

# Simulation of Condensed Matter: Session in memory of Prof. Dr. Kurt Binder



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# Kurt Binder (1944-2022)

computer simulations in condensed matter physics:  
Monte-Carlo simulation, statistical physics of phase transitions,  
finite-size scaling, physics of glasses, interfaces and polymers



2007 - fellow, Gutenberg Forschungskolleg, Universität Mainz  
1983 - 2012 Professor for Theoretical Physics, Universität Mainz  
1977 - 1983 Director, Institute for Solid State Research, Jülich  
1974 - 1977 Professor, Universität des Saarlandes

2018 - 2020 vice chair, scientific council of NIC  
2012 - 2017 vice chair, steering committee of GCS  
2012 - 2017 chair, scientific council of NIC  
2011 vice chair, scientific council of NIC  
2010 - 2012 member, scientific steering committee of PRACE  
1987 - 1992 member, scientific council of HLRZ, Jülich

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some awards:

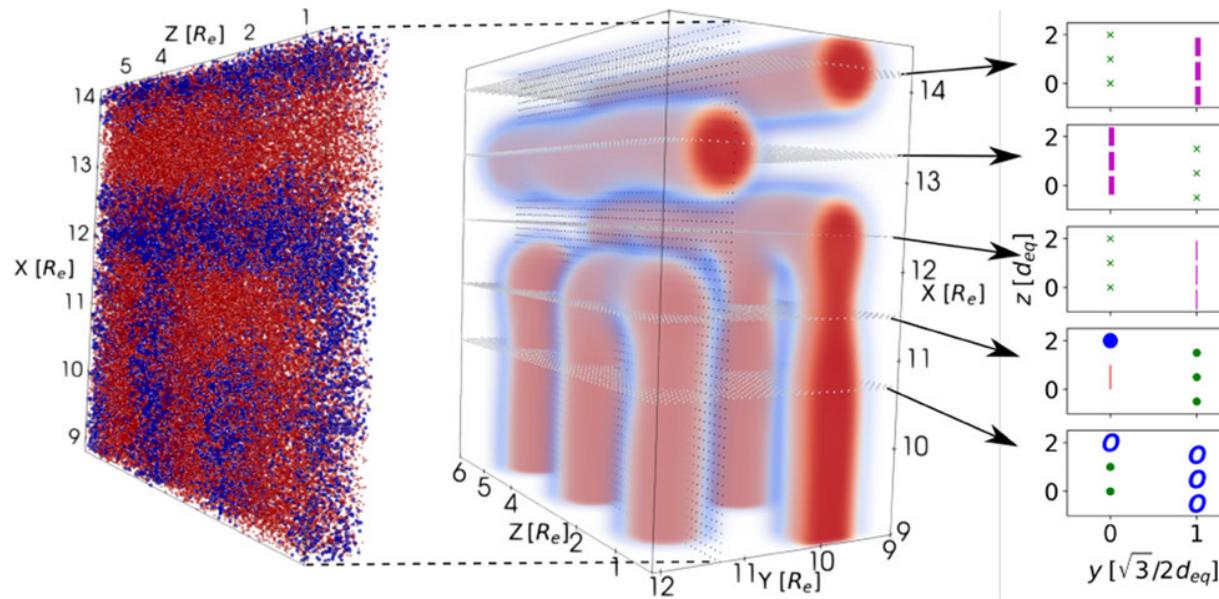
2020 Polymer Physics Prize (APS)  
2009 Lennard-Jones Lecture Award (RSC)  
2007 Boltzmann Medaille (IUPAC)  
2003 Staudinger-Dürrer-Prize (ETH Zürich)  
2001 Berni Alder CECAM Preis  
1993 Max Planck Medaille (DPG)

# Kurt Binder's group (around 1998)



# Multiscale modeling of grain-boundary motion in cylinder-forming block copolymers

Niklas Blagojevic and Marcus Müller



thanks to:

*Gregor Ibbeken,*

*Ludwig Schneider, Oliver Dreyer (Hereon)*

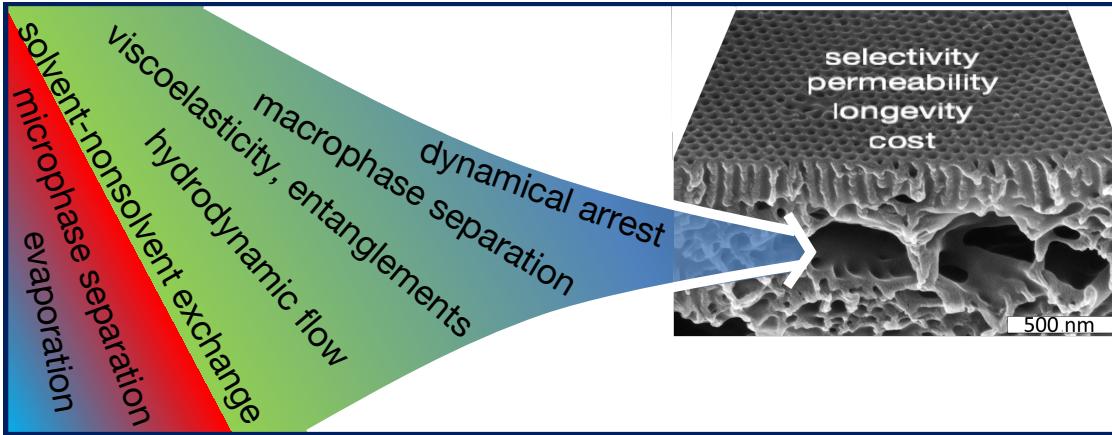
Dreyer, Ibbeken, Schneider, Blagojevic, Radjabian, Abetz, Müller, *Macromolecules* **55**, 7564 (2022)

Müller, *Prog. Polym. Sci.* **101**, 101198 (2020)

Müller, Abetz, *Chem. Rev.* **121**, 14189 (2021)

Blagojevic, Müller, *ACS Polymers AU* **3**, 96 (2023)

# EISA and NIPS in a diblock copolymer film

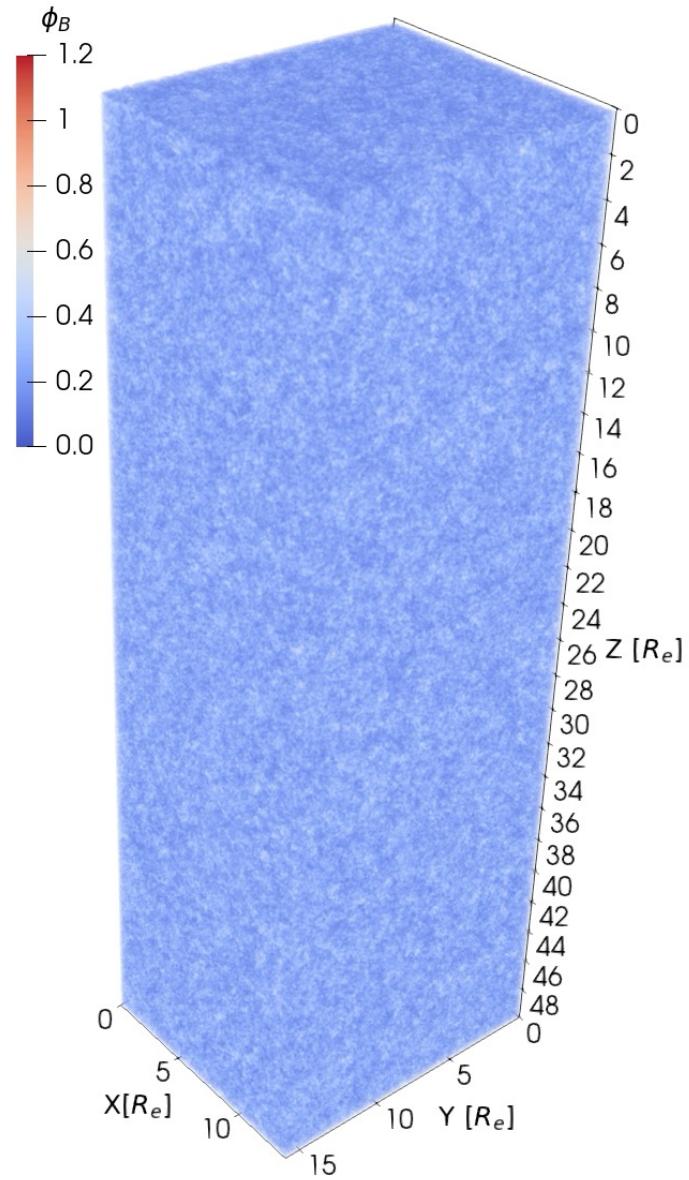


Müller, Abetz, *Chem. Rev.* **121**, 14189 (2021)

**techniques:** particle-based simulation  
and continuum model

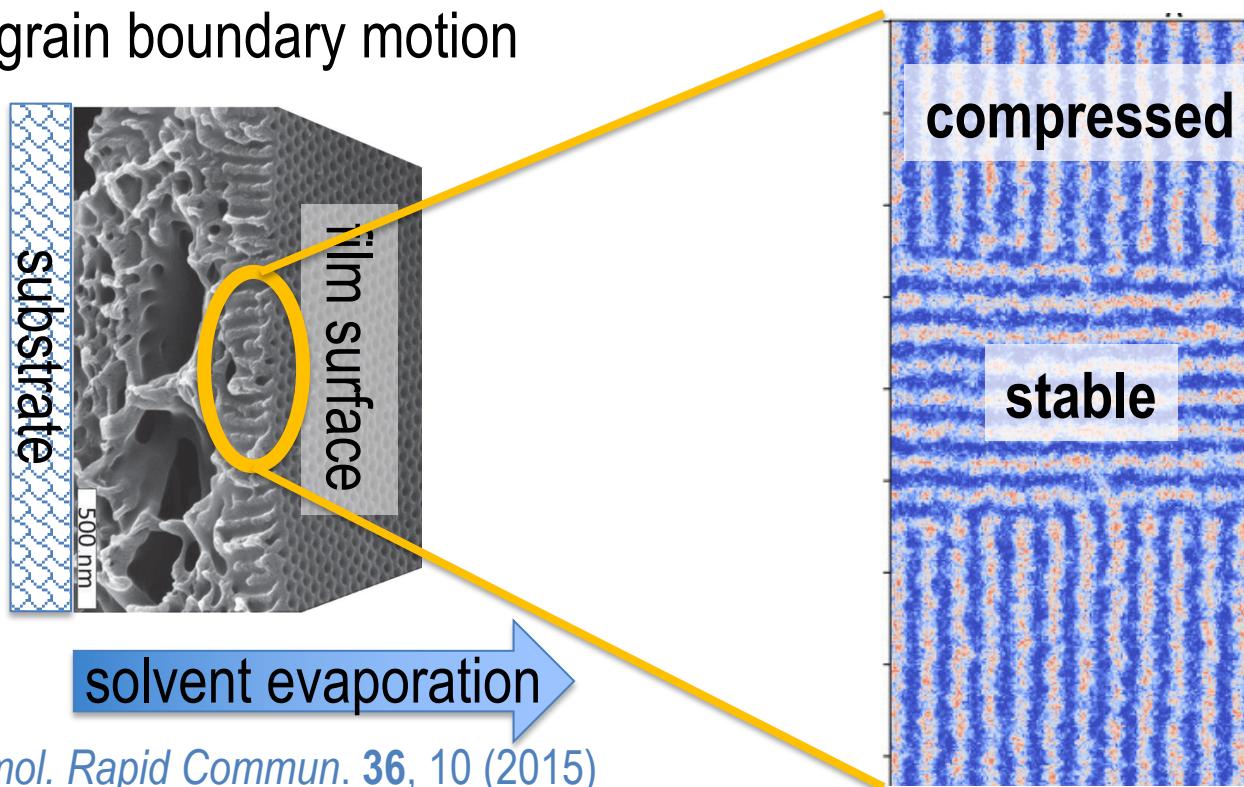
**question:**

- how to fabricate perpendicular cylinders?
- role of solvent selectivity and evaporation rate
- mechanism of structure formation



# solvent evaporation from a diblock copolymer film

- solvent evaporates and polymer density at the film surface increases  
→ skin formation
- microphase separation commences at film surface (EISA)  
domains of cylinders with different orientations form
- as the solvent evaporates further, the film thickness shrinks  
→ anisotropic deformation of domains results in grain coarsening  
via grain boundary motion



