sub-lattices in parallel, here shown for a subset of 4, using automatic embedding [3].

- We construct 2×2 building blocks to be sampled in parallel
- The weight matrix W for SC-LQCD is diagonal (with M a monomer term that is quark-mass dependent term, D_s a term for spatial dimers, D_t a term for temporal dimers which is temperature-dependent).

$$W = \mathrm{diag}(M, M, M, M, D_s, D_s, D_t, D_t)$$

- The constraint (A, b) is weighted by a penalty factor p, A is not diagonal, and the boundary b of each 2×2 sub-lattice depends on external dimers d_{ext}:

$$b = (3 - d_{ext}^{(1)}, 3 - d_{ext}^{(2)}, 3 - d_{ext}^{(3)}, 3 - d_{ext}^{(4)})$$

- Generalization of the QUBO formalism for SU(3) gauge group [4] and away from strong coupling limit is feasible

Quantum Sampling

- Our sampling strategy is based on histograms that D-Wave provides through the QUBO matrix that depend on b
- We compare exact Boltzmann distribution $h_{true}(b)$ to approximate distributions from D-Wave $h_p(b)$.

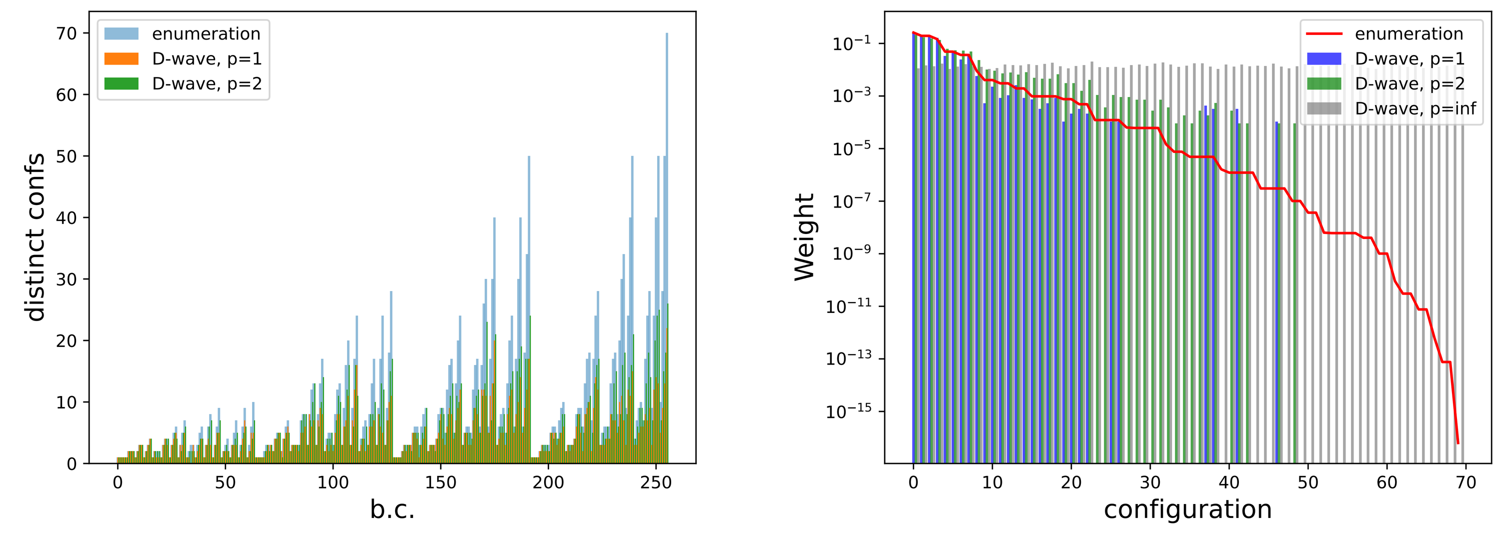

Figure 3: Comparison for 2×2 sub-lattices between exact histograms from enumeration with those generated by D-Wave for $p = 1$ and $p = 2$. *Left:* Comparison of the multiplicities for 256 different boundaries. *Right:* Comparison of the weights for a specific boundary b =(3,3,3,3) for $am_q = 1.0$ and $\gamma = 0.1$,

