Non-perturbative renormalization of gluon and quark flavor singlet operators

Constantia Alexandrou, Simone Bacchio, Martha Constantinou, Jacob Finkenrath, Karl Jansen, Giannis Koutsou, Bhavna Prasad and Gregoris Spanoudes

Abstract

Computations on state-of-the-art supercomputers like the JUWELS Cluster module enables us to study fundamental building blocks of our universe. The allocation was used for our effort on non-perturbative renormalization, an integral part of our precision study of hadronic structure using lattice QCD.

We simulated various ensembles with four degenerate quarks (N_f=4) at six different lattice spacings and multiple values of the quark mass. The renormalization factors are needed to obtain continuum quantities important in hadron structure, including: **i)** nucleon charges and moments of nucleon Parton Distribution Functions [1, 2], **ii)** the gluon momentum fraction renormalized non-perturbatively [3, 4], and **iii)** nucleon form factors [5].

N_f=4 ensembles

Our renormalization program uses the $\mathrm{RI}'_{\mathrm{MOM}}$ scheme [6, 7]. N_f=4 ensembles were generated using JUWELS at the same values of the coupling β as used for the ETMC N_f=2+1+1 ensembles. Multiple values of the quark masses μ_i are used to extrapolate the renormalization factors to the chiral limit. The target is to solve

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D[U] \mathcal{O}(U) e^{-\beta S_g(U)} \prod_i^{N_f=4} \det D(U, \mu_i)$$

which is done in two steps **i)** ensemble generation via Markov Chain Monte Carlo simulations and **ii)** calculations of observables, i.e. correlation and vertex functions, matrix elements etc.. The most computational effort is required for solving $\sim \mathcal{O}(10^6)$ linear equations $D(U,\mu)x = b$. The operator $D(U,\mu)$ is a next-neighbor stencil of dimension $V = 12 \cdot 2L^4$ with $L = [24, \ldots, 128]$.

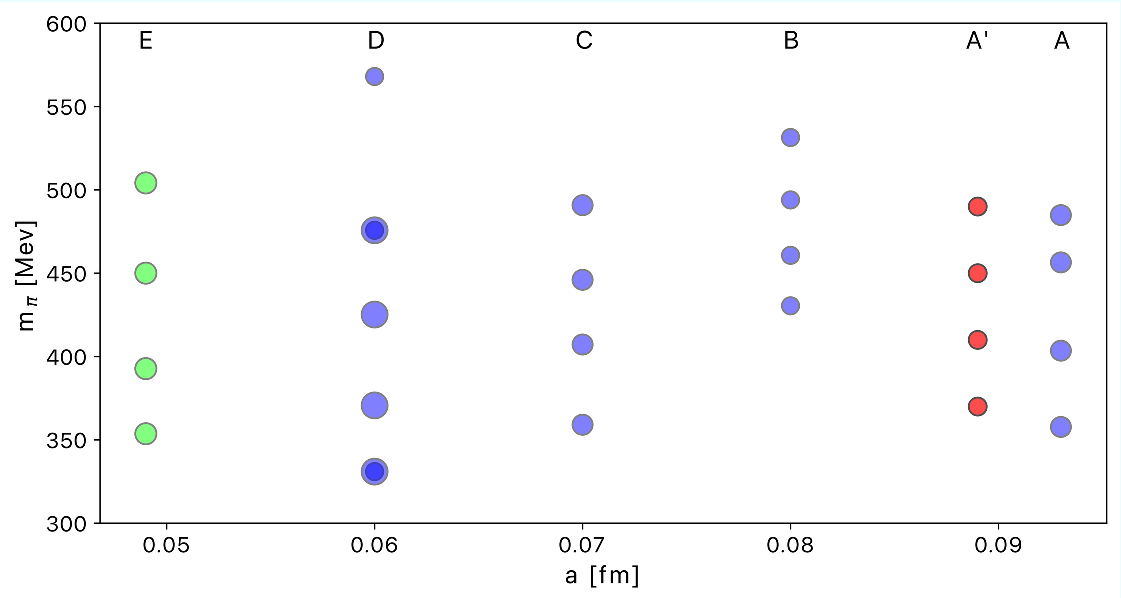

Fig. 1: Generated N_f=4 ensembles at different lattice spacings and pion masses.

Renormalization

We have calculated nonperturbative renormalization factors for the flavor singlet and non-singlet quark bilinear operators [8], and the gluon and quark energy-momentum tensor (EMT) [9]. Our results have been applied in the determination of the quark and gluon momentum fraction in the pion and kaon and the calculation of nucleon charges.