# grain-boundary motion

**system:** cylinder-forming diblock copolymer  $\chi N=35$ ,  $f=1/4$ ,  $N=64$

two grains with orthogonal orientation, one is only metastable

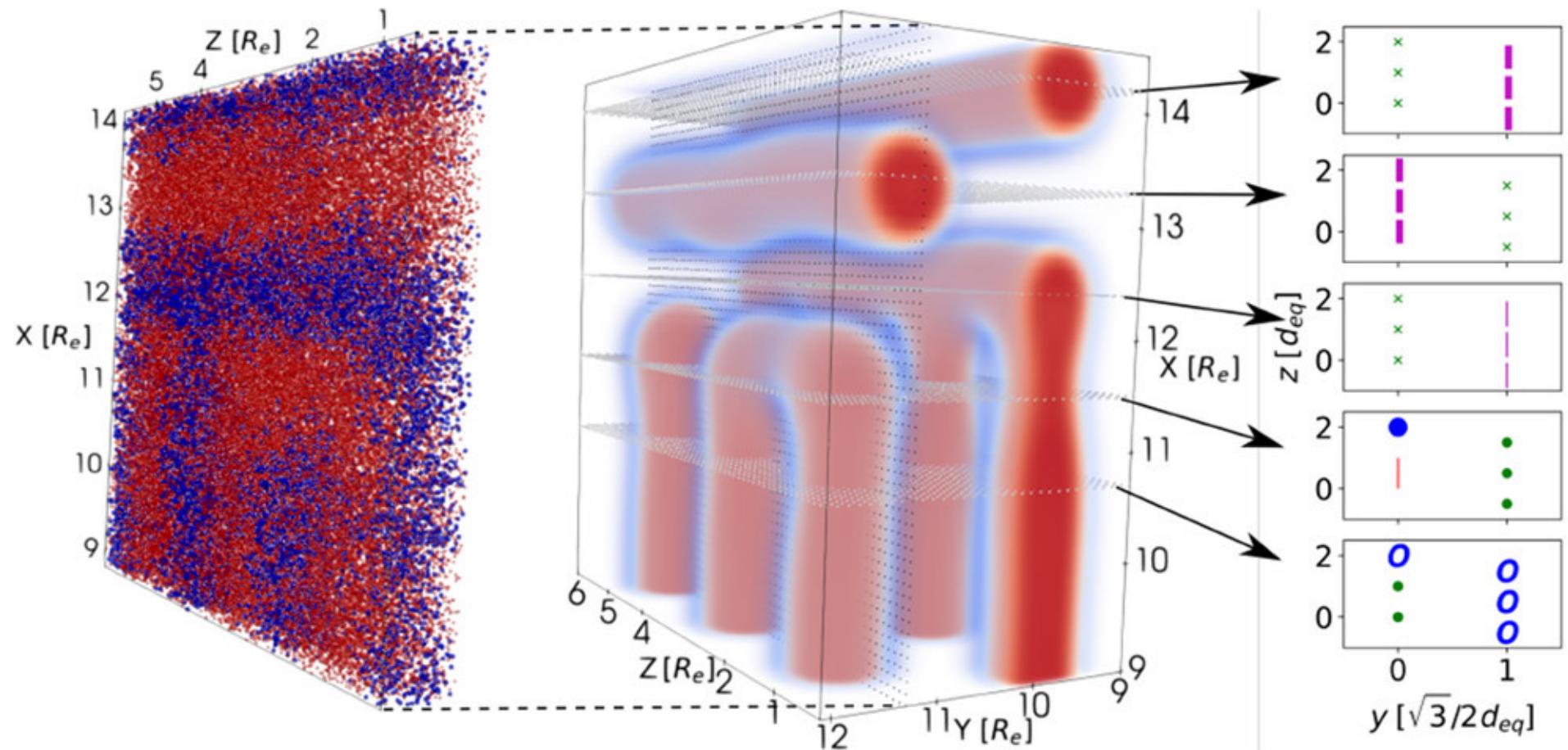
particle simulation



continuum model / MFEP



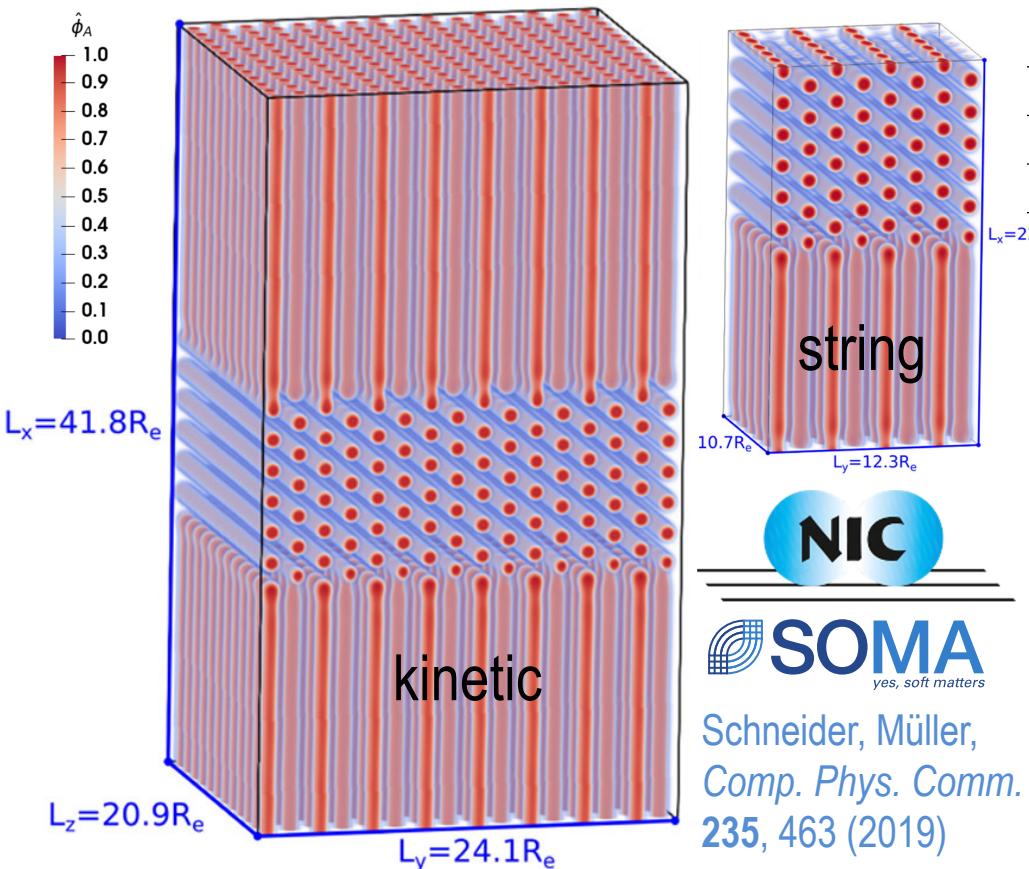
kMC model



# grain-boundary motion

**system:** cylinder-forming diblock copolymer  $\chi N=35$ ,  $f=1/4$

two grains with orthogonal orientation, one is only metastable



NIC  
SOMA  
yes, soft matters  
Schneider, Müller,  
Comp. Phys. Comm.  
235, 463 (2019)

System	$L_x \cdot L_y \cdot L_z [R_e^3]$	$d_{eq} [R_e]$	$\sqrt{N}$
Single-grain	$5 \cdot 12.3 \cdot 10.7$	1.78	750
String	$22.0 \cdot 12.3 \cdot 10.7$	1.78	1300
Kinetic	$41.8 \cdot 24.1 \cdot 20.9$	1.74	60 & 90

typically  $10^8$  segments

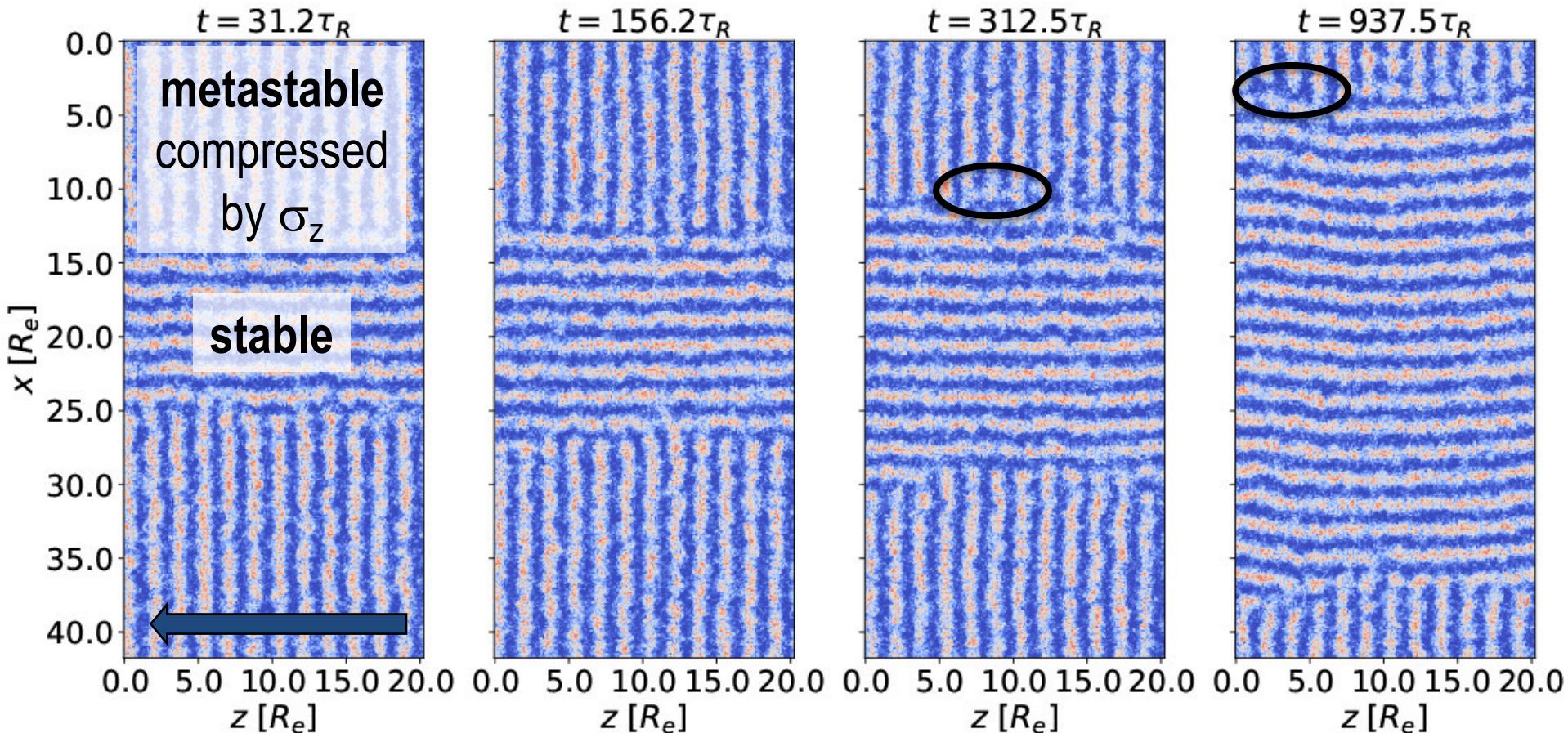
- choice of invariant degree of polymerization  $\sqrt{N} = \rho R_e^3 / N$ :
- small enough so that system overcomes free-energy barriers  $\Delta F \sim \sqrt{N} k_B T$
  - large enough to suppress density fluctuations

compression,  $\sigma_z$ , along z-direction distorts one grain and thereby increases its free energy → deterministic motion of grain boundary

# kinetics of grain-boundary motion

main processes involved in grain-boundary motion (topological changes):

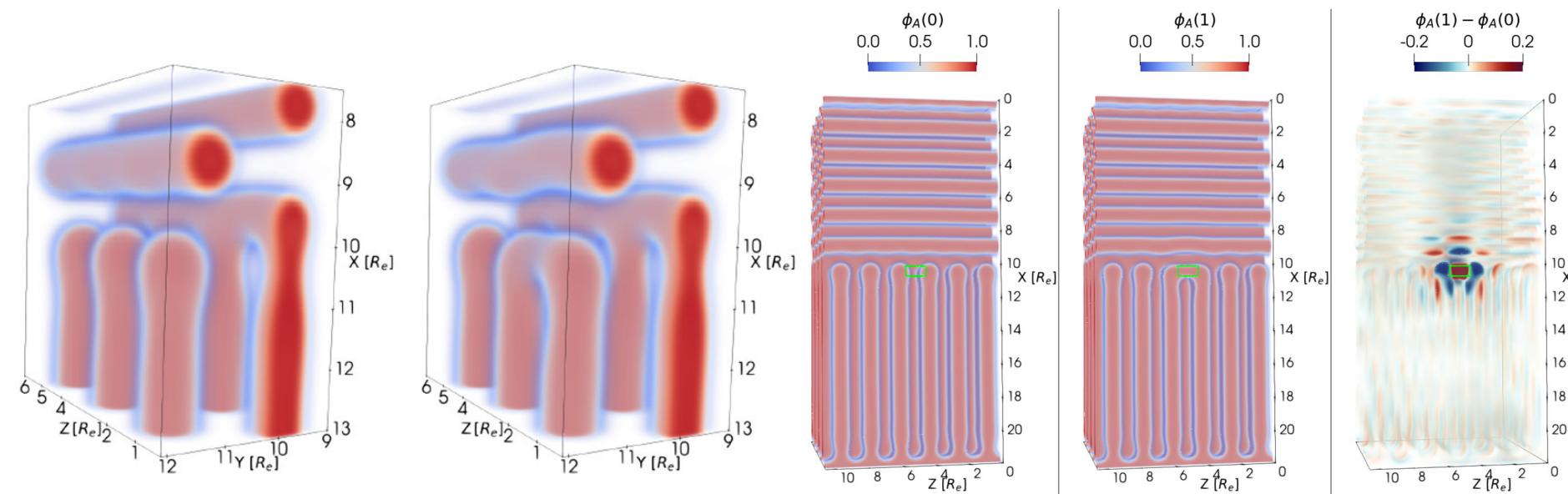
- fusion of heads of perpendicular cylinders with/to parallel cylinders
- rupture of junction between perpendicular and parallel cylinders



# kinetics of grain-boundary motion

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- fusion of heads of perpendicular cylinders with/to parallel cylinders



