Advanced hadron spectroscopy in lattice QCD

Z. Fodor[1,2,3,4], F. Frech[1], K. Szabo[2], F. Stokes[5], G. Wang[6]

(1) Department of Physics, University of Wuppertal, D-42119 Wuppertal, Germany (2) JSC, FZ Jülich, D-52428 Jülich, Germany
(3) Department of Physics, Pennsylvania State University, University Park, PA 16802, USA
(4) Institute for Theoretical Physics, Eötvös University, H-1117 Budapest, Hungary
(5) Department of Physics, University of Adelaide, South Australia 5005, Australia (6) Aix Marseille Univ, Université de Toulon, CNRS, CPT, IPhU, Marseille, France

BERGISCHE UNIVERSITÄT WUPPERTAL

Introduction

Precision calculations of hadronic masses in LQCD have important applications, f.e.:

1. The mass of the Ω^--baryon is known up to high precision ($M^{phys.}_{\Omega^-} = 1672.45(29)\,\text{MeV}$) from experiments. A high-precision lattice computation of aM_{Ω^-} can therefore be used to determine the lattice spacing a. Due to contamination of the lattice correlator with excited states it is not a simple task to extract the ground state energy.
2. The vector (ρ-meson) correlator is the fundamental observable to determine the hadronic contributions to $g-2$ [1, 2]. For long distances the signal-to-noise ration worsens exponentially. The task is to extract the contributing modes at shorter distances to reconstruct the long-distance contributions.

Both of these applications can be achieved through an application of the Generalized Eigenvalue Problem (GEVP).

Generalized Eigenvalue Problems

The correlation function of an operator ($\hat{O}^n$) $\langle \hat{O}^{n\dagger}(t)\hat{O}^n(0)\rangle$ can be decomposed as a sum of different exponential modes (Equation 2). By constructing the correlation matrix (Equation 1) one can solve the Generalized Eigenvalue Problem (GEVP) (Equation 3) and solve for the different exponential modes (Equation 4).

$$\mathbf{C}^{jk}(t) = \langle \hat{O}^{j\dagger}(t)\hat{O}^k(0)\rangle \quad (1)$$

$$\mathbf{C}^{jk}(t) = \sum_i a_i^{jk} \exp(-m_i t) \quad (2)$$

$$\mathbf{C}(t)v_i = \lambda_i(t,t_0)\mathbf{C}(t_0)v_i \quad (3)$$

$$\lambda_i(t,t_0) = \exp(-m_i(t-t_0)) \quad (4)$$

High-precision computation of the Ω^- bayron mass

$$\mathbf{H}(t) = \left(\begin{array}{cc|cccc}
H^{pp}_{t+2t_p+0} & H^{pp}_{t+2t_p+1} & H^{ps}_{t+t_p+0} & H^{ps}_{t+t_p+1} & H^{ps}_{t+t_p+2} & H^{ps}_{t+t_p+3} \\
H^{pp}_{t+2t_p+1} & H^{pp}_{t+2t_p+2} & H^{ps}_{t+t_p+1} & H^{ps}_{t+t_p+2} & H^{ps}_{t+t_p+3} & H^{ps}_{t+t_p+4} \\
\hline
H^{sp}_{t+t_p+0} & H^{sp}_{t+t_p+1} & H^{ss}_{t+0} & H^{ss}_{t+1} & H^{ss}_{t+2} & H^{ss}_{t+3} \\
H^{sp}_{t+t_p+1} & H^{sp}_{t+t_p+2} & H^{ss}_{t+1} & H^{ss}_{t+2} & H^{ss}_{t+3} & H^{ss}_{t+4} \\
H^{sp}_{t+t_p+2} & H^{sp}_{t+t_p+3} & H^{ss}_{t+2} & H^{ss}_{t+3} & H^{ss}_{t+4} & H^{ss}_{t+5} \\
H^{sp}_{t+t_p+3} & H^{sp}_{t+t_p+4} & H^{ss}_{t+3} & H^{ss}_{t+4} & H^{ss}_{t+5} & H^{ss}_{t+6}
\end{array}\right) \quad (5)$$

We use a six-dimensional GEVP, that includes point and smeared sources (p and s) and different time shifts given in subscripts (Equation 5). The point sources have an additional shift t_p.

