Quantum Annealing

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Types of quantum computing hardware

- **Large-scale fault-tolerant quantum computer (FTQC)**

  The "ideal" quantum computer that may be realized in the future.

  Theoretically known to be effective for solving some hard problems efficiently.

  Challenge : To build a device of sufficient scale to realize the "ideal" goal.

No picture yet

- **NISQ** (Noisy intermediate-scale quantum computer)

  Already realized in small to intermediate scales.

  Small-scale experiments have been conducted. Still within the reach of classical computers.

  Challenge: Significant reduction of noise is needed for meaningful results.

- **Quantum annealing (QA)**

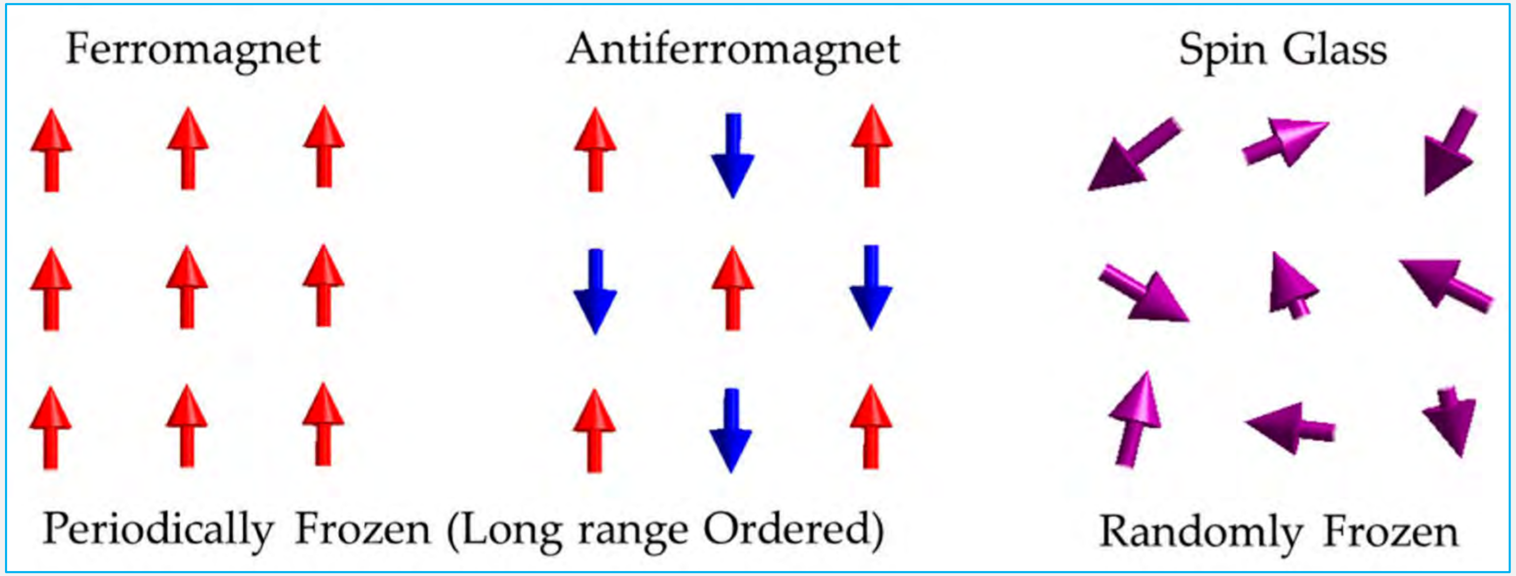
  Already realized in intermediate to moderately-large scales.

  Proof-of-concept experiments already realized with significant scale including some practical use cases.

  Challenge: Limited in the type of problems to be solved, the transverse-field Ising model.

What is quantum annealing?

- Quantum algorithm for the ground-state search of the Ising model, typically the spin glass.