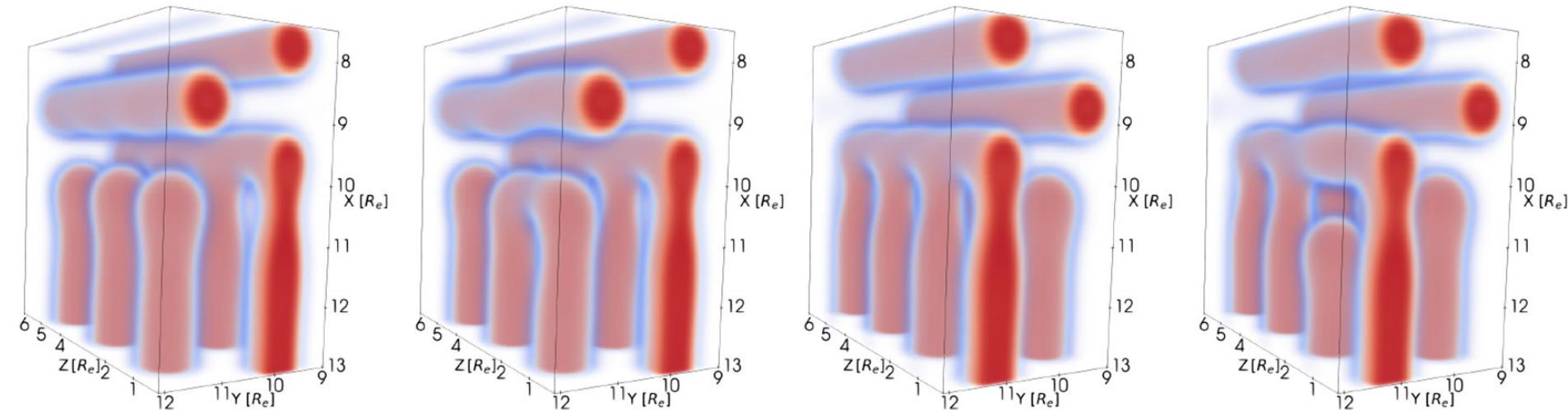
long-range strain field around defect:

- large system size necessary
- dependence of free-energy barrier on environment

# kinetics of grain-boundary motion

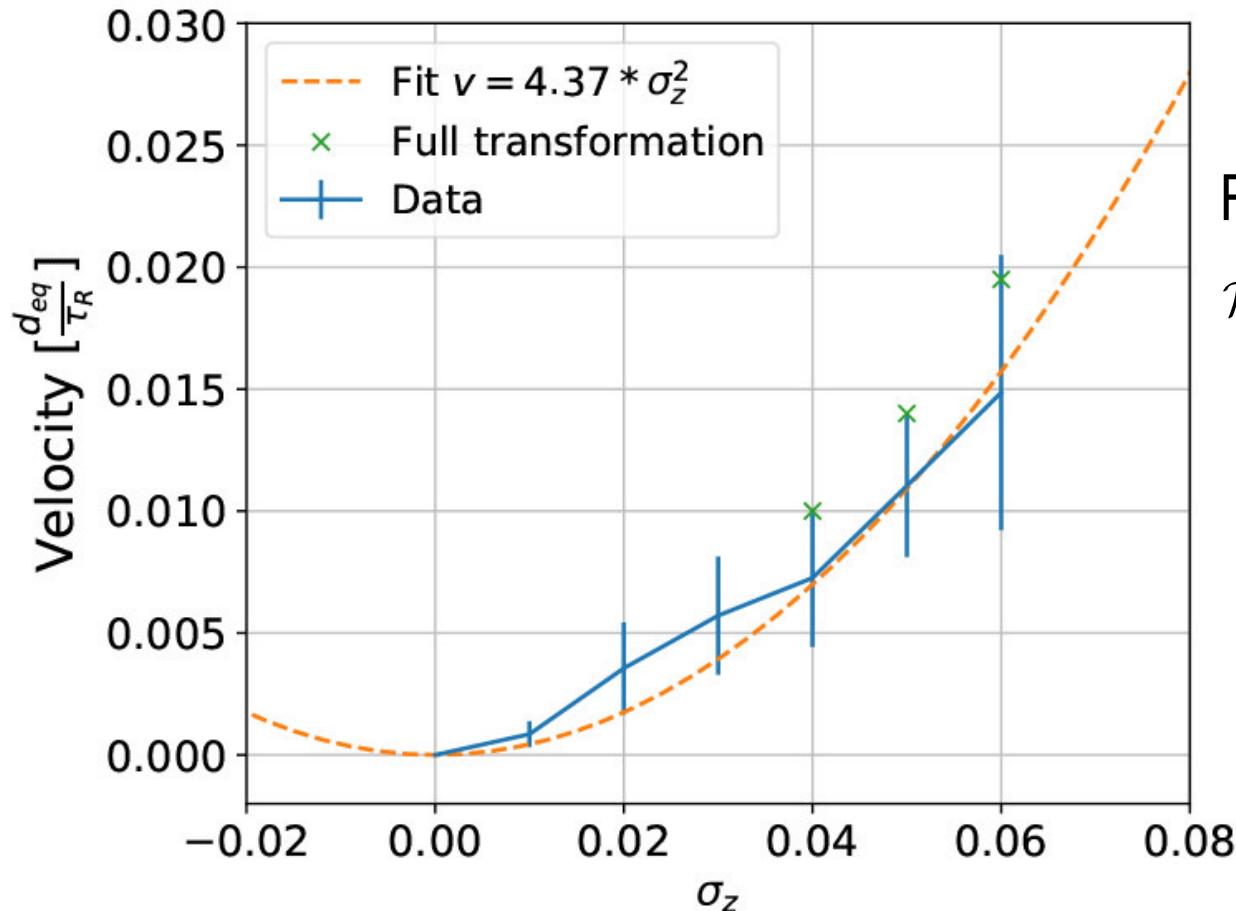
main processes involved in grain-boundary motion (topological changes):

- fusion of heads of perpendicular cylinders with/to parallel cylinders
- rupture of junction between perpendicular and parallel cylinders



# velocity of grain boundary at $\sqrt{\mathcal{N}} = 65$

fast motion at small invariant degree of polymerization  $\sqrt{\mathcal{N}} = \rho R_e^3 / N$   
grain-boundary velocity  $\sim$  free-energy difference  $\Delta f \sim \sigma_z^2$

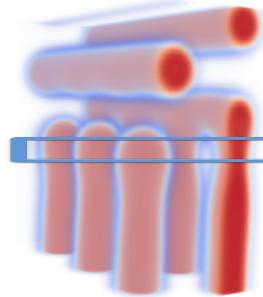
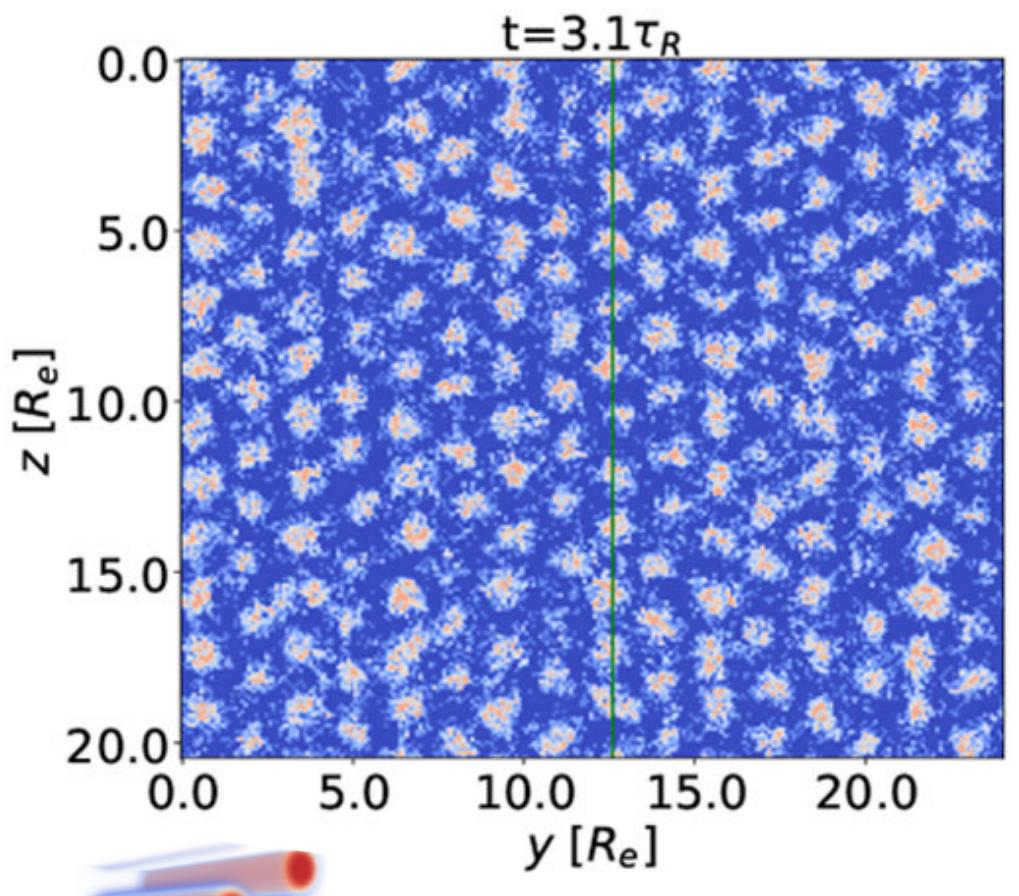


Rayleighian:

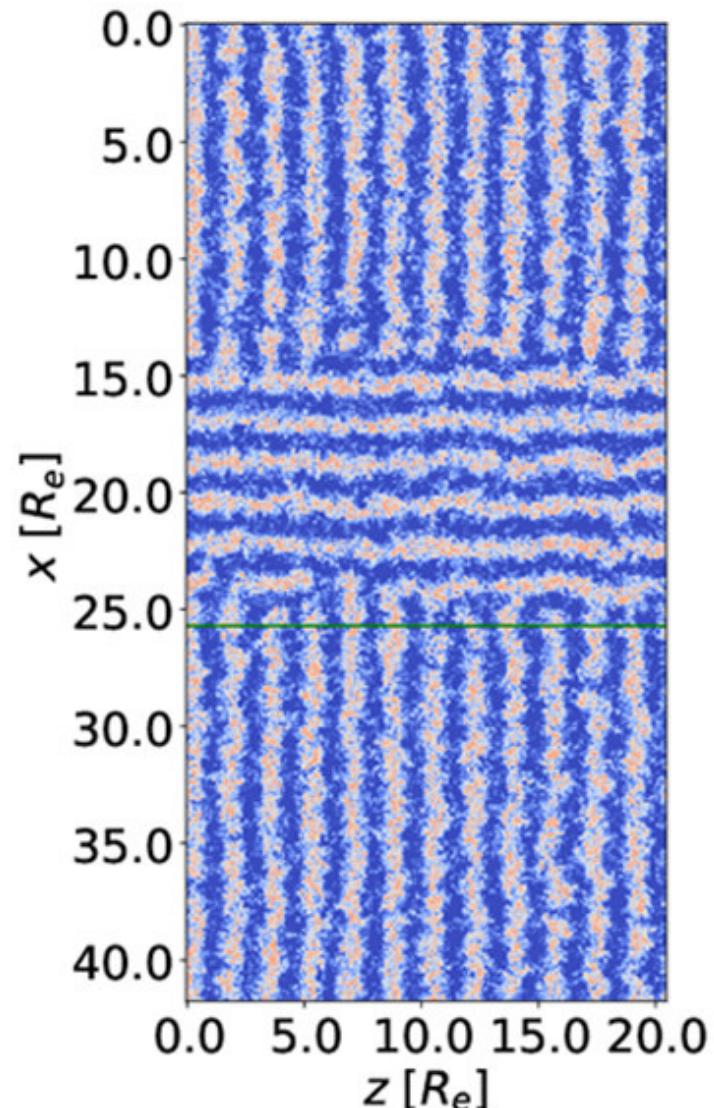
$$\begin{aligned}\mathcal{R} &= \dot{F} + \frac{1}{2} \sum T \\ &= -\Delta f A v + \frac{1}{2} A \zeta v^2 \rightarrow \min \\ v &= \Delta f / \zeta\end{aligned}$$

study sequence of topological changes and concomitant free-energy barriers  
that are required for grain-boundary motion

