Nucleon structure from lattice QCD

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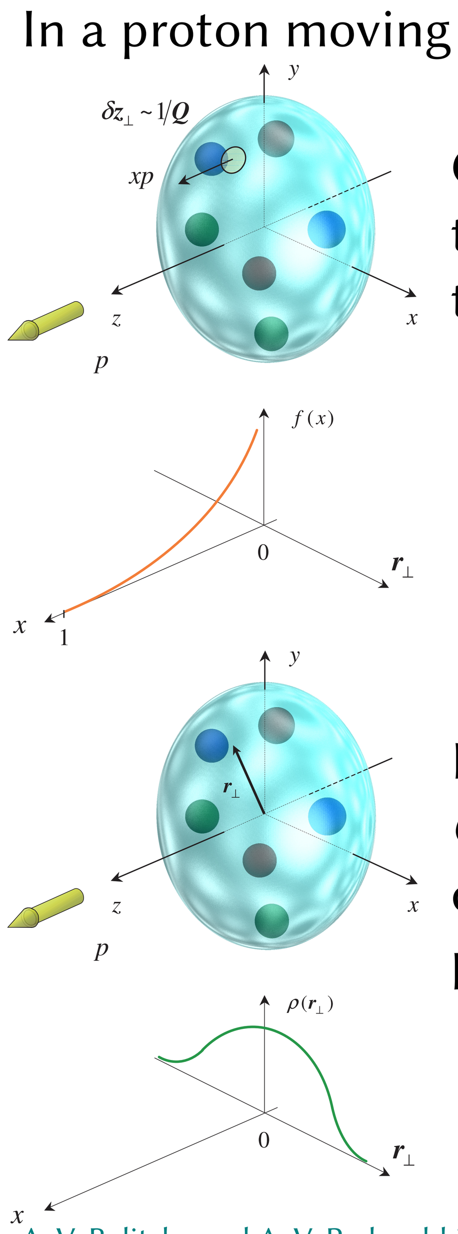
Quark substructure of the proton

In a proton moving in the z direction:



Quark parton distribution functions (PDFs) describe the distribution of quarks and antiquarks with respect to longitudinal momentum fraction x:

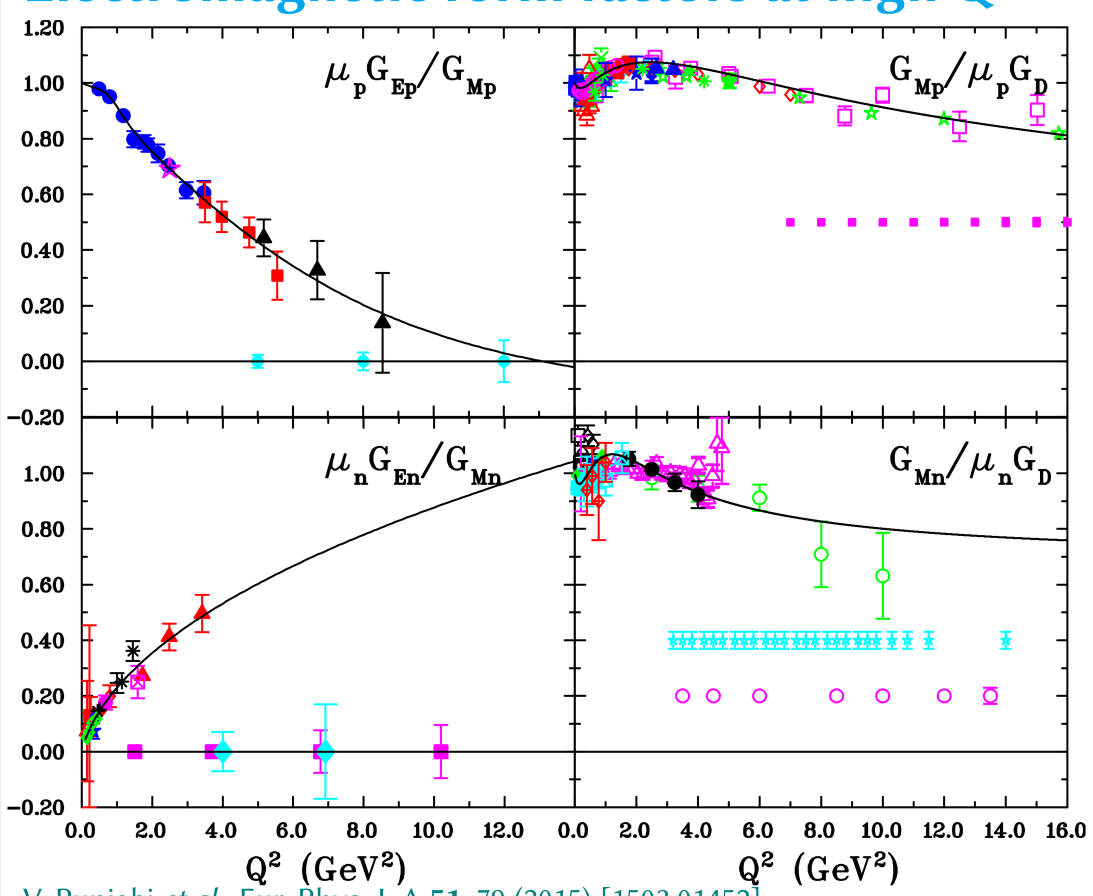
$$\begin{aligned} q(x), \bar{q}(x) &\quad \text{unpolarized,} \\ \Delta q(x), \Delta\bar{q}(x) &\quad \text{helicity (longitudinally polarized),} \\ \delta q(x), \delta\bar{q}(x) &\quad \text{transversity.} \end{aligned}$$