- We use a hybrid approach, a classical Metropolis-Hastings algorithm follows the QUBO determination of the histograms:

$$P_{\text{accept}} = e^{-S_{\text{new}} + S_{\text{old}}} \frac{h_{\text{old}}}{h_{\text{new}}}$$

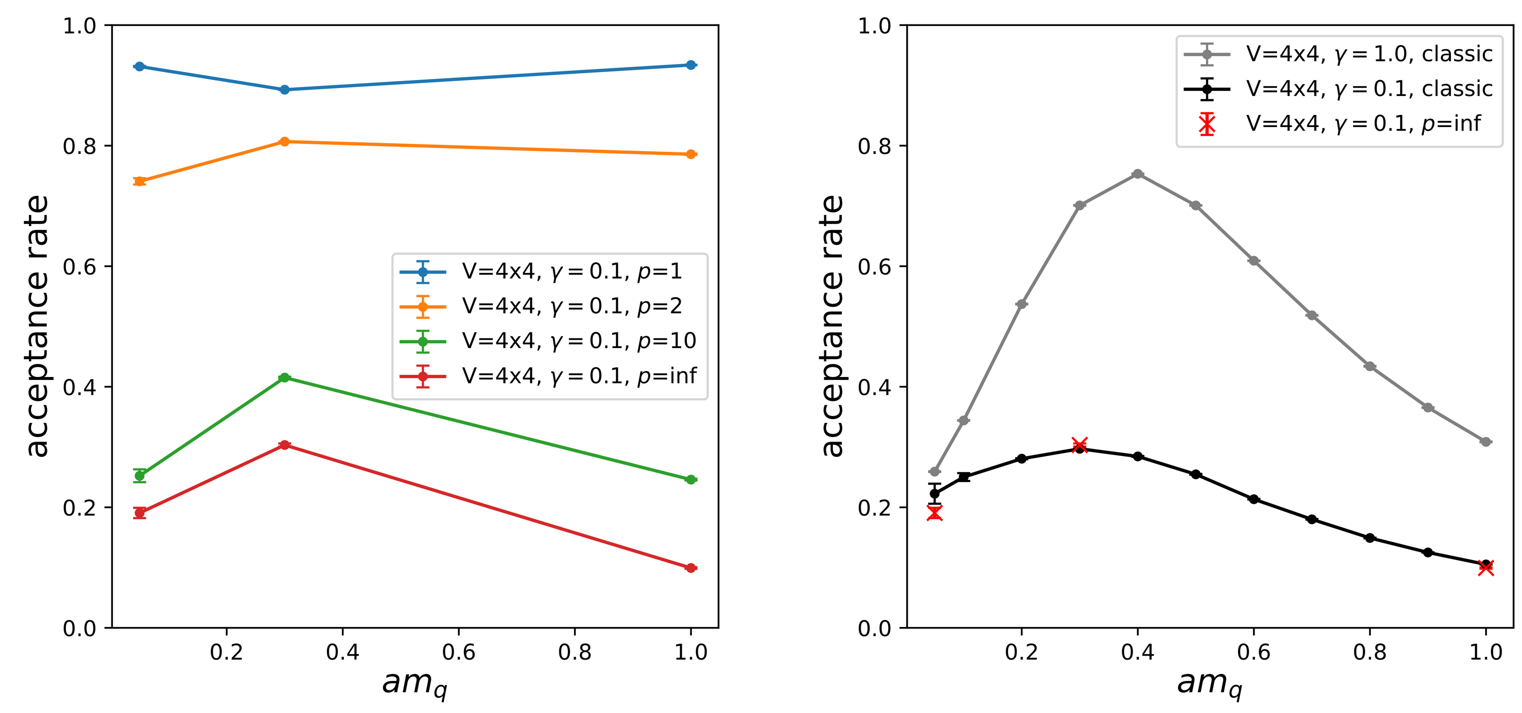

Figure 4: Comparison of the acceptance rate. *Left:* for a 4×4 lattice from Metropolis-Hastings, for various penalty factors p. *Right:* Metropolis from classical computation. Metropolis-Hastings for $p = \infty$ reproduces the classical Metropolis acceptance rate.