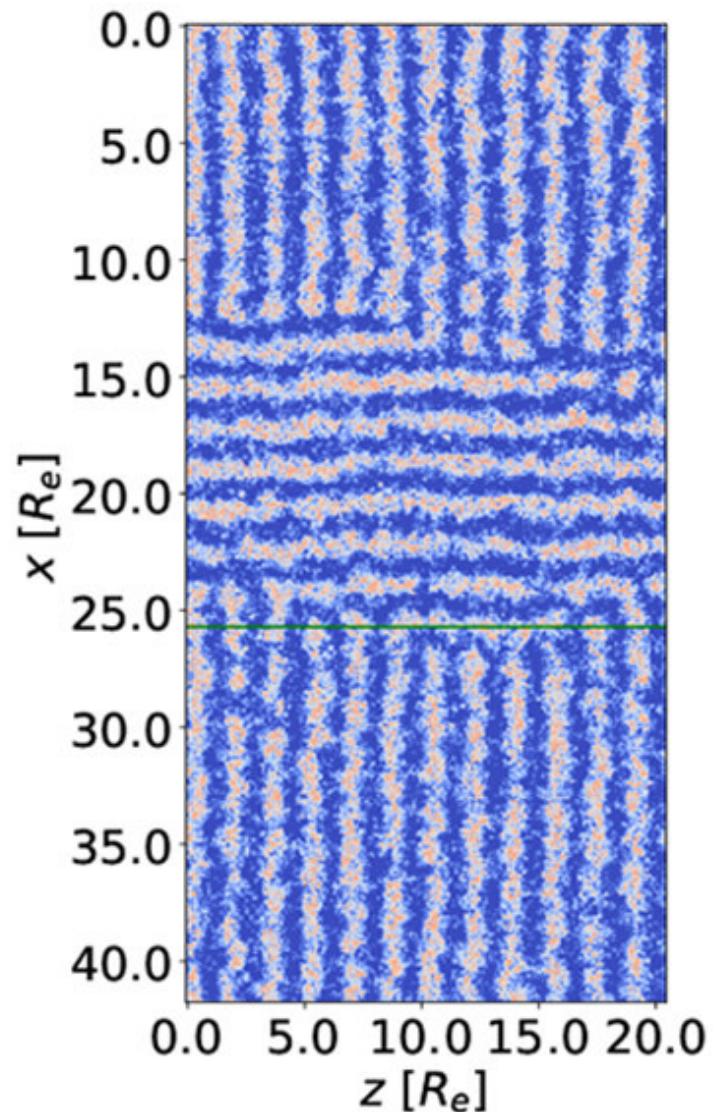
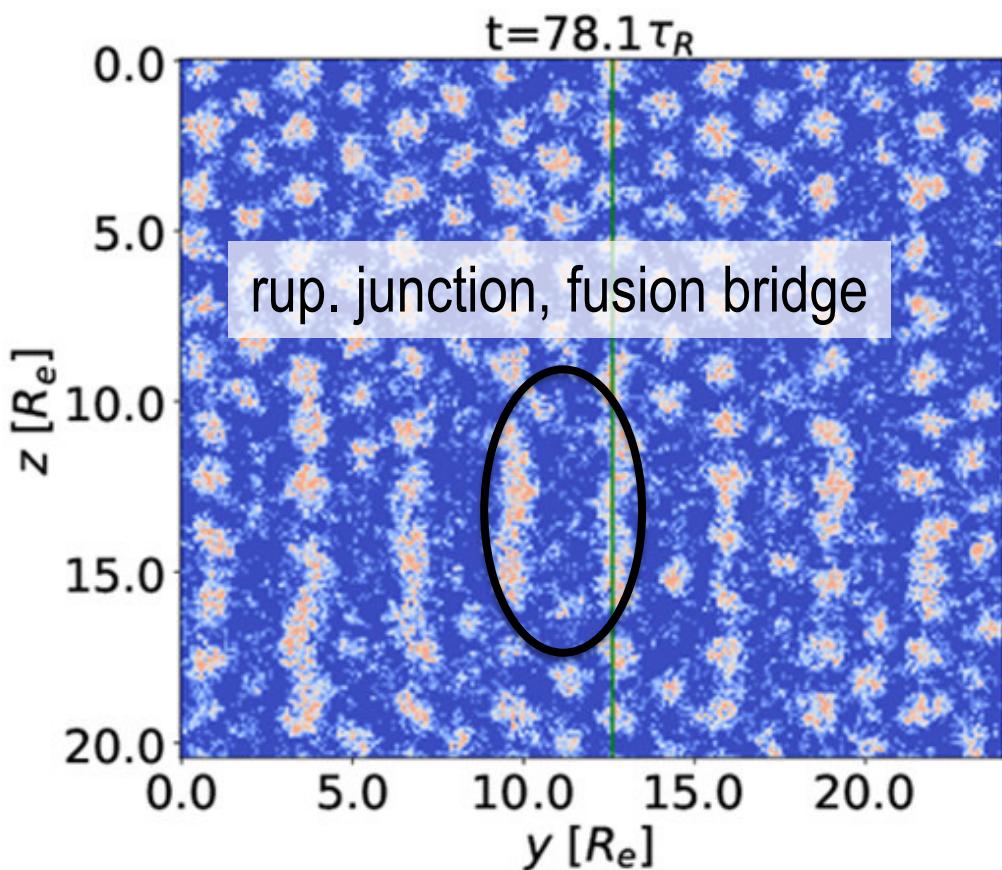
sequence of topology changes, correlated in space and time



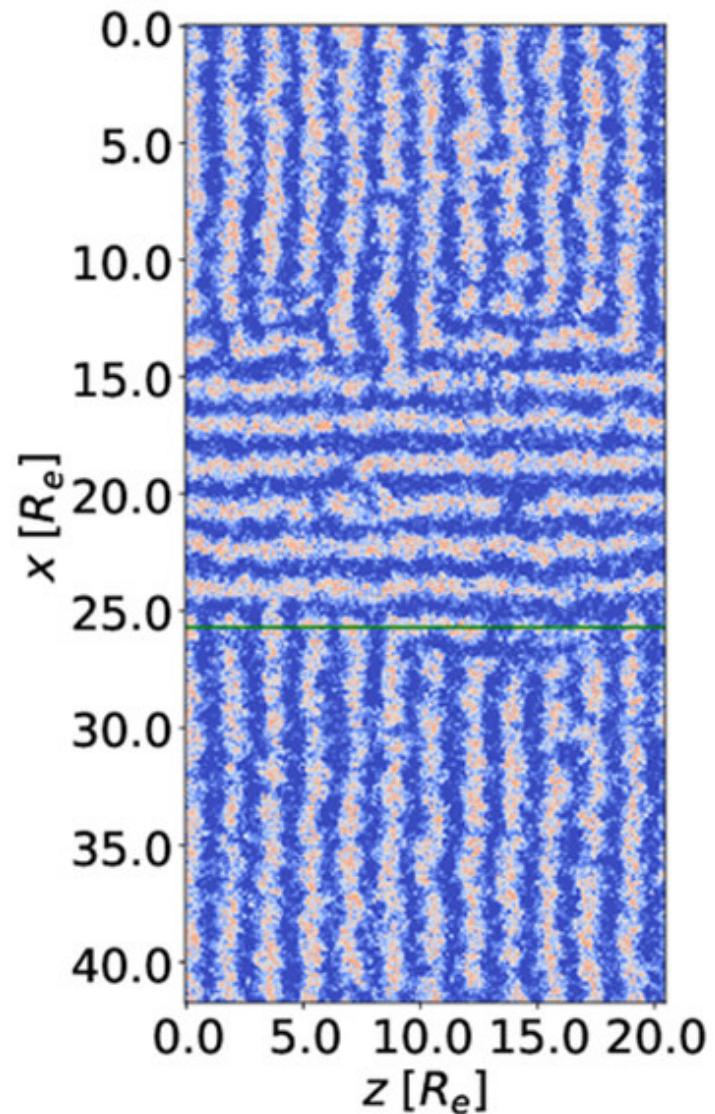
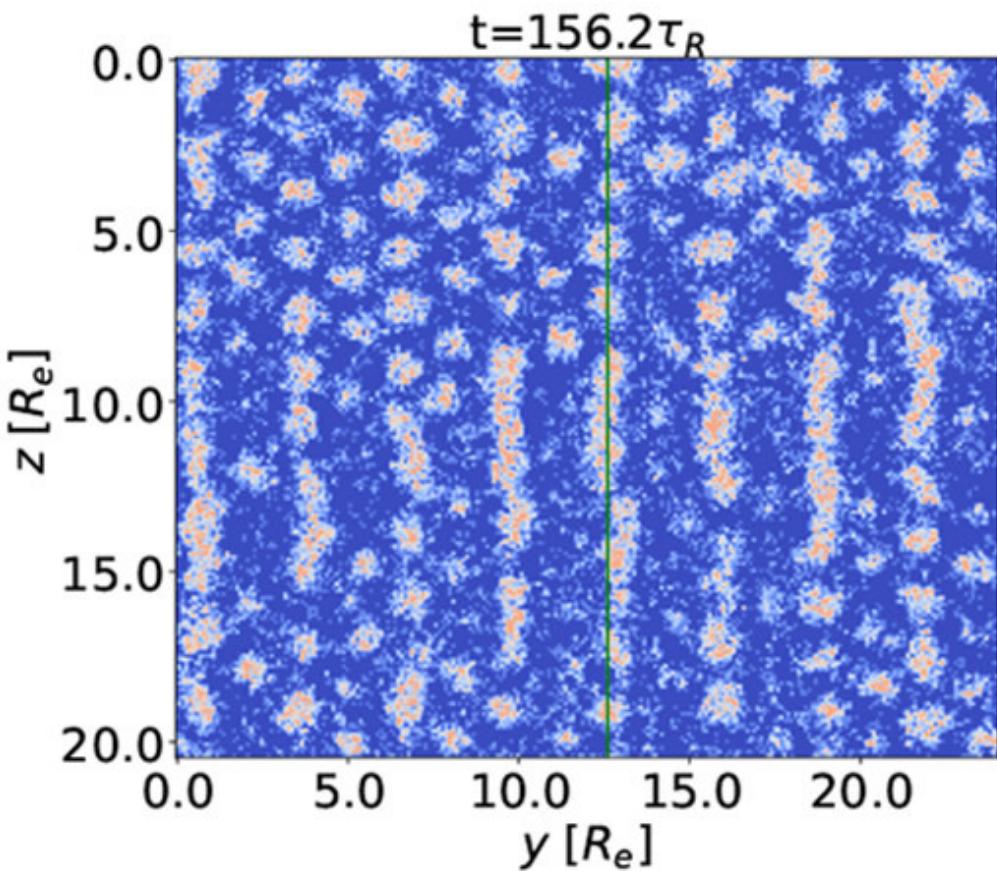
cylinder heads and junctions



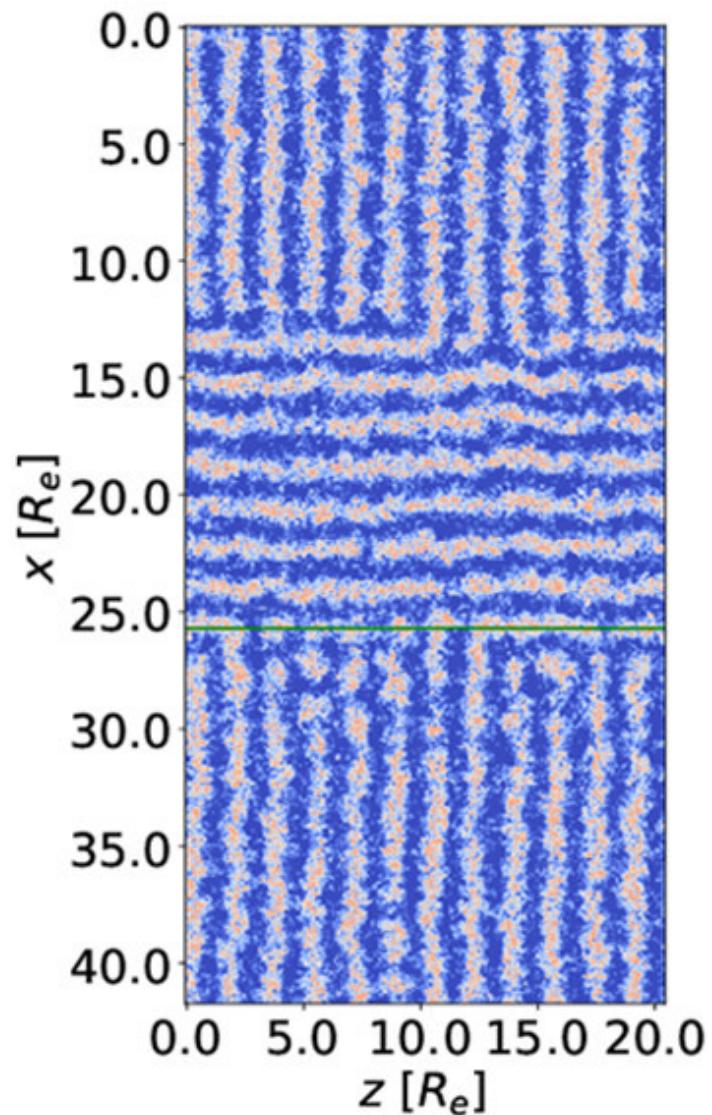
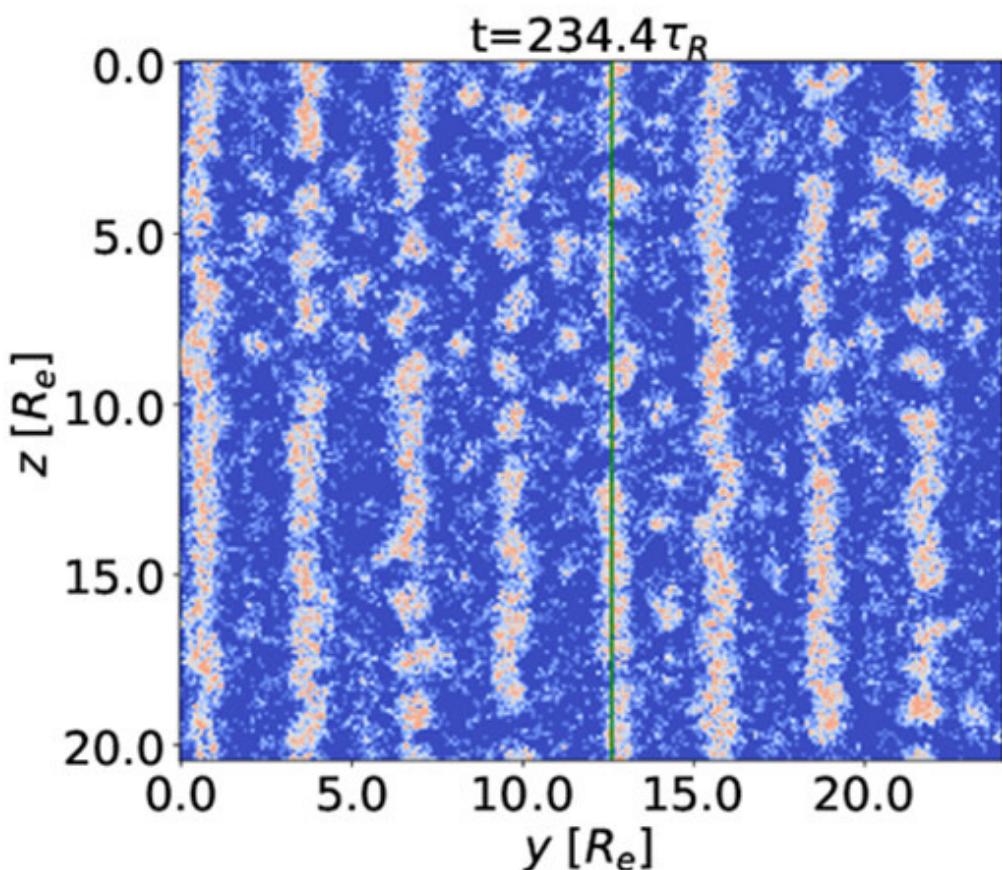
sequence of topology changes, correlated in space and time



