Valence leading isospin breaking contributions to $a_\mu^{\rm HVP}$

A. Evangelista[a], S. Bacchio[b], J. Finkenrath[c], R. Frezzotti[a], G. Gagliardi[d], M. Garofalo[e], B. Kostrzewa[e], V. Lubicz[f], F. Pittler[b] F. Sanfilippo[d], S. Simula[d], N. Tantalo[a], C. Urbach[e]

[a] Dipartimento di Fisica and INFN, Università di Roma "Tor Vergata", Via della Ricerca Scientifica 1, I-00133 Roma, Italy
[b] Computation-based Science and Technology Research Center, The Cyprus Institute, 20 Kavafi Str., Nicosia 2121, Cyprus
[c] Department of Theoretical Physics, European Organization for Nuclear Research, CERN, CH-1211 Genève 23, Switzerland
[d] Instituto Nazionale di Fisica Nucleare, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy
[e] Helmholtz-Institut für Strahlen- und Kernphysik (Theorie) and Bethe Center for Theoretical Physics, Universität Bonn
[f] Dipartimento di Matematica e Fisica, Università Roma Tre and INFN, Sezione di Roma Tre, Via della Vasca Navale 84, I-00146 Rome, Italy

Extended Twisted Mass Collaboration

Introduction

The Hadronic Vacuum Polarization (HVP), arising from vacuum fluctuations in which a virtual photon temporarily converts into quarks and gluons, plays a crucial role in Standard Model (SM) predictions of the muons anomalous magnetic moment, a_μ. Lattice Quantum Chromodynamics (LQCD) provides a powerful approach to compute $a_\mu^{\rm HVP}$. With the increasing precision of lattice calculations, it becomes necessary to include the leading-order Isospin Breaking (LIB) corrections to $a_\mu^{\rm HVP}$ coming from the electromagnetic and strong isospin-breaking effects.

Setup

We compute the LIB effects on observables using isosymmetric QCD (isoQCD) gauge ensembles from ETMC Table 1. A local mixed lattice action for QCD+QED with dynamical u, d, s, c quarks is defined [1] using maximally twisted mass Wilson clover fermions [2]:

$$S_{\rm mix} = S_{\rm TM}[\vec{g}^0] + S_{\rm ghost}[\vec{g}^0] + S_{\rm QCED}[\vec{g}]\,,$$

Where $S_{\rm TM}$ is the automatically $\mathcal{O}(a)$ improved fermionic lattice action used in ETMC ensembles production [3]. The bare parameters $\vec{g}^0$ and $\vec{g}$ refer to the isoQCD and to the QCD + QED theories. In the electro-quenched (EQ) approximation we assume $a = a^0$ (equal lattice spacing in the two theories) with a change in the bare strong coupling. The QCD+QED action with U denote the SU(3) gauge link-field and A the QED non-compact gauge potential.

name	L [fm]	a [fm]	M_π [MeV]	$M_\pi L$
B48	3.84	∼0.08	≈ 135	2.63
B64	5.12	∼0.08	≈ 135	3.51

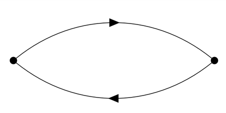
Table 1: ETMC's $N_f = 2+1+1$ gauge ensembles analysed in this project. The time extent is always set to $T = 2L$.

$$S_{\rm QCED}[U, A, q_f, \bar{q}_f] = \sum_{f=u,d,s,c} \sum_x \bar{q}_f(x) \left\{ \gamma_\mu \tilde{\nabla}_\mu [U, A] - i\gamma_5 r_f \left(W_{cl}^f[U] + m_{\rm cr}^f \right) + \mu_f \right\} q_f(x)$$