Strategy: The RI'/MOM formula for determining the renormalization factor $Z_{\mathcal{O}}^{\mathrm{RI}'}$ of a quark bilinear operator $\mathcal{O}$ is: $(Z_q^{\mathrm{RI}'})^{-1} Z_{\mathcal{O}}^{\mathrm{RI}'} \frac{1}{12} \mathrm{Tr}[\Lambda_{\mathcal{O}}(p)(\Lambda_{\mathcal{O}}^{\mathrm{tree}}(p))^{-1}]|_{p^2=\mu_0^2} = 1$, where $\Lambda_{\mathcal{O}}$ is the amputated vertex function of $\mathcal{O}$ with external quark fields and $Z_q^{\mathrm{RI}'}$ is the renormalization factor of the quark field. A similar condition is set for gluon operators. The calculation of the vertex functions have been performed using the momentum source approach, and the one-end trick for disconnected quark loops. Our strategy contains chiral extrapolations with four quark masses, subtractions of lattice artifacts calculated in one-loop lattice perturbation theory, perturbative conversion to $\overline{\mathrm{MS}}$ scheme at 2 GeV, and momentum extrapolations to eliminate residual dependence on the scale μ_0.

Results: We show in Fig. 2 some selected results of our computation. We stress that due to the presence of large gauge noise, high statistics of $\mathcal{O}(10,000)$ are required for the calculation of gluon vertex functions.

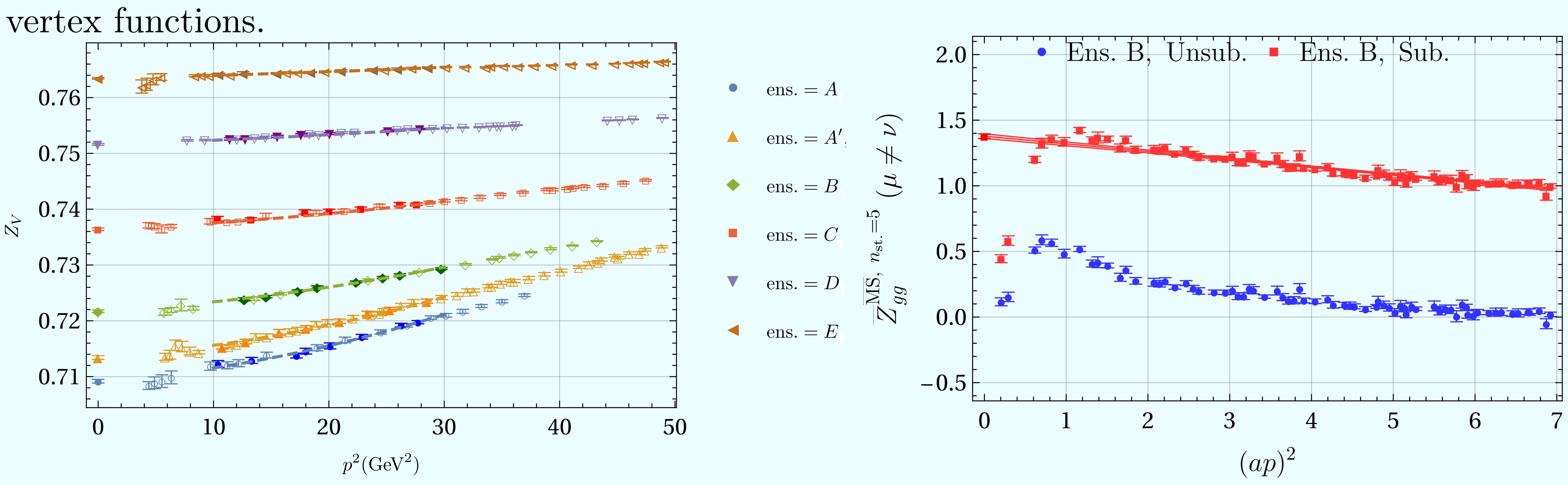

Fig. 2: The renormalization factor of the vector (left) and the gluon EMT operator (right) as a function of $p^2 = \mu_0^2$, for different ensembles.

Quark and gluon momentum fraction in the pion and kaon

The renormalization factors enabled us to calculate the first complete momentum decomposition for both the pion and the kaon. This was done in terms of their quark and gluon constituents, performed within lattice QCD at the physical point [9] on three ensembles with N_f=2+1+1 extracting from matrix elements of the EMT via ratios of 3- and 2-point functions.