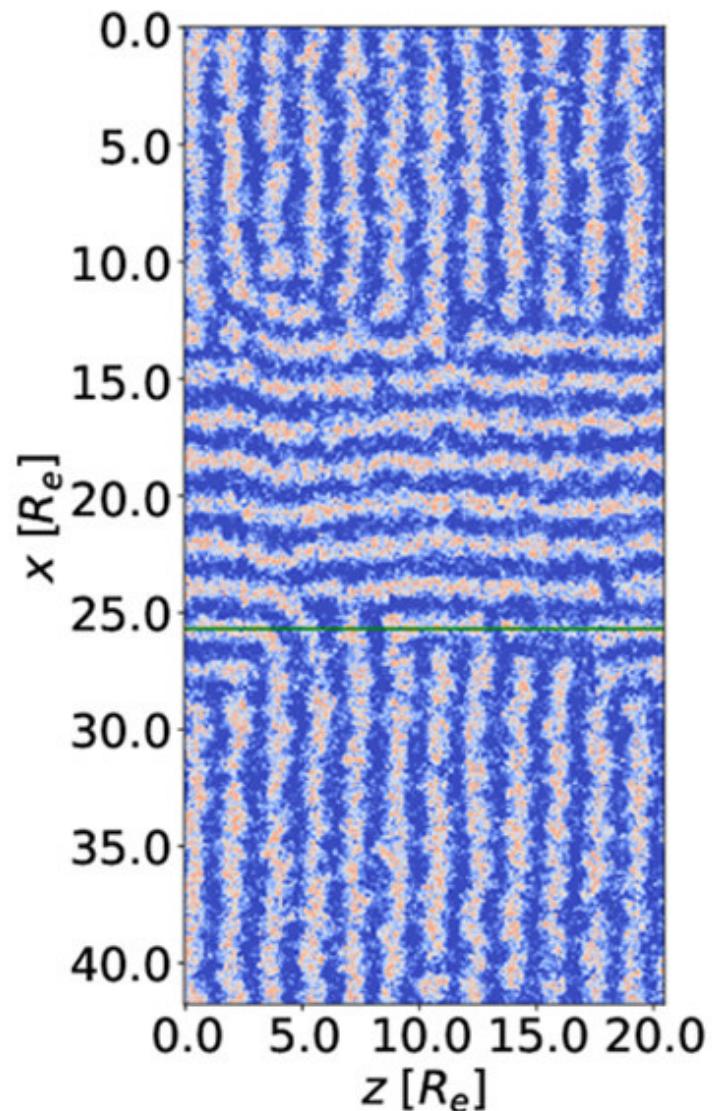
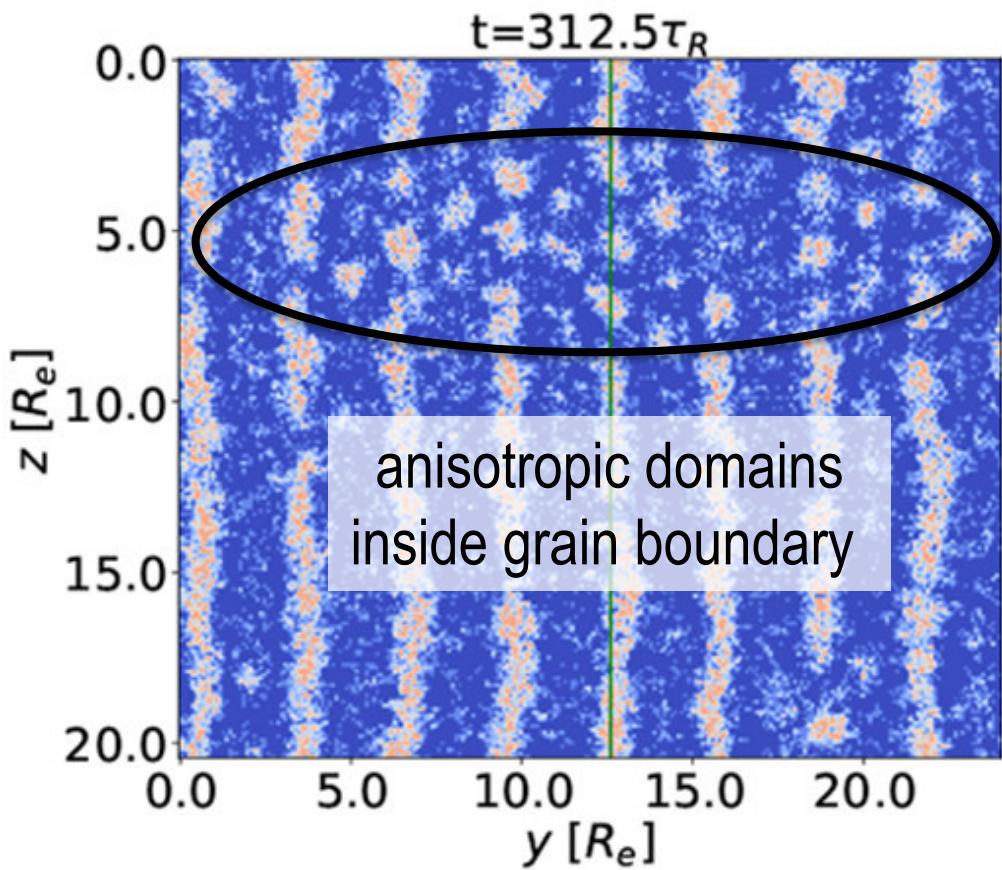
sequence of topology changes, correlated in space and time



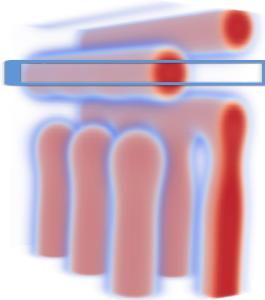
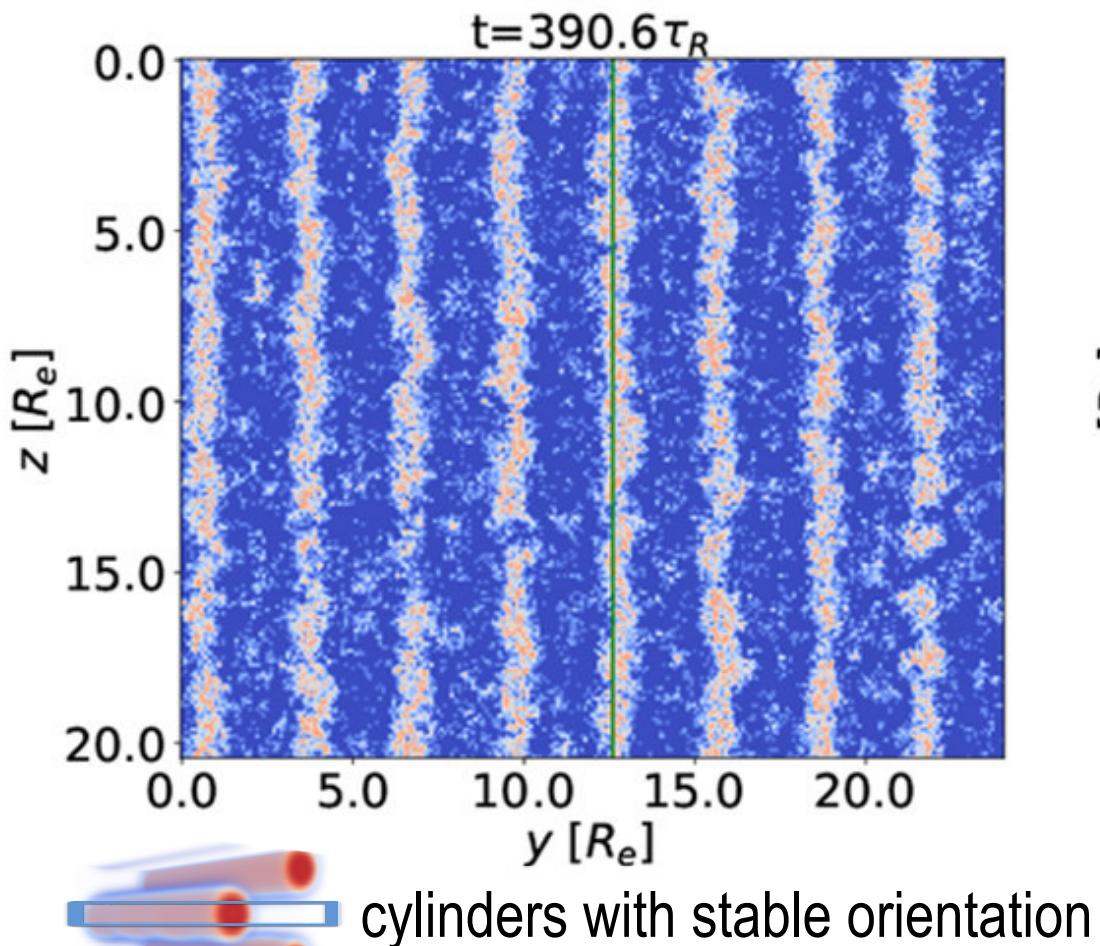
sequence of topology changes, correlated in space and time



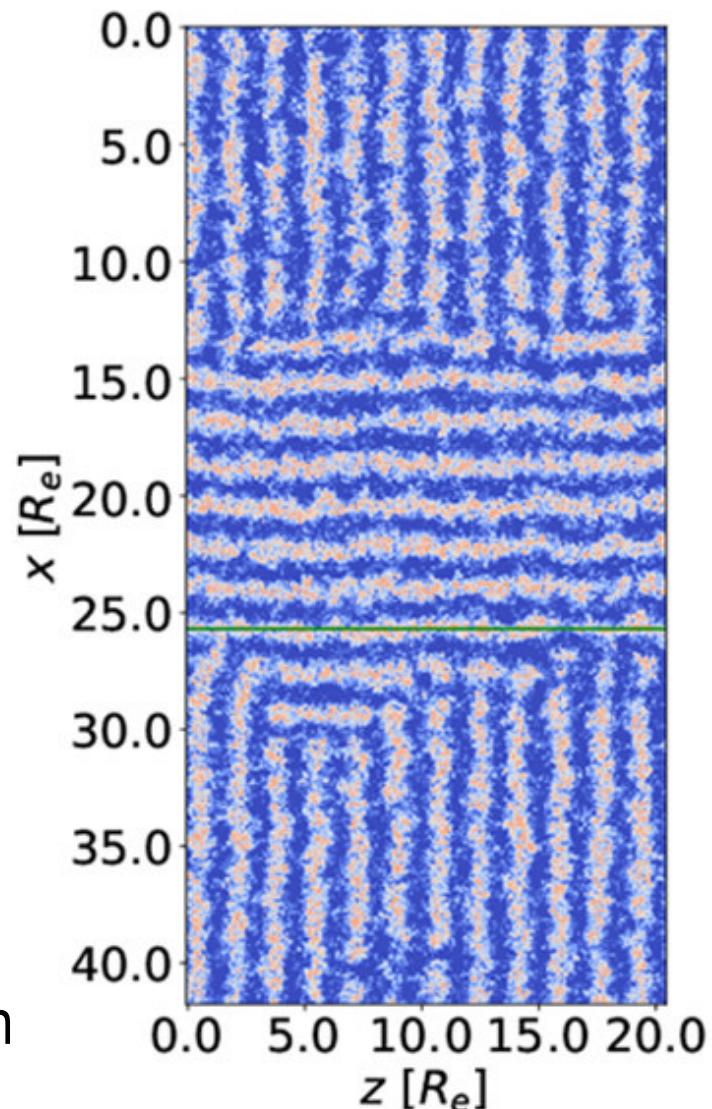
# sequence of topology changes, correlated in space and time



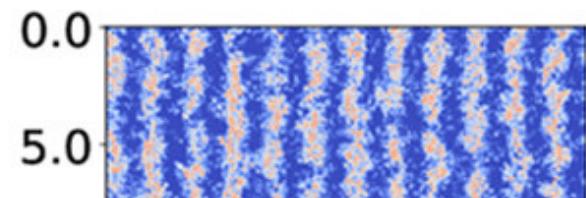
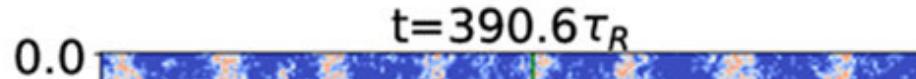
# sequence of topology changes, correlated in space and time



cylinders with stable orientation



# sequence of topology changes, correlated in space and time



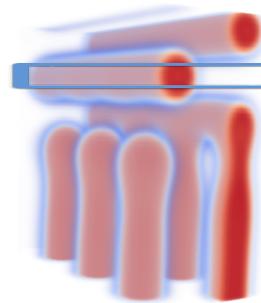
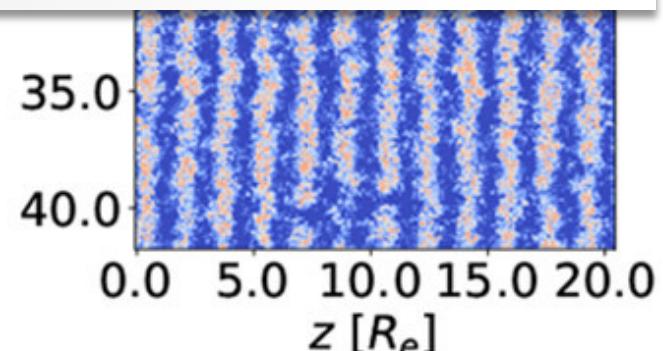
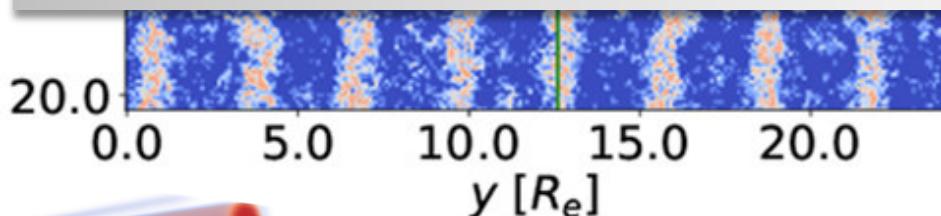
## *grain-boundary motion by particle simulations:*