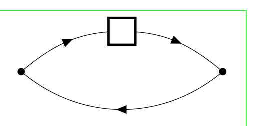
where $W_{cl}^f[U, A]$ is the clover Wilson term, m_{cr}^f the critical mass for flavour f, while $r_u = r_c = 1$ and $r_d = r_s = -1$ Furthermore, $m_{\rm cr}^u = m_{\rm cr}^c$ and $m_{\rm cr}^d = m_{\rm cr}^s$ since critical masses differ only due to QED effects. Moreover, $S_{\rm QCED}$ is automatically $\mathcal{O}(a)$-improved [4]. Lastly, $S_{\rm ghost}$ has the same structure as $S_{\rm QCED}$ and the field content needed to cancel the quark determinant coming from $S_{TM}[g_0]$ up to $\mathcal{O}(a^2)$. We compute the LIB corrections using the RM123 method expanding in $\vec{g} - \vec{g}^0$

$$\langle \mathcal{O}\,[U, A; \vec{g}] \rangle^{\vec{g}} = \langle \mathcal{O}\,[U; \vec{g}_0] \rangle^{\vec{g}_0} + \Delta \langle \mathcal{O}[U, A; \vec{g}] \rangle^{\vec{g}_0} + \mathcal{O}\left(\alpha_{em}^2, \delta_{ud}^2, \alpha_{em}\delta_{ud} \right)\,,$$

Wick contraction:



Pseudoscalar $\bar{q}_f \gamma_5 q_f$ insertion

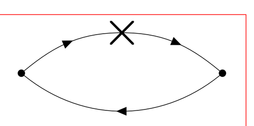


$$\Delta \langle \mathcal{O} \rangle^{\vec{g}_0} = \frac{1}{2} e^2 \frac{\partial \langle \mathcal{O} \rangle^{\vec{g}}}{\partial e^2}\bigg|_{\vec{g}=\vec{g}_0} + \sum_{\substack{f=u,d\\ s,c}} (\mu_f - \mu_f^0) \frac{\partial \langle \mathcal{O} \rangle^{\vec{g}}}{\partial \mu_f}\bigg|_{\vec{g}=\vec{g}_0} + \sum_{\substack{f=u,d\\ s,c}} (m_{\rm cr}^f - m_{\rm cr}^0) \frac{\partial \langle \mathcal{O} \rangle^{\vec{g}}}{\partial m_{\rm cr}^f}\bigg|_{\vec{g}=\vec{g}_0}$$

Numerical derivatives: $\mathcal{U}_\mu(x) = U_\mu(x)\, e^{ie\, e_f A_\mu(x + a\hat{\mu}/2)}$

$$\frac{\partial \langle \mathcal{O} \rangle^{\vec{g}}}{\partial e^2} = \frac{\langle \mathcal{O}(e) \rangle - 2\langle \mathcal{O}(0) \rangle + \langle \mathcal{O}(-e) \rangle}{e^2}$$

Scalar $\bar{q}_f \mathbb{1} q_f$ insertion



In this way local gauge symmetries are preserved, lattice artifacts for physical observables are $O(a^{2n}), n \in \mathbb{N}$ and the QCD+QED results to first order in α_{em} and δm_{ud} are obtained in terms of correlations functions evaluated in isoQCD. In our calculation we implemented a hadronic renormalization scheme for QCD+QED [5–7]. To set the scale in isosymmetric QCD (isoQCD), we adopt the Edinburgh/FLAG consensus values:

$$f_\pi = 130.5\ \text{MeV} \qquad M_\pi = 135.0\ \text{MeV} \qquad M_K = 494.6\ \text{MeV} \qquad M_{D_s} = 1967\ \text{MeV}\,.$$