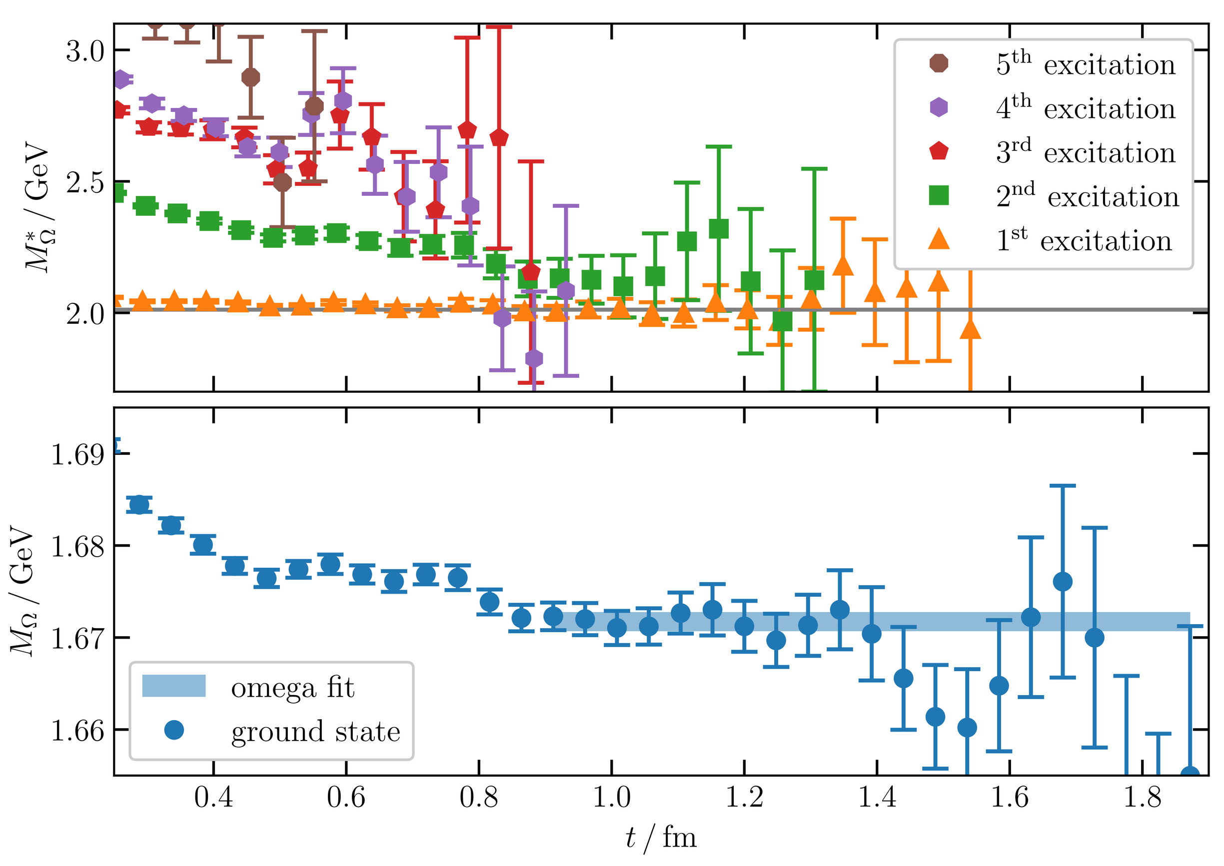


Figure 1. Effective mass $M_{eff}(t) = \frac{1}{\Delta t}\log\frac{\lambda(t,t_0)}{\lambda(t+\Delta t,t_0)}$ plateaus from the GEVP in Equation 5. Figure is taken from [2].