Electromagnetic form factors $F_1(Q^2)$ and $F_2(Q^2)$ [or $G_E(Q^2)$ and $G_M(Q^2)$] describe the distribution of quarks minus antiquarks with respect to transverse position $r_\perp$. In the unpolarized case:

$$\rho(r_\perp) = \int \frac{d^2q}{(2\pi)^2} e^{iq\cdot r_\perp} F_1(q^2).$$

A. V. Belitsky and A. V. Radyushkin, Phys. Rept. **418** (2005) [hep-ph/0504030]

Electromagnetic form factors at high Q^2



Past and planned measurements of proton and neutron electromagnetic form factors.

Does proton $G_E(Q^2)$ have a zero at $Q^2 \approx 10$ GeV^2?

V. Punjabi *et al.*, Eur. Phys. J. A **51**, 79 (2015) [1503.01452]

Reaching high momentum on the lattice

High-momentum nucleons are challenging for two reasons:

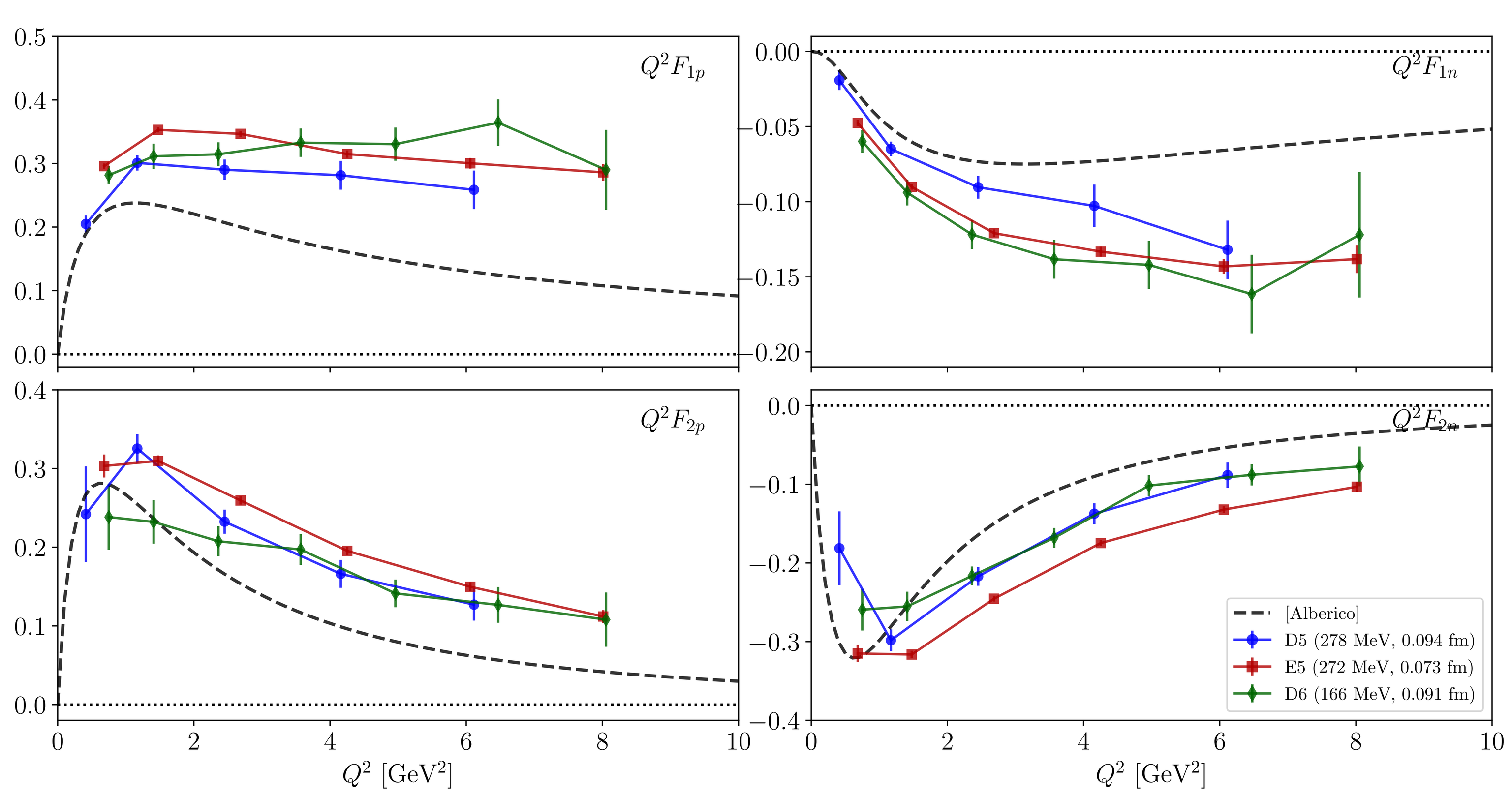
1. worse signal-to-noise ratio $\sim \exp(-[E_N(p) - \frac{3}{2}m_\pi]t)$,
2. smaller energy gap $\Delta E(p) \equiv E_{\text{excited}}(p) - E_N(p)$.