- *small*  $\bar{\mathcal{N}}$ :

grain-boundary velocity  $\sim$  free-energy difference between grains  
elementary transitions: fusion of cylinder heads / rupture of junctions  
sequence of topological changes, correlated in space and time

- *large*  $\bar{\mathcal{N}}$ :

distortion of grains or elimination of cylinders to relieve stress



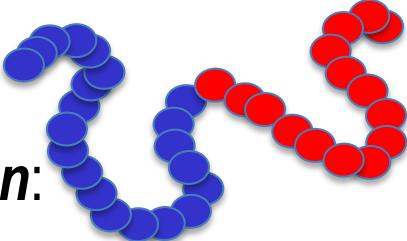
cylinders with stable orientation

# from particle simulation to continuum description

## *degrees of freedom:*

particle coordinates,  $N(n_A + n_B)$

$$\{\mathbf{R}\} = \mathbf{R}(s)$$



## *model definition:*

intra- and intermolecular potentials

(here: soft, coarse-grained model, SCMF)

single-chain dynamics

(here: Rouse dynamics)

segmental friction,  $\gamma$

**projection:**  $\hat{\phi}_A(\mathbf{r}|\{\mathbf{R}\}_i) = \frac{1}{\rho_0} \sum_{i=1}^n \sum_{s=1}^{fN} \delta(\mathbf{r} - \mathbf{R}_i(s))$

$$\frac{\mathcal{F}[m]}{k_B T} = -\ln \int \mathcal{D}[\{\mathbf{R}\}_i] e^{-\mathcal{H}/k_B T} \delta[m - \hat{m}(\mathbf{r}|\{\mathbf{R}\}_i)]$$

## *challenge:*

- sampling of free-energy differences or chemical potential  $\mu[m](\mathbf{r}) = \frac{\delta \mathcal{F}[m]}{\delta m(\mathbf{r})}$
- construction of reversible paths (reaction coordinates)

order parameters

segment composition field (and density)

$$m(\mathbf{r}) = \phi_A(\mathbf{r}) - \phi_B(\mathbf{r})$$

$$\rho(\mathbf{r}) = \phi_A(\mathbf{r}) + \phi_B(\mathbf{r}) \approx \rho_o \quad (\text{not } \rho(\{\mathbf{R}\}))$$

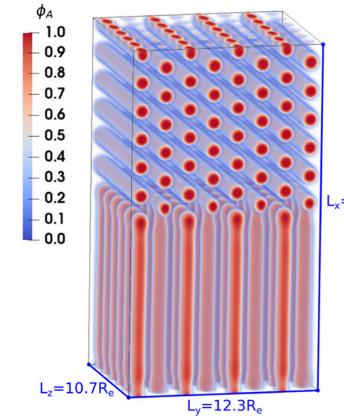
free-energy functional,  $\mathcal{F}[m]$

(SCFT, Ohta-Kawasaki)

model-B dynamics

(Hohenberg & Halperin)

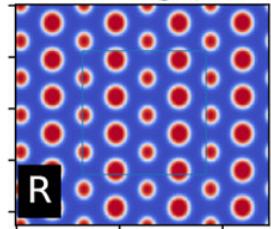
Onsager coefficient



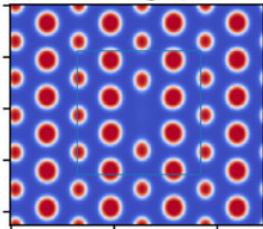
# MFEP of fusion and rupture transitions

- identify minimal set of 18 transitions (fusion/rupture in different environments)
- obtain chemical potential by field-theoretic umbrella sampling
- calculate the Minimum Free-Energy Path (MFEP) via string method

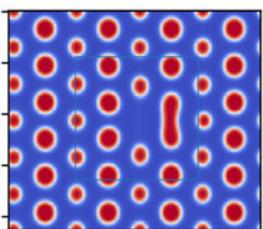
starting  $\phi_{A0}$



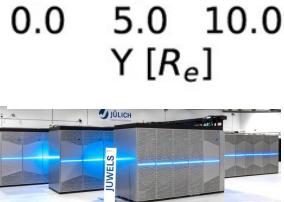
ending  $\phi_{A1}$



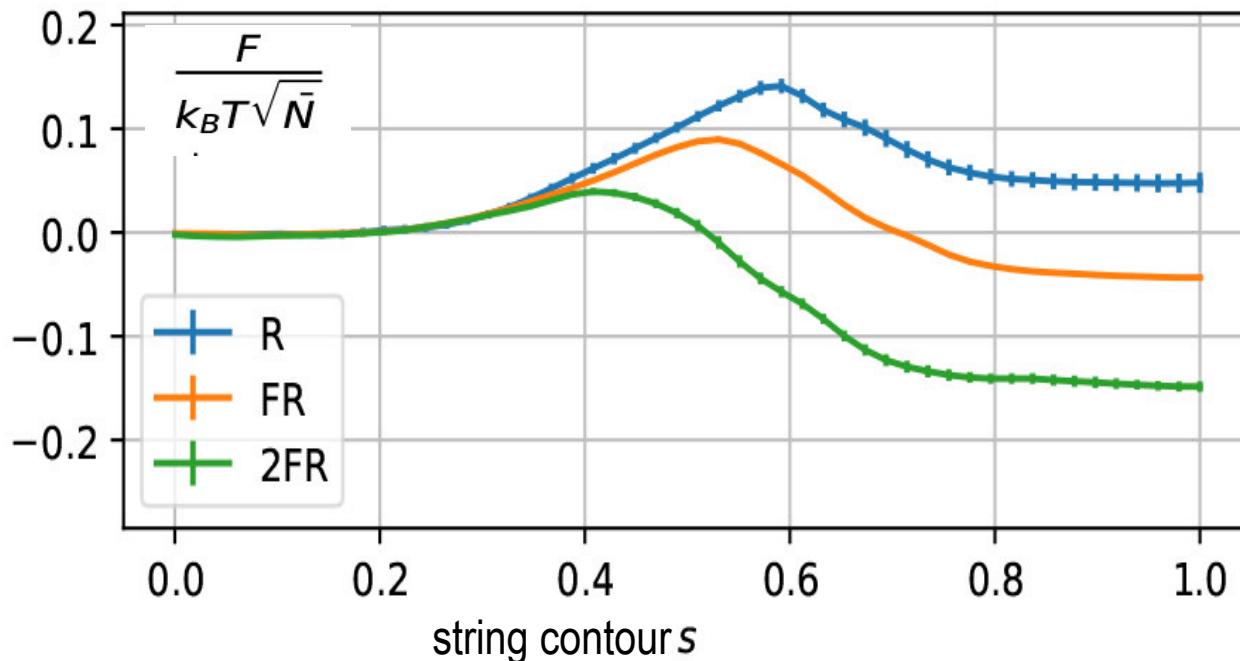
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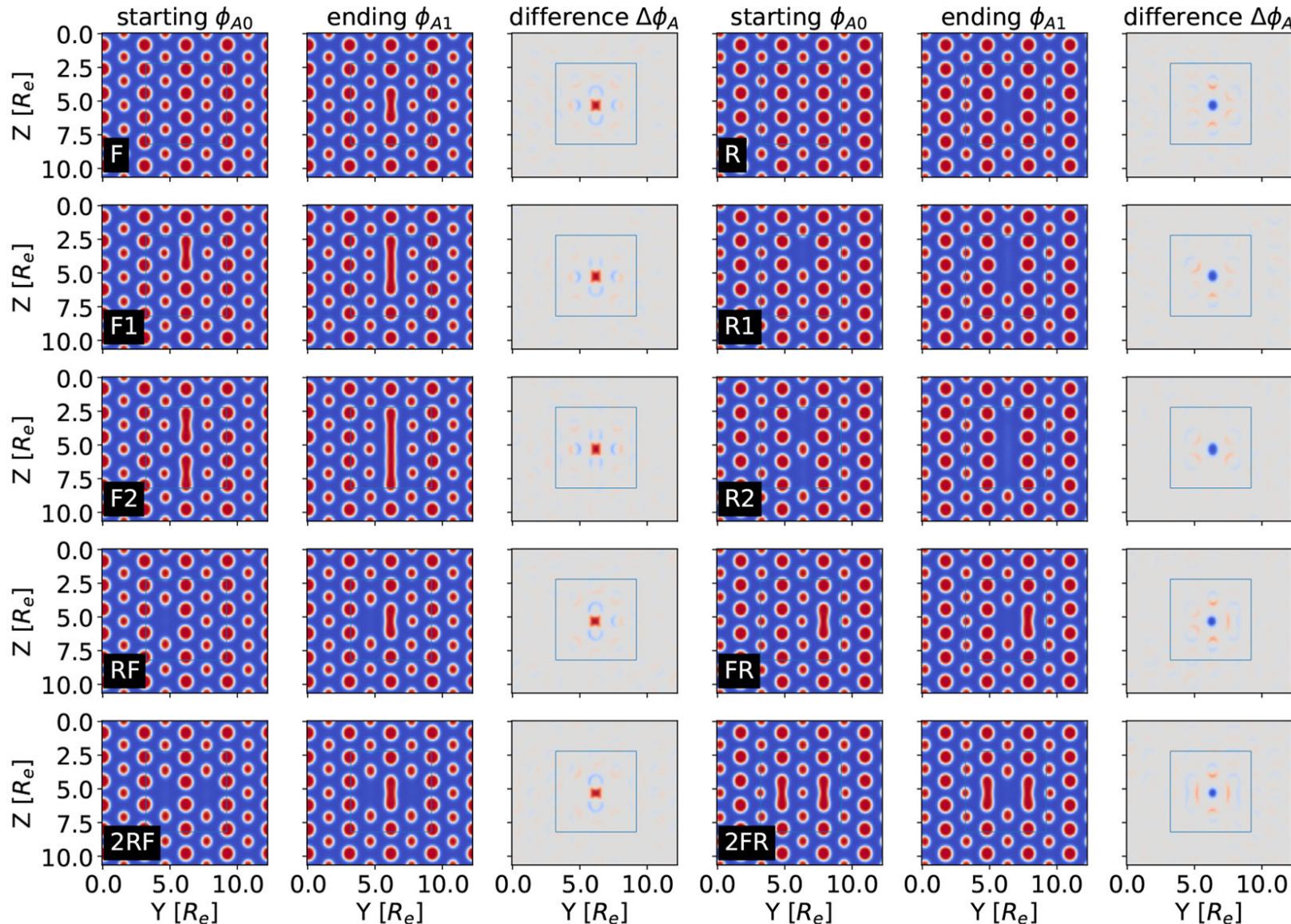


E, Ren, Vanden-Eijnden, *J. Chem. Phys.* **126**, 164103 (2007)