The GEVP with the correlation matrix from Equation 5 is solved to extract six different states. The effective energies of these states are shown in Figure 1. We choose the fit ranges $[0.9\,\text{fm}, 2.0\,\text{fm}]$ and $[1.0\,\text{fm}, 2.0\,\text{fm}]$ according to the method in Figure 2. The final masses for most of our ensembles are shown in Figure 3.
With this high-precision computation we are able to improve our scale setting value of [1]

$$w_0^{old} = 0.17236[70]\,\text{fm} \quad (6)$$

to the much more precise value [2]

$$w_0^{new} = 0.17245[51]\,\text{fm}. \quad (7)$$

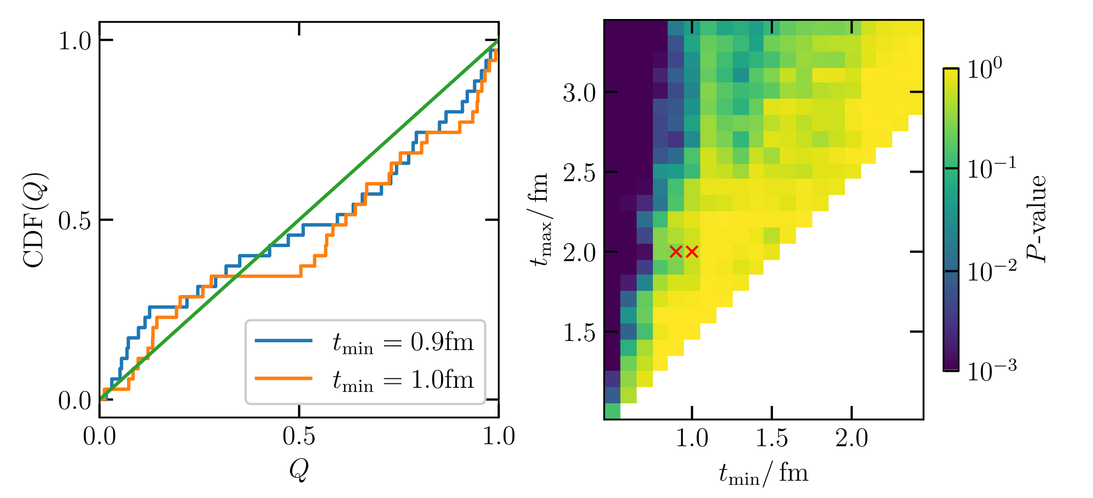


Figure 2. We estimated the optimal range to fit the correlator by measuring the Q-value [3] for each range and ensemble and performing KS-tests along the ensembles. Large P-values correspond to good fit qualities. Figure is taken from [2].