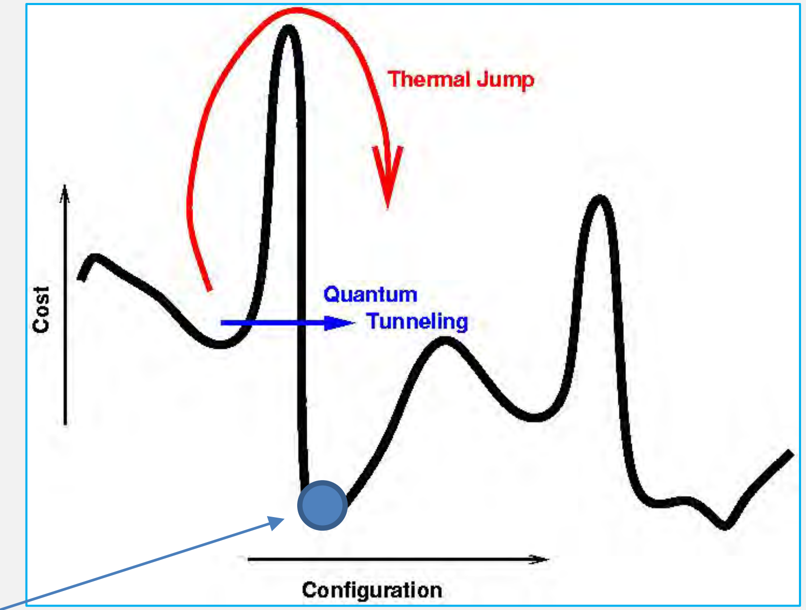


It is hard to find the ground state (lowest-energy state) of spin glass systems with complex interactions.

S. Sato et al, Symmetry **12** (2020)

What is quantum annealing?

- Quantum algorithm for the ground-state search of the Ising model, typically the spin glass.
- Quantum counterpart of simulated annealing.



To find the lowest-energy state

From Wikipedia

Classical simulated annealing uses thermal hopping.
Quantum annealing employs tunneling.

What is quantum annealing?

- Quantum algorithm for the ground-state search of the Ising model, typically the spin glass.
- Quantum counterpart of simulated annealing.
- Spin glass problems = Combinatorial optimization problems

VOLKSWAGEN

trafic optimization
with a simulation using 10 000 cabs in Beijing.
Later implemented at the Lisbon Web Summit
source : D-Wave, Volkswagen

accenture

trucks routing
trucks routing optimization
source: Accenture, D-Wave

FERROVIE DELLO STATO ITALIANE

trains station optimization
to reduce passengers connecting time
source: D-Wave

DENSO

fleet optimization
Denso and Toyota, presented at CES 2017 on Denso booth.
source: D-Wave, Denso

DLR

aircraft gate allocation in airports
to minimize passagers transit time
source: DLR, D-Wave

ExxonMobil

containers shipment optimization
using VQE, MIP, QUBO
source: IBM, ExxonMobil

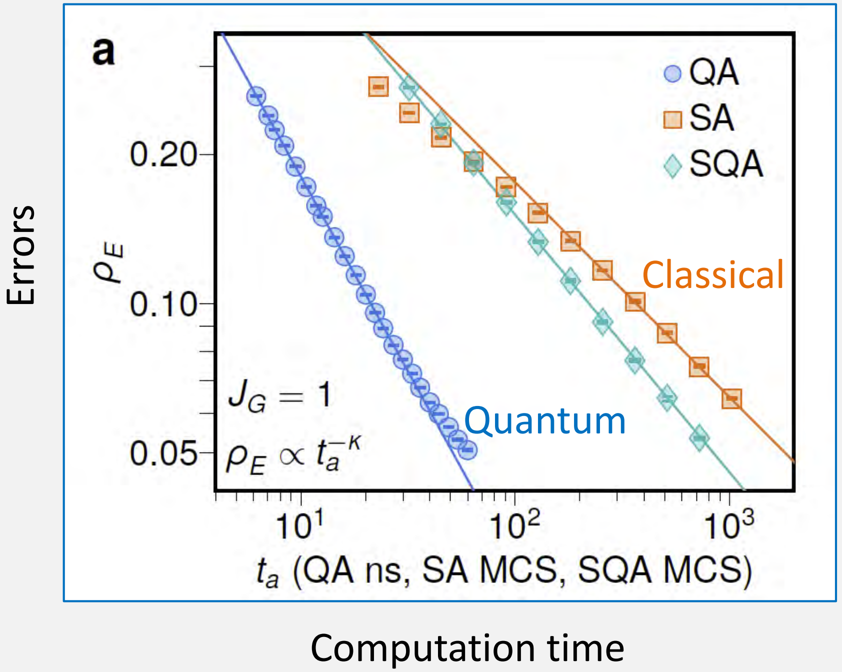
Figure 675: a sampler of quantum computing use cases in the transportation industry. (cc) Olivier Ezratty, 2022.