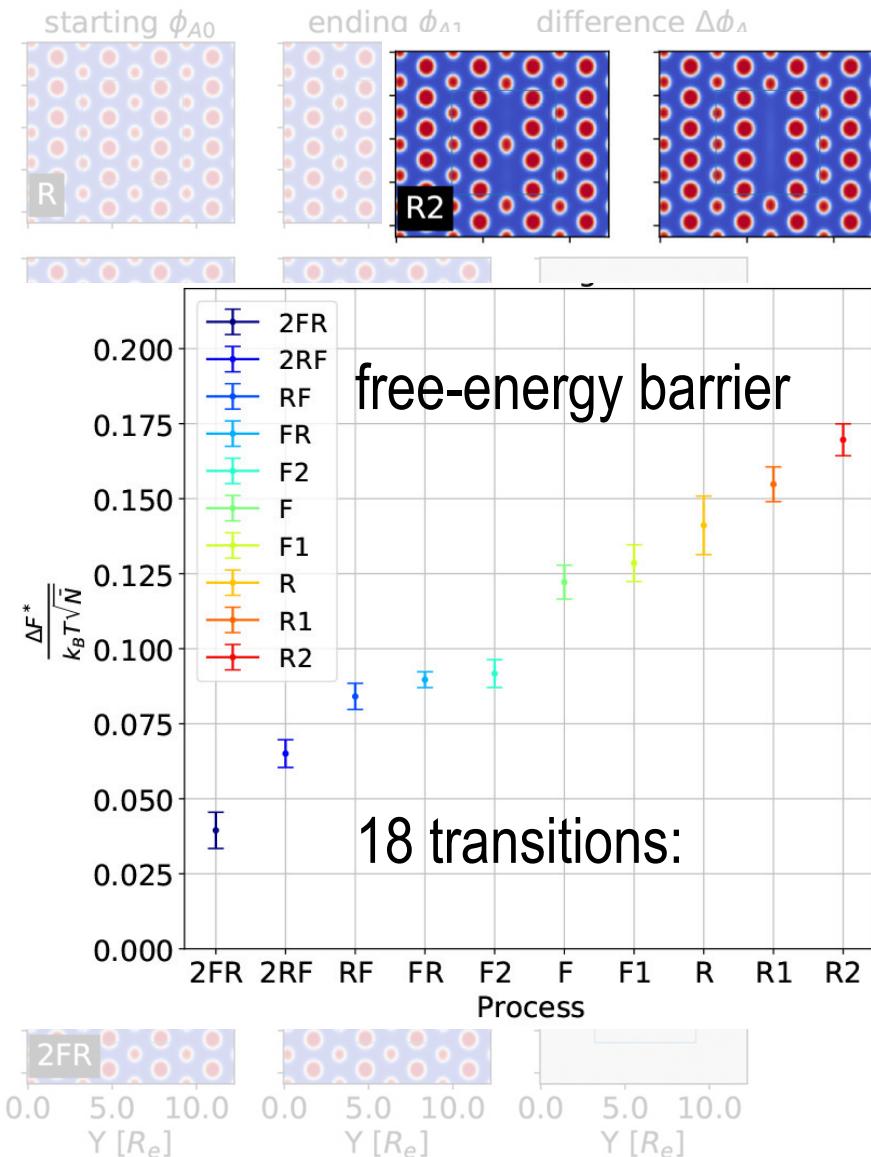
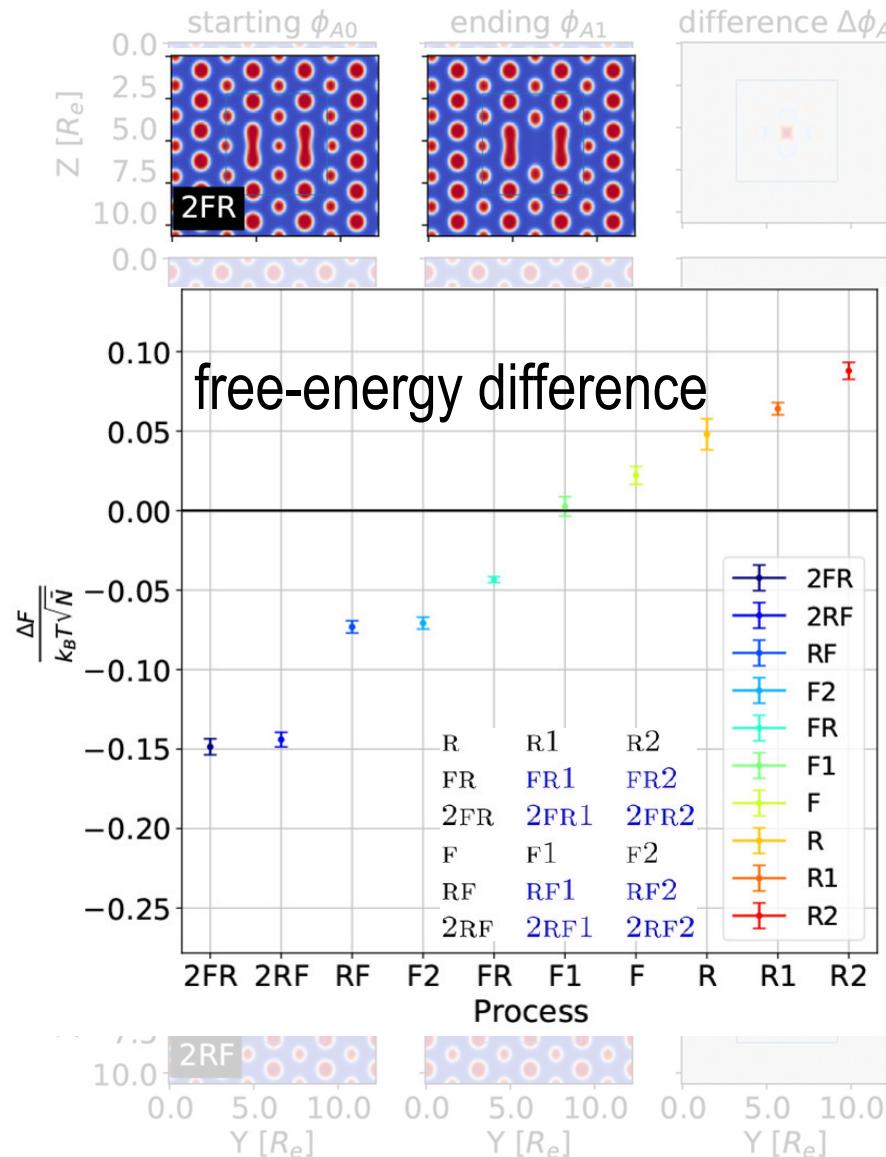


→ free-energy difference and barrier depend on local environment (spatiotemporal correlation)

# MFEP of 10 fusion and rupture transitions

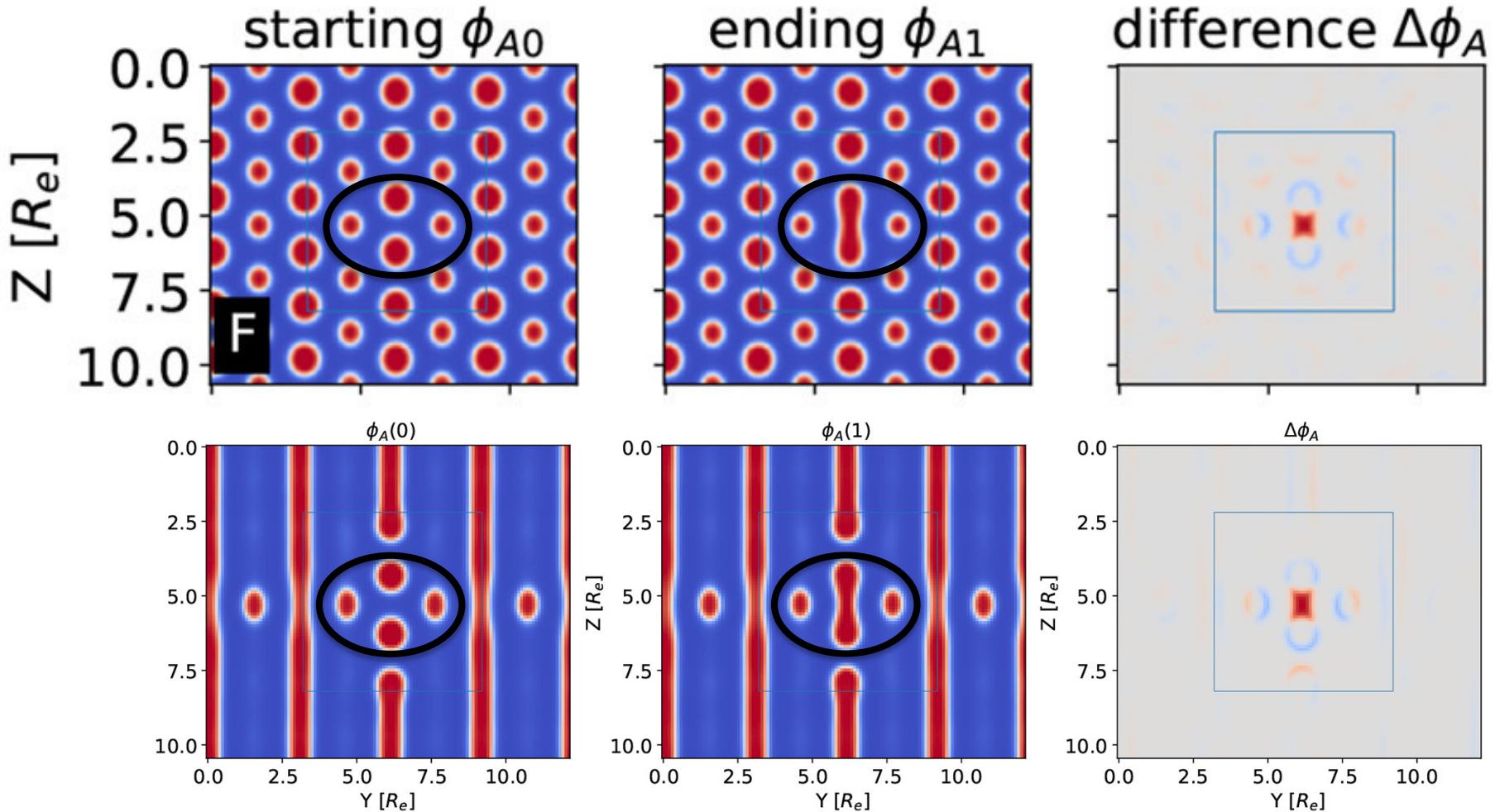


# MFEP of fusion and rupture transitions



# MFEP of fusion and rupture transitions

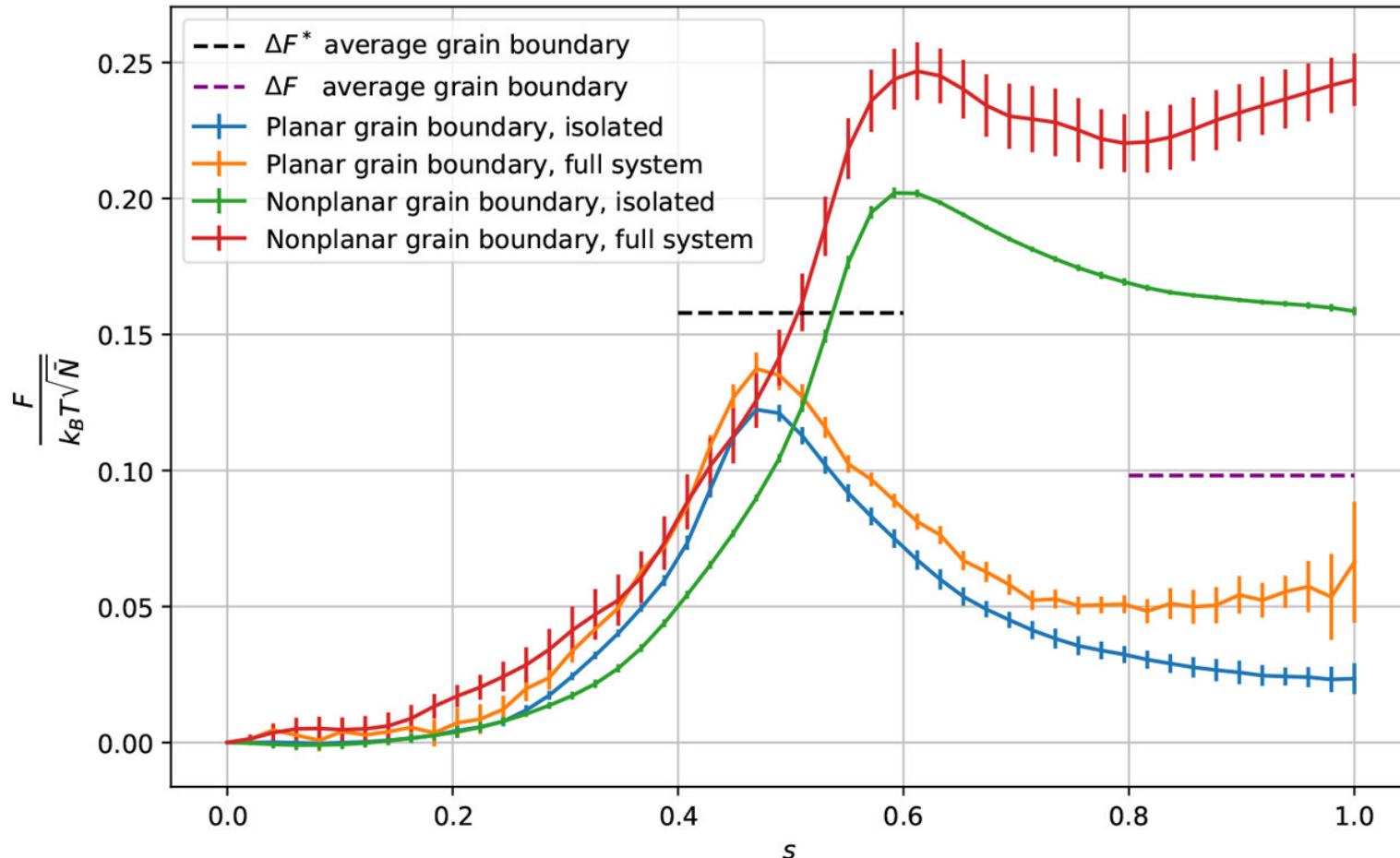
**question:** Do 18 transitions suffice to capture the environment dependence?



# MFEP of fusion and rupture transitions

**question:** Do 18 transitions suffice to capture the environment dependence?

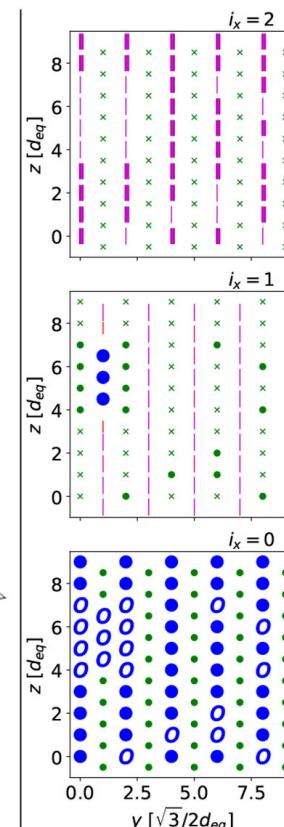
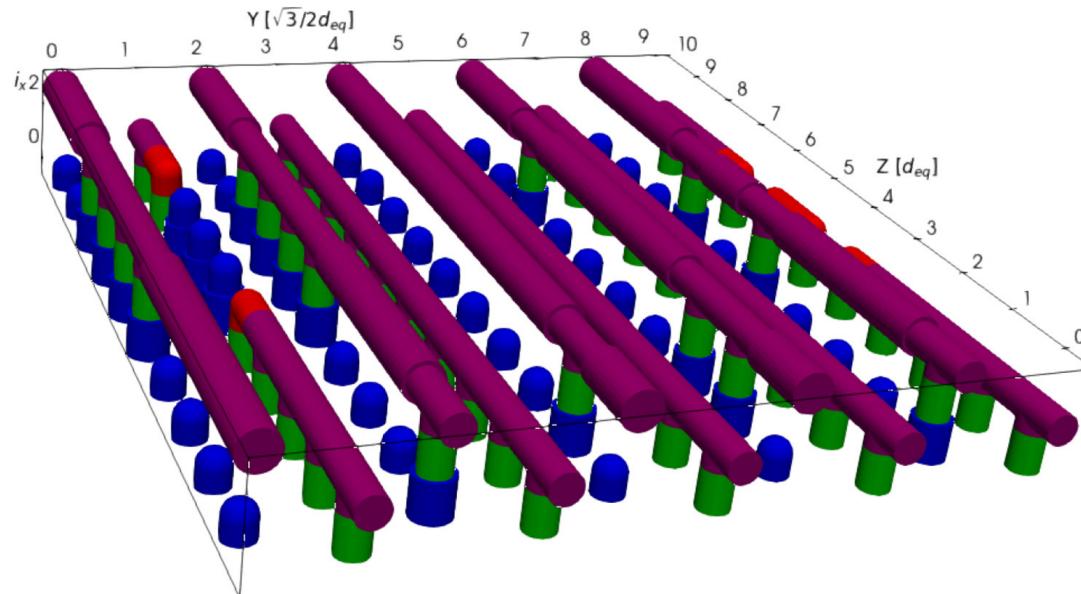
**answer:** unfortunately, no