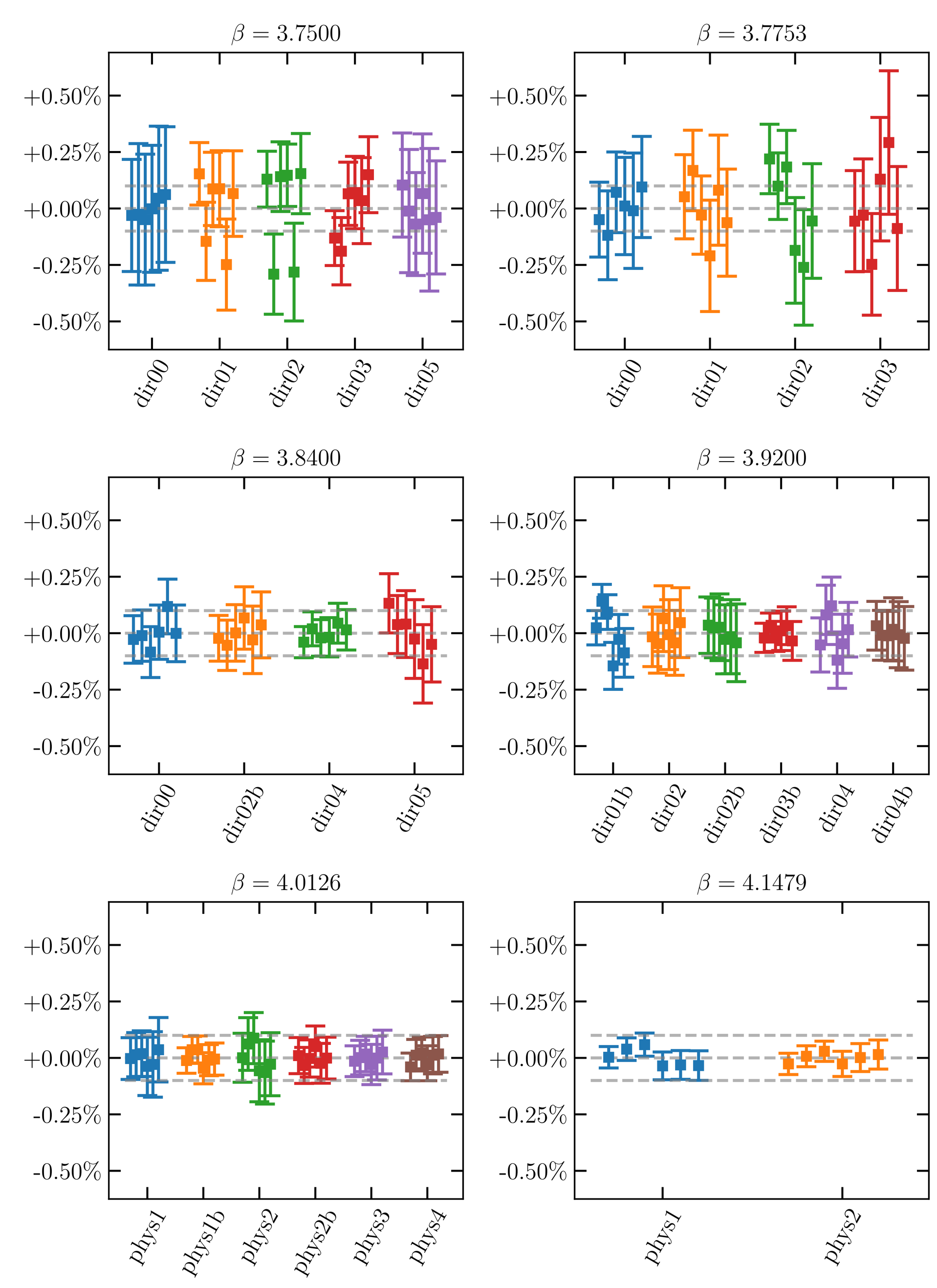
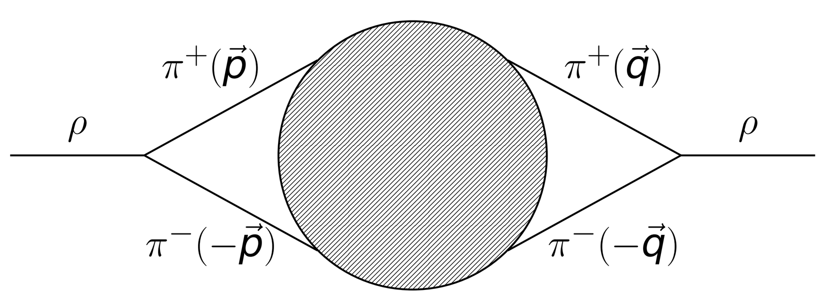


Figure 3. Distribution of the Ω^- baryon masses around their mean value on different ensembles. The six numbers per ensembles are associated with three operators and two fit ranges. Figure is taken from [2].

Tail reconstruction of the vector meson



Construction of two-pion states with the same quantum numbers as the ρ-meson (except for the energy):

$$C^0(t) = \langle \hat{\rho}^\dagger(t)\hat{\rho}(0)\rangle \quad (8)$$

$$C^i(t) = \sum_j G^{ij}_{\rho\pi\pi} \times \langle (\hat{\pi\pi})^\dagger_j(t)(\hat{\pi\pi})_j(0)\rangle \quad (9)$$

The indices j of the two-pion correlators correspond to taste/momentum-choices. The computation of the coefficients in Equation 9 can be found in [4].