Counterterm and scale setting

The critical mass shifts are generated only from QED effects can be determined, as in [8], imposing

$$\bar{C}(t) \equiv \partial_0 \sum_{\vec{x}} \langle V_0^1(x) P^2(0) \rangle^{\vec{g}} \Big/ 2 \sum_{\vec{x}} \langle P^2(x) P^2(0) \rangle^{\vec{g}} = 0 \tag{1}$$

where $V_0^1(y) = (\bar{\psi}_\ell \gamma^0 \tau_1 \psi_\ell)(y)$ and $P^2(y) = (\bar{\psi}_\ell \gamma^5 \tau_2 \psi_\ell)(y)$ with $\psi_\ell = (u, d)^T$. Since we are working in EQ approximation $\Delta m_{\rm cr}^d = (q_d/q_u)^2 \Delta m_{\rm cr}^u$ and can define $\Delta \bar{m}_{cr} \equiv (q_u^2 + q_d^2)\left(\Delta m_{\rm cr}^f / 2q_f^2 \right)$. Imposing Eq.(1) we get $\Delta \bar{m}_{cr}$ as in Figure 1. To determine the shift in the quark masses $\Delta\mu_f = \mu_f - \mu_f^0$, we compute the corrections to the mesons masses M_P with $P = \pi^+, K^+, K^0$ and D_s and impose their experimental value. Furthermore, we used the QED Finite Size Effects (FSE) given in [9] to correct for the dominating hadron structure independent FSE. This renormalization scheme holds in EQ approximation. Beyond the EQ approximation, LIB effects change also the lattice spacing and five independent hadronic inputs will be used for QCD+QED renormalization, such as $M_{\pi^+}, M_{K^+}, M_{K^0}, M_{D_s}, M_{D^0}$ (The proposal on the ongoing computer allocation was to use the Ω baryon mass M_Ω for the computer allocation. However, due to a significant reduction in resources, this subproject had to be aborted).

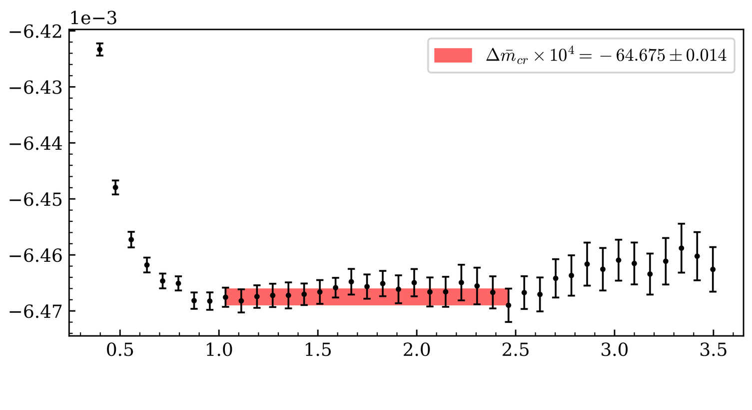


Figure 1: Critical mass shift $\Delta\bar{m}_{cr}$ determinations on the cB211.072.64 ensemble.

LIB contibution to $a_\mu^{\rm HVP}$

With all counterterms determined, we compute corrections to $a_\mu^{\rm HVP}$ in the time-momentum representation [10]:

$$a_\mu^{\rm HVP} = 2\alpha_{\rm em}^2 \int_0^\infty dt\, t^2 K(m_\mu t)\, \hat{C}_{JJ}(t)\,, \tag{2}$$

where $K(m_\mu t)$ is an analytic kernel and $\hat{C}_{JJ}(t)$ is the renormalized two-point vector-vector correlator

$$\hat{C}_{JJ}^f(t) = Z_{V,f}^2 C_{JJ}^f(t) \qquad \hat{C}_{JJ}(t) = \frac{1}{3} \sum_{i=1,2,3} \int d^3x \langle J_i(\vec{x}, t) J_i^\dagger(0) \rangle\,,$$

with $J_i(\vec{x}, t) = \sum_{fu,d,s,c,...} q_f \bar{f}(x) \gamma_i f(x)$ and $Z_{V,f}$ is the renormalization constant for the flavour f. The correction to the renormalized two-point vector-vector correlator reads

$$\Delta\hat{C}_{JJ}^f(t) = Z_{V,f}^2 \Delta C_{JJ}^f(t) + 2Z_V \Delta Z_{V,f} C_{JJ}^{f,\rm iso}(t)$$

where $\Delta Z_{V,f}$ is the QED correction for the flavour f to the isoQCD renormalization constant Z_V. The relation used to determine $Z_{V,f}$ normalization constant in isoQCD keeps holding in our QCD + QED mixed action setup [11], we can compute $\Delta Z_{V,f}$

$$\Delta Z_{V,f} = \Delta \left\{ 2\mu_f C_{PP}(t) / \partial_0 C_{A_0 P}(t) \right\}$$