Large Volume Results

- On large volumes and low temperatures, D-Wave produces smaller errors compared to classical Worm simulations

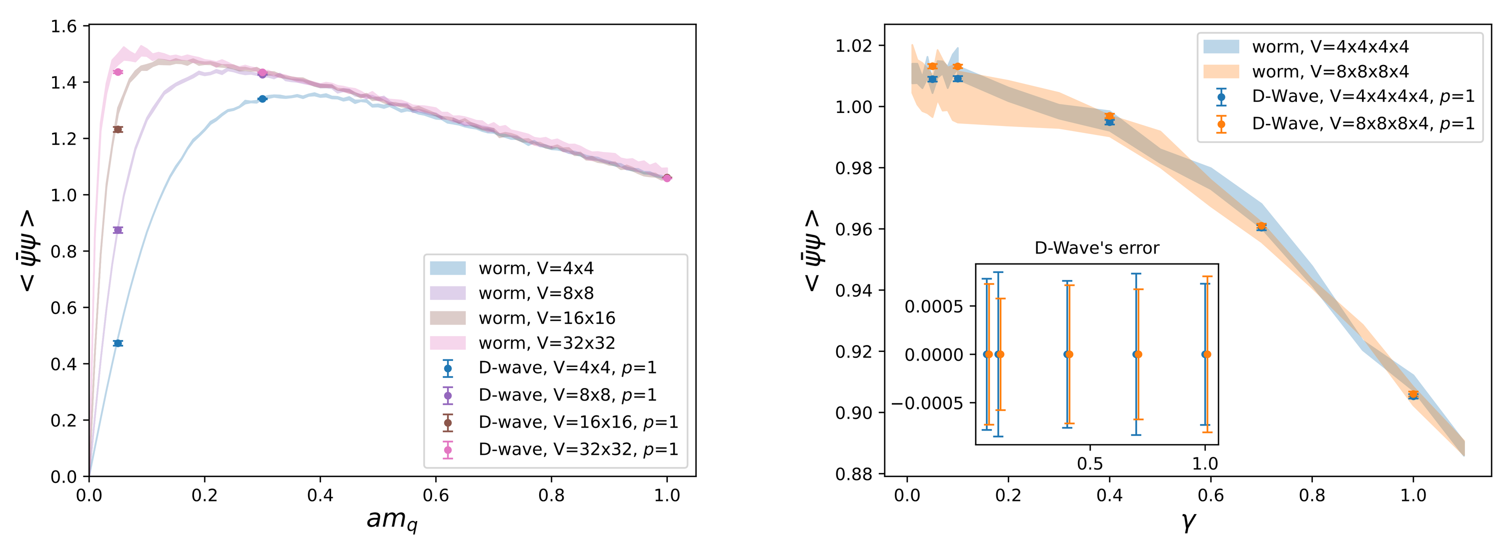

Figure 5: Chiral condensate as a function of the quark mass am_q (left) and γ (right)

Acknowledgement & Literature

The authors gratefully acknowledge the Jülich Supercomputing Centre for funding this project by providing computing time on the D-Wave Advantage™ System JUPSI through the Jülich UNified Infrastructure for Quantum computing (JUNIQ).

[1] P. Rossi and U. Wolff, Lattice QCD With Fermions at Strong Coupling: A Dimer System,

[2] S. A Rahman, R. Lewis, E. Mendicelli, and S. Powell, SU(2) lattice gauge theory on a quantum annealer, Phys. Rev. D 104, 034501 (2021),

[3] J. Kim, T. Luu, and W. Unger, U(N) gauge theory in the strong coupling limit on a quantum annealer, Phys. Rev. D 108, 074501 (2023),

[4] J. Kim, T. Luu, and W. Unger, Testing importance sampling on a quantum annealer for strong coupling SU(3) gauge theory, in 40th International Symposium on Lattice Field Theory (2023)