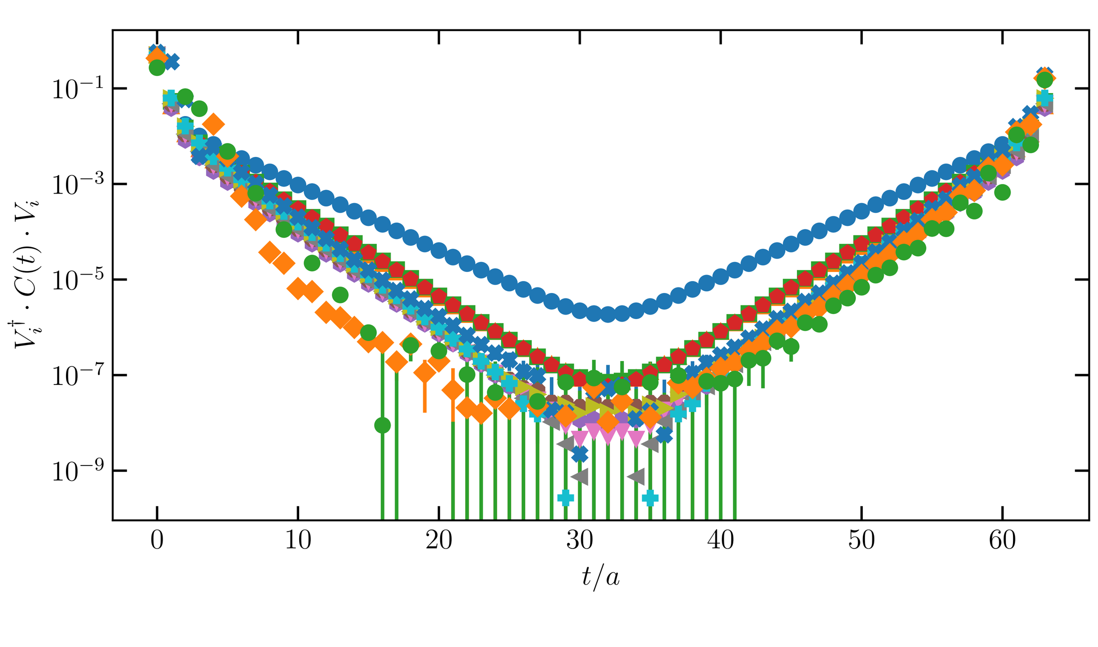


Figure 4. The eigenvalues $\lambda(t, t_0)$ shown as a function of t. Due to periodic boundary conditions there are also back-propagating contributions. Figure is taken from [5].

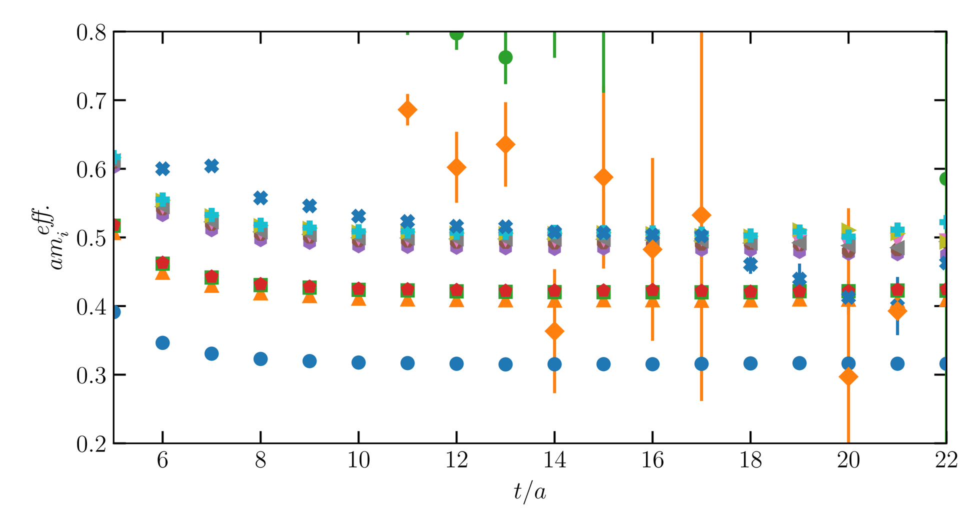


Figure 5. Effective energies which corresponds to the slope of the data in Figure 5. Figure is taken from [5].