We make this more manageable via

- Breit-frame kinematics $\vec{p}' = -\vec{p}$ to minimize $|\vec{p}|$ for a given Q^2,
- momentum smearing to optimize nucleon operator at the chosen $\vec{p}$.

G. S. Bali, B. Lang, B. U. Musch, A. Schäfer, Phys. Rev. D **93**, 094515 (2016) [1602.05525]

Preliminary high-momentum form factors

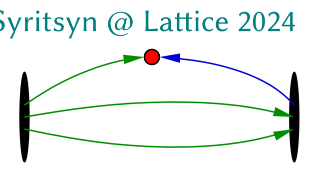


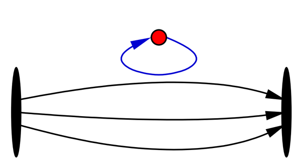
Contribution from connected diagrams to proton and neutron F_1 and F_2.

- Reasonable signal up to $Q^2 \approx 8$ GeV^2!
- Removing excited states is a challenge. Here: two-state fit.
- Discretization effects may also be substantial.

Disconnected diagrams also computed:
consistent with zero and $\lesssim 20\%$ of connected.

S. Syritsyn @ Lattice 2024


connected


disconnected

Second Mellin moment

Simplest way to study PDFs on the lattice is via their lowest Mellin moments: e.g. the second moments