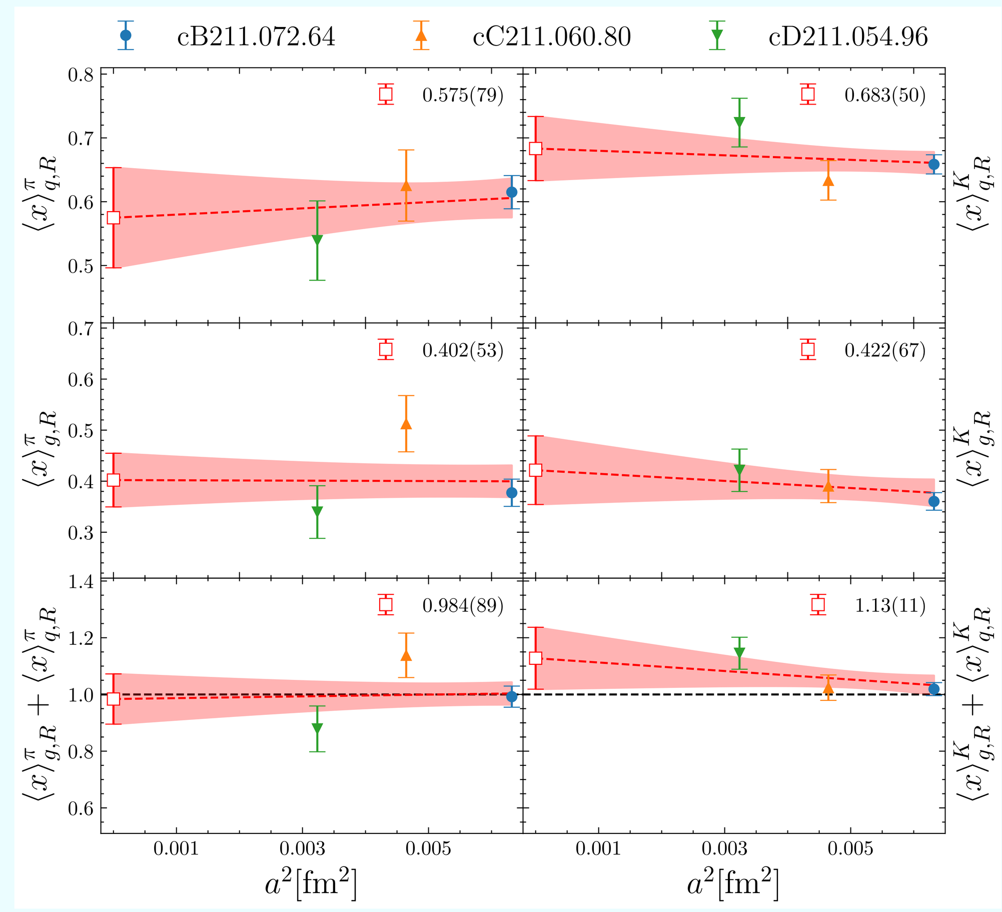

Fig. 3: Continnum limit extrapolation are showing on the right panel for the pion (left part) and the kaon (right part). We present our results for the total quark and gluon contributions, as well as the momentum sum rule.

We found that the total momentum fraction carried by quarks is 0.575(79) and 0.683(50) and by gluons 0.402(53) and 0.422(67) in the pion and in the kaon, respectively, in the $\overline{\mathrm{MS}}$ at 2 GeV.

Our results for $\langle x \rangle_{g,R}^{\pi,K}$ indicate a similar momentum fraction carried by gluons in the kaon and the pion, while $\langle x \rangle_{q,R}$ tends to be smaller in the pion.

We found that the momentum sum is 0.984(89) for the pion and 1.13(11) for the kaon. This is the first decomposition into quark and gluon parts available of $\langle x \rangle^K$ using a first-principles calculation.