The integral in Eq. 2 is performed as a finite sum up to a value t_{cut}

$$\Delta a_\mu^{\rm HVP}(f; t_{\rm cut}) = 2\alpha_{\rm em}^2 \sum_{n=0}^{n_{\rm cut}} n^2 K(m_\mu a n)\, \Delta\hat{C}_{JJ}^f(an)\,,$$

where $n_{\rm cut} = t_{\rm cut}/a$ is the Euclidean time in lattice units. To reduce the noise on the corrections to $a_\mu^{\rm HVP}(\ell)$ we calculate them at larger value of the quark mass $m_\ell = r_m \mu_\ell$ with $r_m = 3, 5, 7, 9$ and extrapolate to the isoQCD light point $r_m = 1$ for each values of $t_{\rm cut}$, Figure 2 left panel.

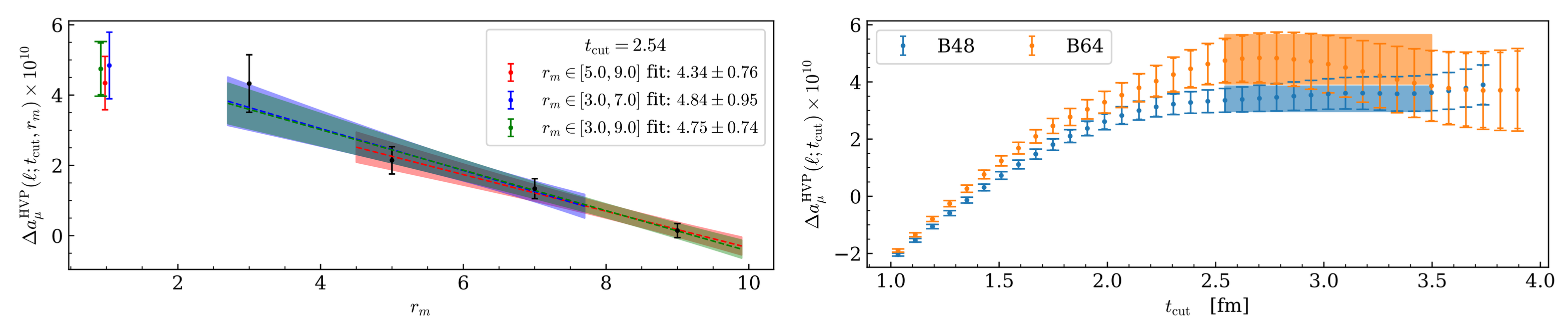


Figure 2: **(left)** Chiral extrapolation of $\Delta a_\mu^{\rm HVP}(\ell; t_{\rm cut})$ for the cB211.072.64 (B64) ensemble. **(right)** The time dependence of the extrapolated $\Delta a_\mu^{\rm HVP}(\ell; t_{\rm cut})$ on the two volumes $L \sim 3.8$ fm (B48 in blue) and $L \sim 5.1$ fm (B64 in orange).

The extrapolated results for each $t_{\rm cut}$ for the two volumes are presented in Figure 2 right panel.

The analysis of the larger volume B96 gauge ensemble, that we propose in the continuation of the project, together with an increase of statistics on the B48 and B64 ensembles, will allow us to clarify whether the marginal tension seen in Figure 2 right panel is a due to a statistical fluctuation or to a genuine finite size effect. Our errors here are preliminary and merely statistical and a careful assessment of systematic error is in progress, we are planning to repeat the study at three more value of the lattice spacing and to evaluate the EQ quark disconnected and electro-unquenched contributions in the future in separate projects of ETMC.