# Monte-Carlo simulation of grain-boundary motion

**model:** describe diblock copolymer morphology by 7 states on a lattice

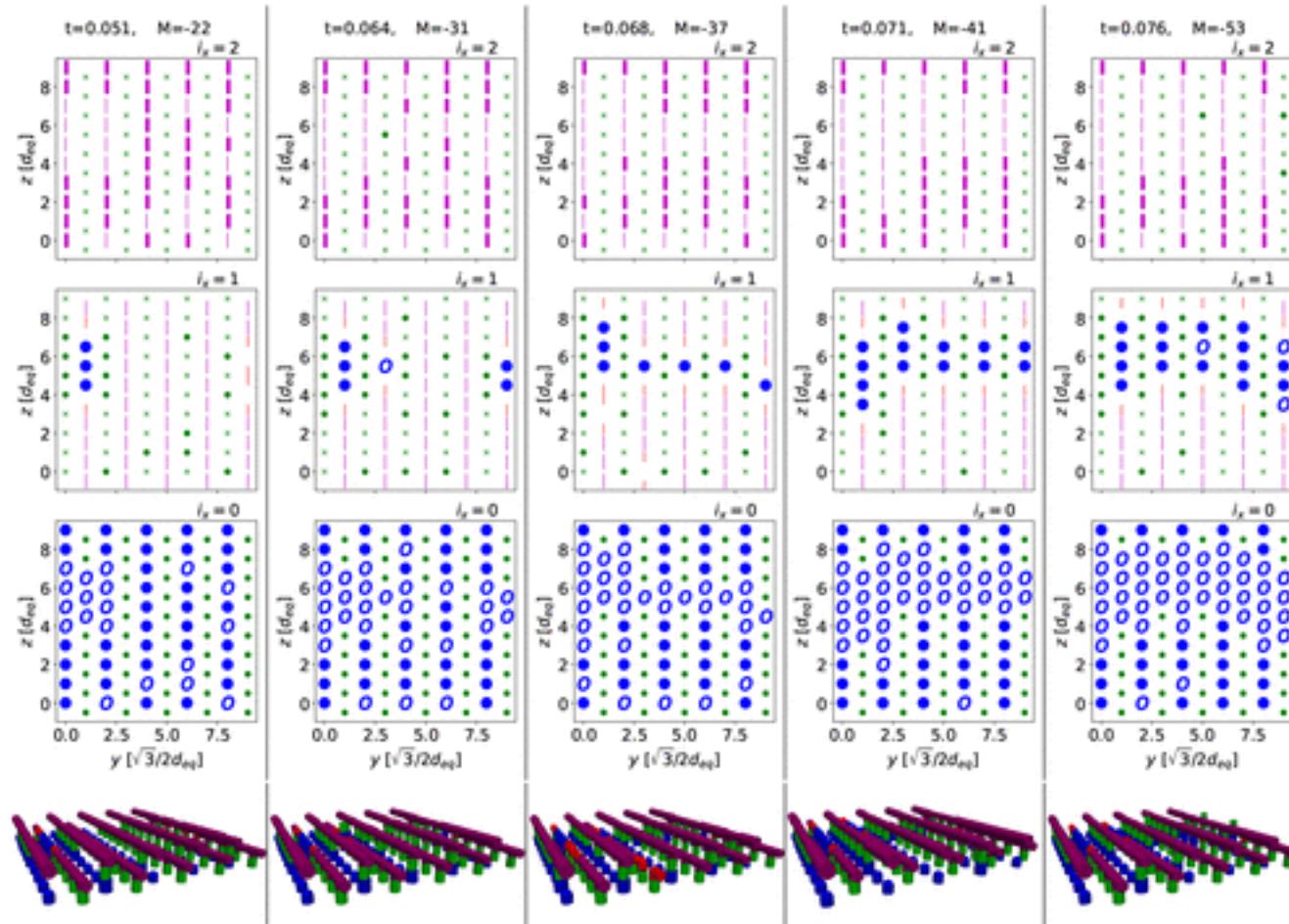
state	icon	may transition to
(1) parallel inner cylinder	■	■
(2) perpendicular inner cylinder	○	● ●
(3) cylinder head	●	○ ■
(4) bridge center	■	■ ■
(5) junction	●	○ ✕
(6) void	✖	●
(7) bridge end	■	● ■



# Monte-Carlo simulation of grain-boundary motion

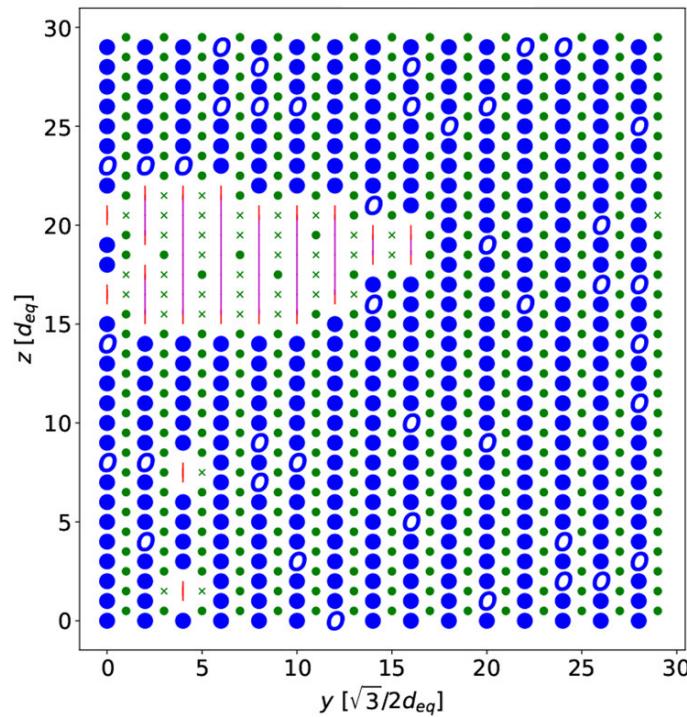
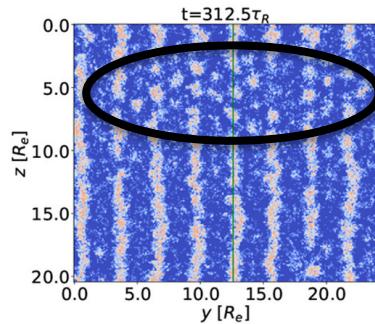
**model:** describe diblock copolymer morphology by 7 states on a lattice  
and use consistent transition rates from MFEP + optimization

nucleation of an anisotropic cluster at  $t=0.051\tau_{kMC}$        $\Delta t=0.02\tau_{kMC}\approx 10^6$  MCS

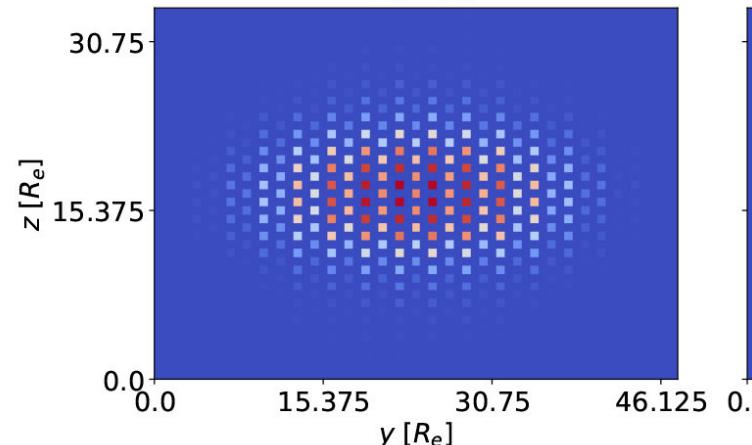


# Monte-Carlo simulation of grain-boundary motion

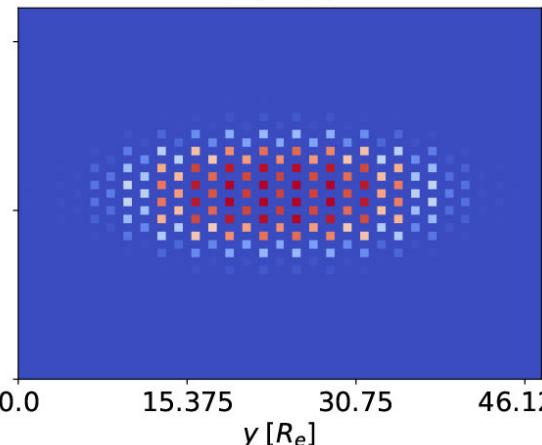
anisotropy of clusters in grain boundary



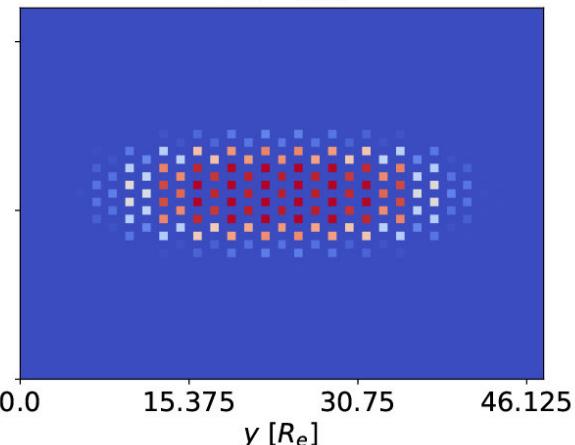
$$\sqrt{N} = 50$$



$$\sqrt{N} = 75$$

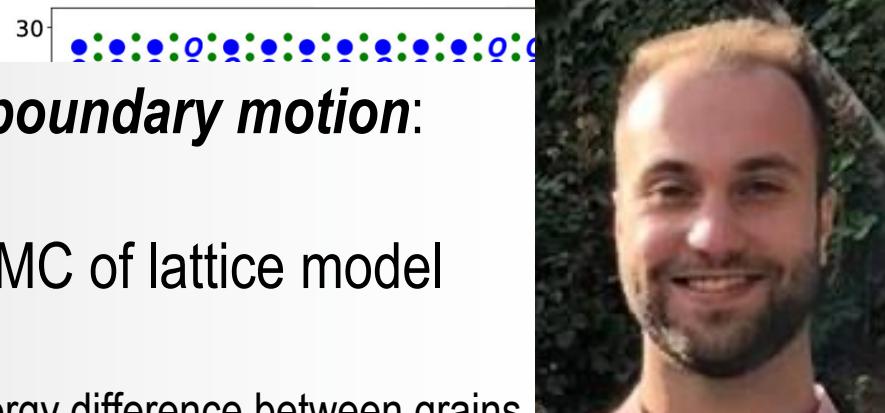


$$\sqrt{N} = 100$$



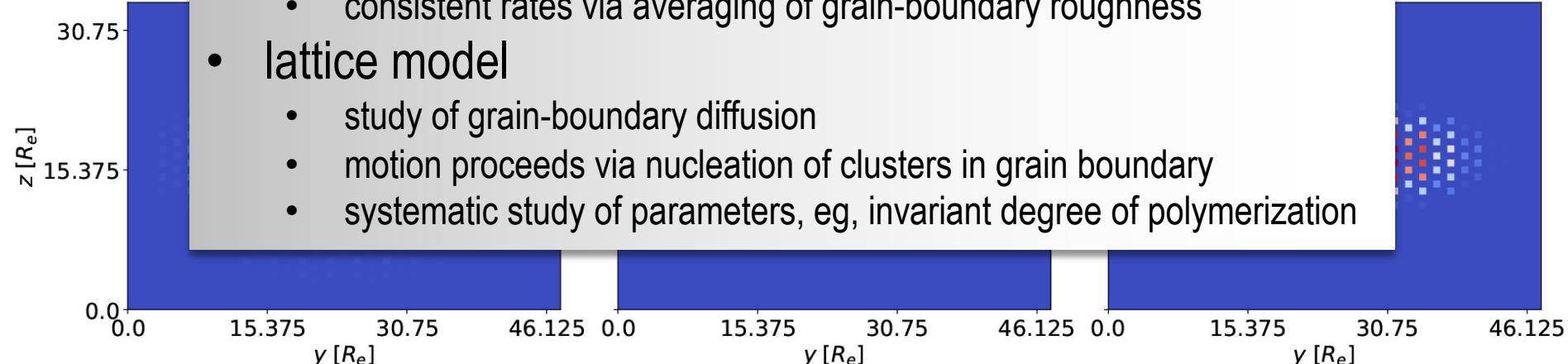
# Monte-Carlo simulation of grain-boundary motion

anisotropy of clusters in grain boundary



## *multiscale modelling of grain-boundary motion:*

- triple-scale modelling:  
particle simulation – MFEP – MC of lattice model
- particle simulation:
  - grain-boundary velocity ~ free-energy difference between grains
  - class of transitions: fusion of cylinder heads and rupture of junctions
  - sequence of topological changes, correlated in space and time
- continuum model and MFEP
  - free-energy differences and barriers
  - free-energy profiles of transitions depend on environment
  - consistent rates via averaging of grain-boundary roughness
- lattice model
  - study of grain-boundary diffusion
  - motion proceeds via nucleation of clusters in grain boundary
  - systematic study of parameters, eg, invariant degree of polymerization



# Prof. Dr. Kurt Binder



10. Februar 1944 – 27. September 2022



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