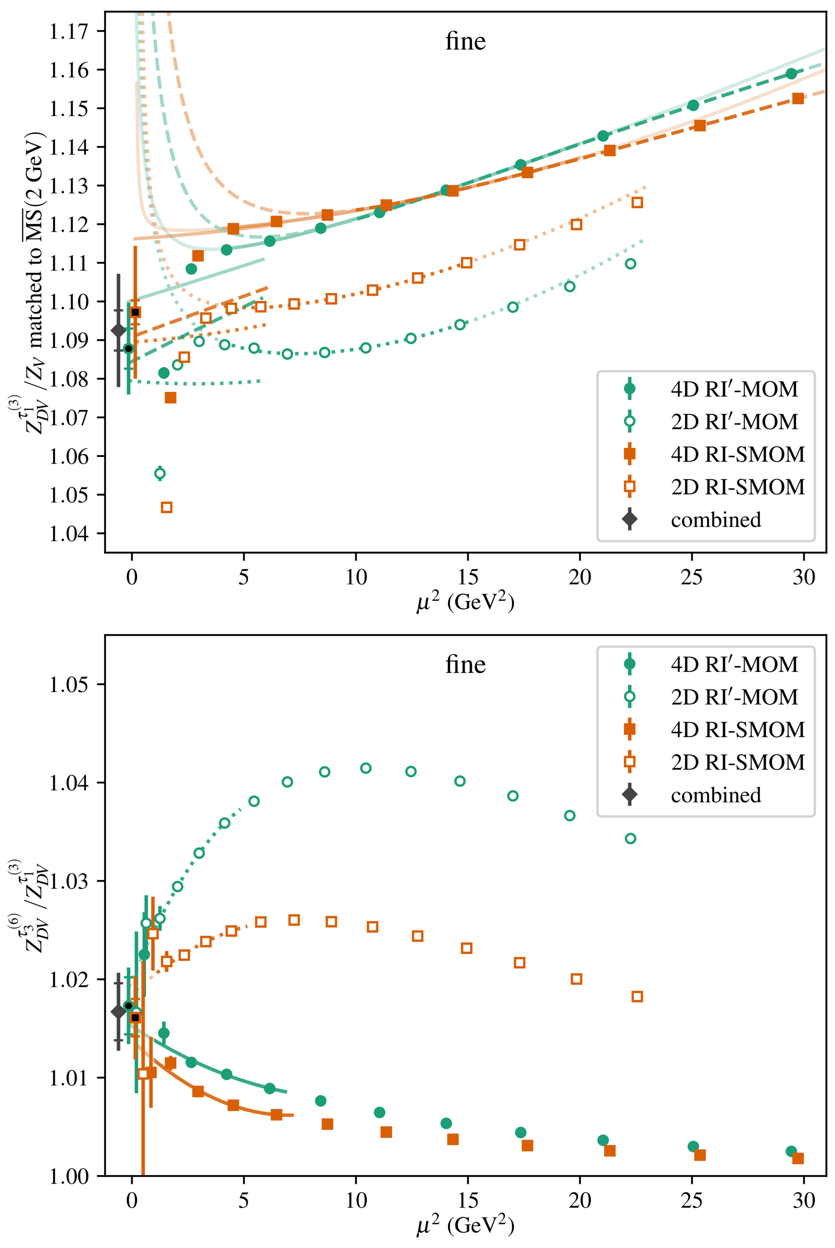
$$\langle x\rangle_{q^+} \equiv \int_0^1 x[q(x) + \bar{q}(x)]\,dx,$$
$$\langle x\rangle_{\Delta q^-} \equiv \int_0^1 x[\Delta q(x) - \Delta\bar{q}(x)]\,dx,$$
$$\langle x\rangle_{\delta q^+} \equiv \int_0^1 x[\delta q(x) + \delta\bar{q}(x)]\,dx.$$

Obtained from forward matrix elements of twist-two operators: e.g.

$$\langle N(p)|\bar{q}\gamma_{\{\mu}D_{\nu\}}q|N(p)\rangle = \langle x\rangle_{q^+}\bar{u}_N(p)\gamma_{\{\mu}ip_{\nu\}}u_N(p),$$

where {} denote the symmetric traceless part. Varying p, μ, ν, we get different matrix elements with nonzero kinematic factor.

Nonperturbative renormalization



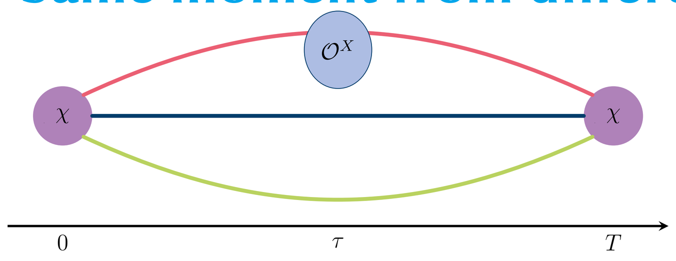
Proceed in three steps using Rome-Southampton method:

1. Z_V from hadronic scheme (already computed).
2. $Z_O(\mu)/Z_V$ using RI′-MOM and RI-SMOM schemes, converted to $\overline{\text{MS}}$ using perturbation theory. Need to fit at high μ^2.
3. $O(4)$-breaking $Z_{O'}/Z_O$ for two different hypercubic irreps. No perturbation theory needed; can use small μ^2.