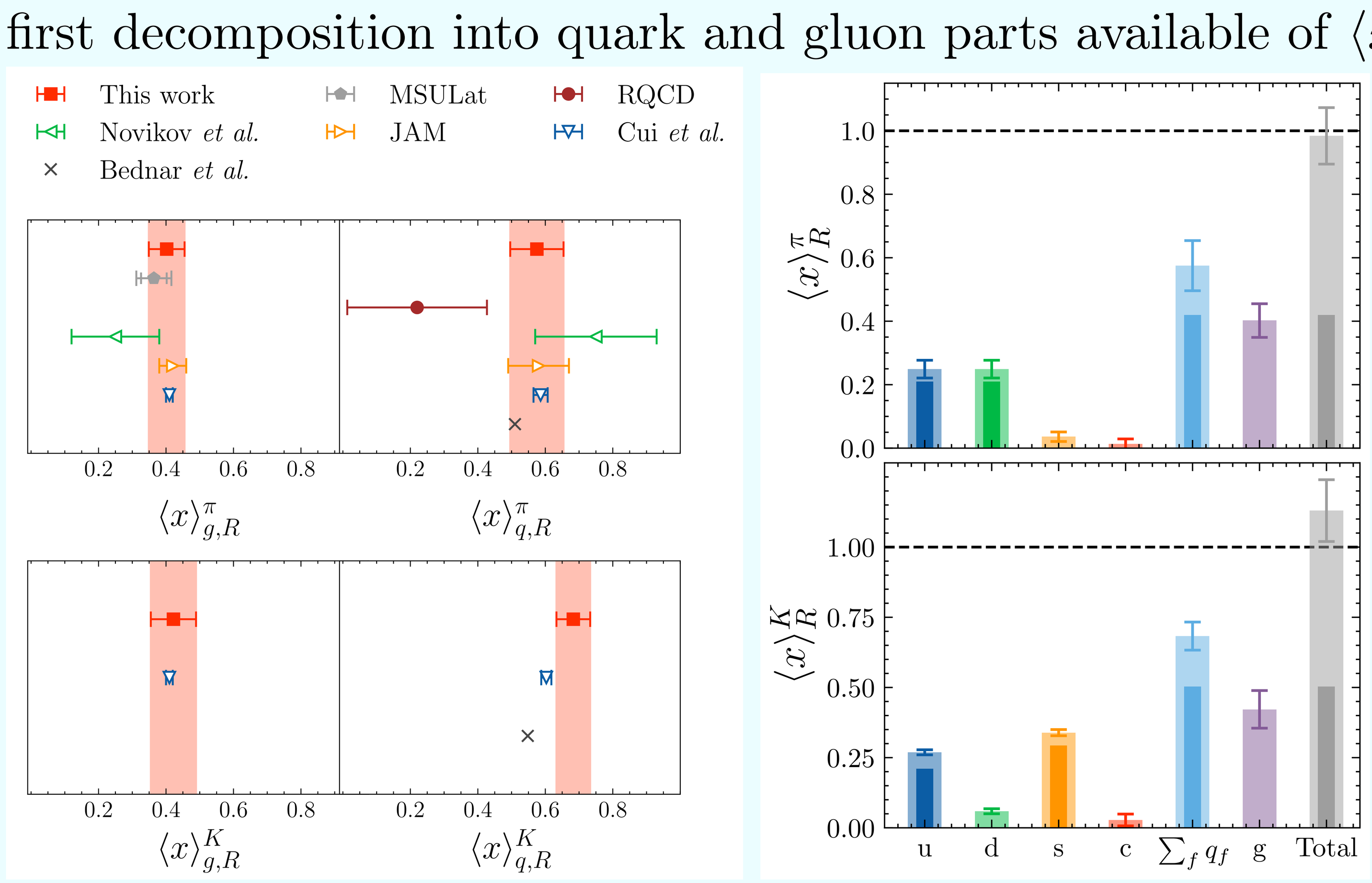

Fig. 4: The left panel shows comparison of our results with other available data, for the pion (upper panels) and kaon gluon and quark fractions (lower panels). The right panel shows the quark and gluon momentum fractions for the pion (upper panel) and kaon (lower panel) obtained in the continuum. Inner bars represent only the connected contributions, while the outer bars show the total, including disconnected contributions.

With the computational capacity of the next generation of supercomputers, such as JUPITER will deliver, more complicated renormalization of higher moments of PDFs can be addressed by enhancing statistical outcomes. We also plan to extend the calculations to finer lattices. This will further deepening our understanding of the building blocks of our universe, the structure of hadrons.

References

[1] C. Alexandrou *et al.* [arXiv:1908.10706 [hep-lat]].
[2] C. Alexandrou *et al.* [arXiv:2202.09871 [hep-lat]].
[3] C. Alexandrou *et al.* [arXiv:2003.08486 [hep-lat]].
[4] C. Alexandrou *et al.* [arXiv:2109.10692 [hep-lat]].
[5] C. Alexandrou *et al.* [arXiv:2309.05774 [hep-lat]].
[6] G. Martinelli *et al.* [arXiv:hep-lat/9411010 [hep-lat]].
[7] C. Alexandrou *et al.* [arXiv:1201.5025 [hep-lat]].
[8] C. Alexandrou *et al.* [arXiv:2412.01535 [hep-lat]].
[9] C. Alexandrou *et al.* [arXiv:2405.08529 [hep-lat]].

Acknowledgements

We thank all members of ETMC for the most enjoyable collaboration. J.F. acknowledges financial support by the Next Generation Triggers project. This work has received funding through the Inno4scale project, funded by the EuroHPC-JU under GA No 101118139.