Acknowlogments

We thank all members of ETMC for the most enjoyable collaboration. S.B. received financial support from the Inno4scale project, which received funding from the European High-Performance Computing Joint Undertaking (JU) under Grant Agreement No. 101118139. The JU receives support from the European Union's Horizon Europe Programme. A.E., R.F., G.G, F.S. and N.T. are supported by the Italian Ministry of University and Research (MUR) under the grant PNRR-M4C2-I1.1-PRIN 2022-PE2 Non-perturbative aspects of fundamental interactions, in the Standard Model and beyond F53D23001480006 funded by E.U.- NextGenerationEU. M.G. and C.U. are supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) as part of the CRC 1639 NuMeriQS – project no. 511713970. F.S. is supported by ICSC – Centro Nazionale di Ricerca in High Performance Computing, Big Data and Quantum Computing, funded by European Union -NextGenerationEU and by Italian Ministry of University and Research (MUR) projects FIS 00001556 and PRIN 2022N4W8WR. The authors gratefully acknowledge the Gauss Centre for Supercomputing e.V. (www.gauss-centre.eu) for funding this project by providing computing time on the GCS Supercomputer JUWELS [12, 13] at Jülich Supercomputing Centre (JSC). We thank CINECA and EuroHPC JU for the computer time on Leonardo-Booster provided to us through the Extreme Scale Access Call grant EHPC-EXT-2024E01-027 and the specific initiative INFN-LQCD123.

References

[1] Roberto Frezzotti, Giancarlo Rossi, and Nazario Tantalo. Sea quark qed effects and twisted mass fermions, 2016.
[2] R. Frezzotti and G.C. Rossi. Chirally improving Wilson fermions. 1. O(a) improvement. *JHEP*, 0408:007, 2004.
[3] C. Alexandrou et al. Strange and charm quark contributions to the muon anomalous magnetic moment in lattice QCD with twisted-mass fermions. 11 2024.
[4] R. Frezzotti, G. Martinelli, M. Papinutto, and G. C. Rossi. Reducing cutoff effects in maximally twisted lattice QCD close to the chiral limit. *JHEP*, 04:038, 2006.
[5] Nazario Tantalo. Isospin breaking effects on the lattice, 2013.
[6] Antonin Portelli. Inclusion of isospin breaking effects in lattice simulations, 2015.
[7] Simone Bacchio, Antonio Evangelista, Roberto Frezzotti, Giuseppe Gagliardi, Marco Garofalo, Nikolaos Kalntis, Simone Romiti, Francesco Sanfilippo, and Nazario Tantalo. Valence leading isospin breaking contributions to $a_\mu^{\rm HVP-LO}$. 1 2025.
[8] Roberto Frezzotti, Giancarlo Rossi, and Nazario Tantalo. Sea quark QED effects and twisted mass fermions. *PoS*, LATTICE2016:320, 2016.
[9] Sz. Borsanyi et al. Ab initio calculation of the neutron-proton mass difference. *Science*, 347:1452–1455, 2015.
[10] David Bernecker and Harvey B. Meyer. Vector Correlators in Lattice QCD: Methods and applications. *Eur. Phys. J. A*, 47:148, 2011.
[11] C. Alexandrou et al. Lattice calculation of the short and intermediate time-distance hadronic vacuum polarization contributions to the muon magnetic moment using twisted-mass fermions. *Phys. Rev. D*, 107(7):074506, 2023.
[12] Jülich Supercomputing Centre. JUWELS: Modular Tier-0/1 Supercomputer at the Jülich Supercomputing Centre. *Journal of large-scale research facilities*, 5(A135), 2019.
[13] Jülich Supercomputing Centre. JUWELS Cluster and Booster: Exascale Pathfinder with Modular Supercomputing Architecture at Juelich Supercomputing Centre. *Journal of large-scale research facilities*, 7(A138), 2021.