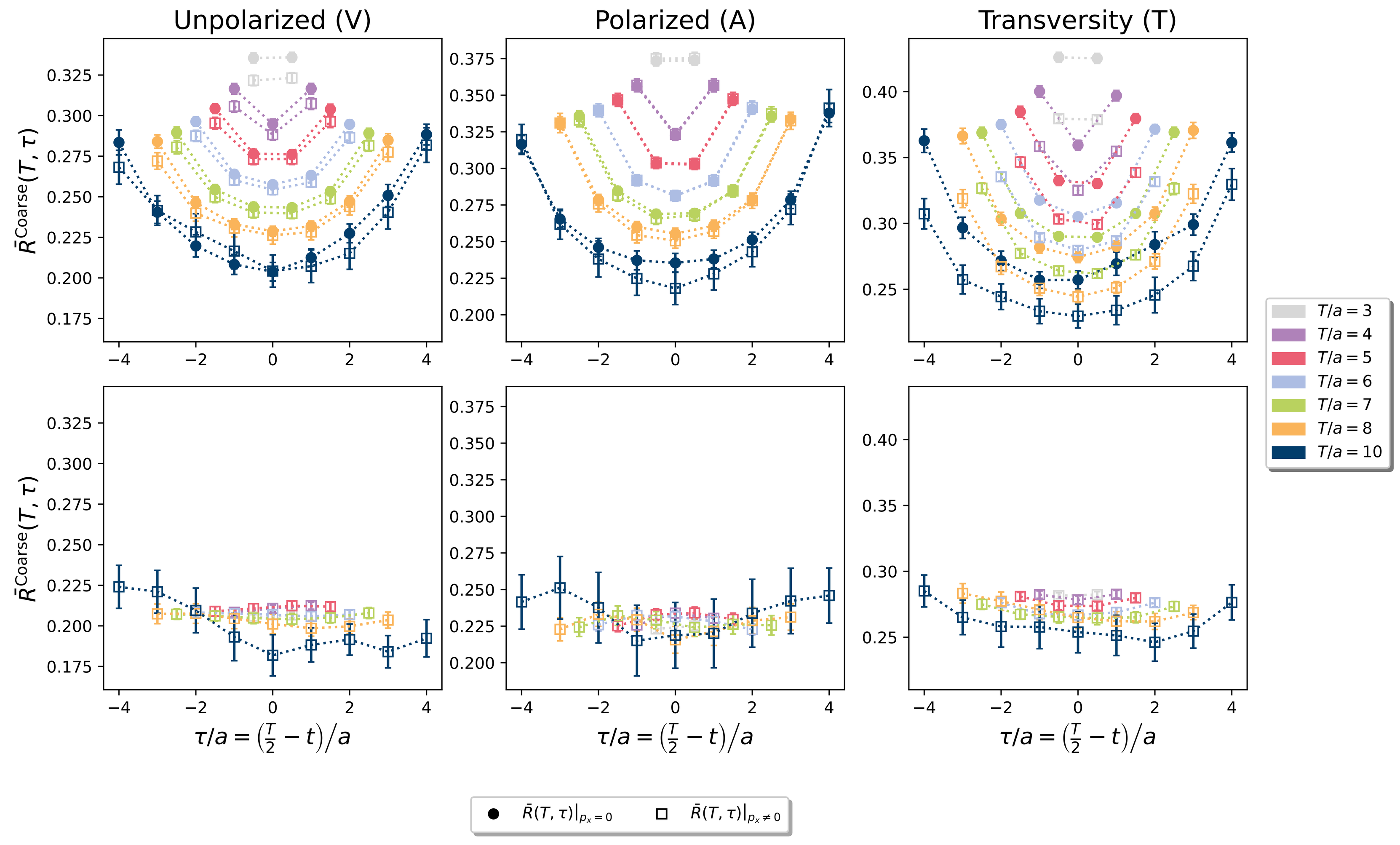
Further control systematics via different kinematics:
"4D" $\sim \frac{1}{2}(p,p,p,p)$ and
"2D" $\sim \frac{1}{\sqrt{2}}(p,p,0,0)$.

M. Rodekamp, M. Engelhardt, JRG, S. Krieg *et al.*, Phys. Rev. D **109**, 074508 (2024) [2401.05360]

Same moment from different matrix elements



Isolate nucleon matrix element via Euclidean time evolution by time separations τ and $T-\tau$.



Some choices of operator with nonzero momentum have less excited-state contamination. Calculate at physical pion mass with two lattice spacings. Weight many different analyses to get final results:

$$\langle x\rangle_{u^+-d^+} = 0.200(17), \quad \langle x\rangle_{\Delta u^- - \Delta d^-} = 0.213(16), \quad \langle x\rangle_{\Delta u^- - \Delta d^-} = 0.219(21).$$

M. Rodekamp, M. Engelhardt, JRG, S. Krieg *et al.*, Phys. Rev. D **109**, 074508 (2024) [2401.05360]

Next step: third and fourth moments

Twist-two operators with two or three derivatives are more strongly affected by lattice symmetry breaking $O(4) \to H_4$. Need to choose components $\mu_1 \ldots \mu_n$ more carefully.
Nonzero momentum needed to get nonzero kinematic factor.