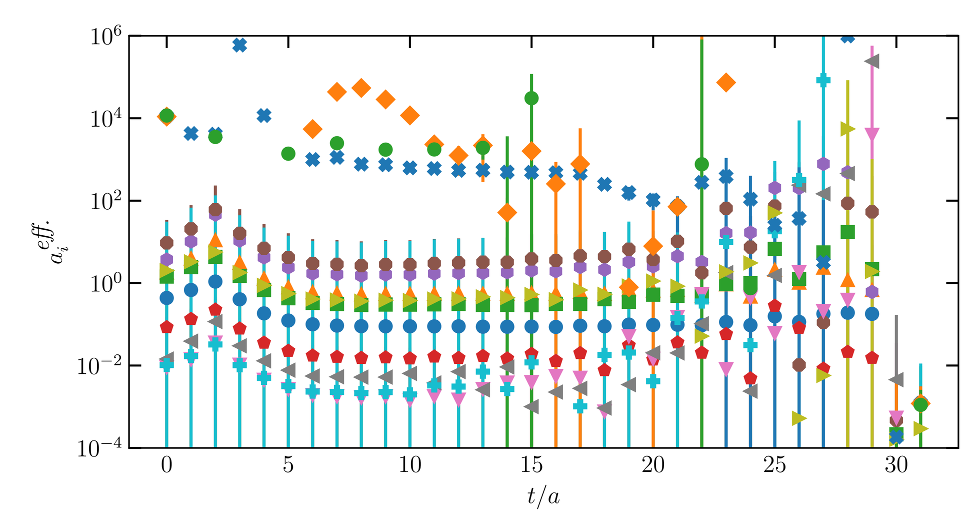


Figure 6. Plateaus for the prefactor a_i^{00} from Equation 2. The states that do not display a clear plateau (e.g. orange) correspond to $E > m_\rho$ and are heavily suppressed for the tail reconstruction. Figure is taken from [5].

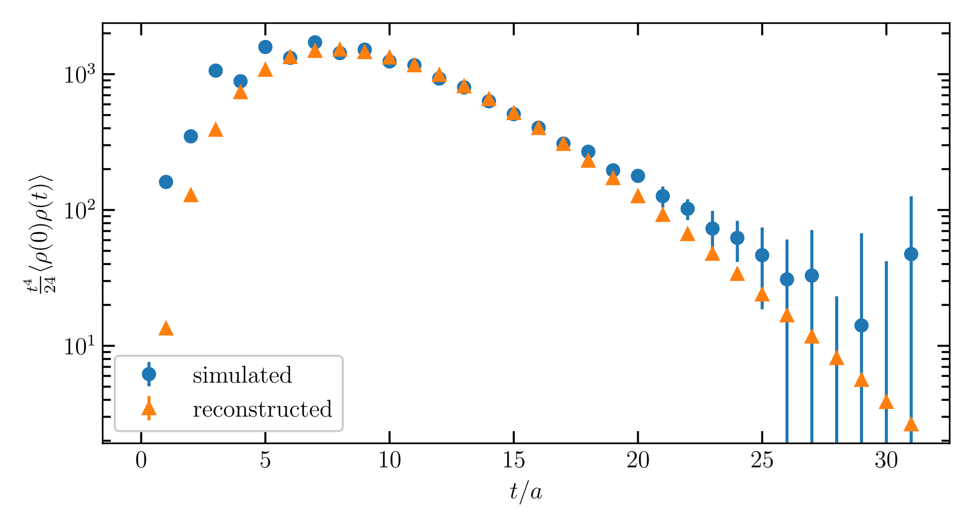


Figure 7. Reconstruction of the first moment compared to the direct simulation. Since states with $E > m_\rho$ are left out, the reconstruction is incomplete at small t, however at large t these contributions are suppressed and the reconstruction converges to the true value and the reconstruction is much more precise. Figure is taken from [5].

References

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[5] Frech *et al.*, "Reconstruction of the vector meson propagator using a generalized eigenvalue problem"