Many examples exist for proof-of-concept experiments for industrial as well as scientific problems.

O. Ezratty "Understanding Quantum Technologies" (2022)

What is quantum annealing?

- Quantum algorithm for the ground-state search of the Ising model, typically the spin glass.
- Quantum counterpart of simulated annealing.
- Generic quantum algorithm (metaheuristic) for combinatorial optimization problems.
- Likely to be more effective than classical SA for some problems.



Quantum annealing reduces errors more quickly than classical simulated annealing and simulated quantum annealing for 3D spin glass problem.

King et al, arXiv:2207.13800

What is quantum annealing?

- Generic quantum algorithm (metaheuristic) for combinatorial optimization problems.
- Equivalently, a solver of the physics problem of the ground-state search of spin glasses.
- Quantum version of simulated annealing.
- Expected (but not yet rigorously proven) to be effective for some problems.
- Hardware implementation of moderately-large scale (5000 qubits) is available.

Forschungszentrum Jülich

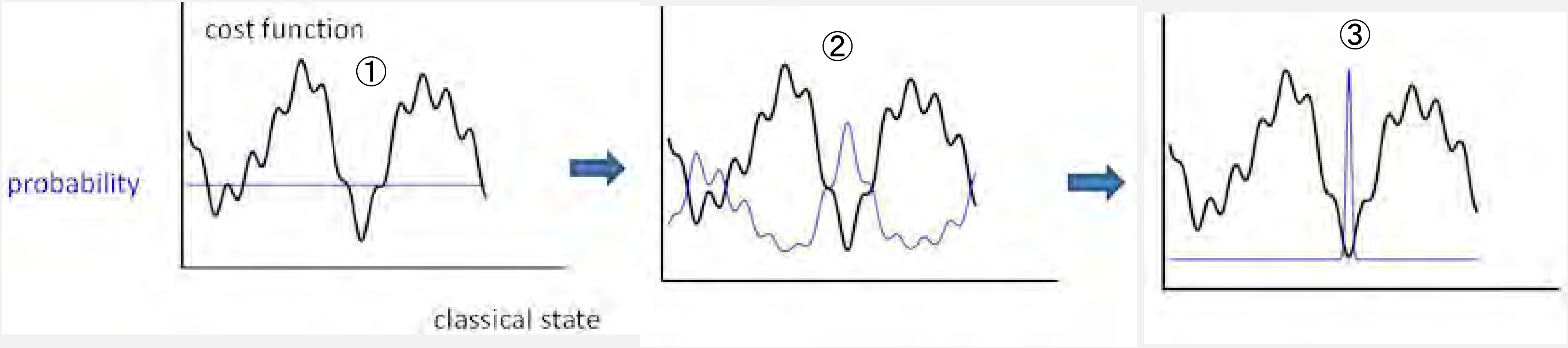
Google+ NASA

D-Wave's hardware is installed at a number of institutions.

How does quantum annealing work?

① Prepare a quantum wave function with all states with equal probability.
② Gradually decrease quantum effects to increase the probability of the ground state.
③ Measurement to find the ground-state with high probability.

① Uniform wave function → ② Control quantum effects toward the ground state → ③



All states with equal probability

Less quantum effects
= higher probability of
the ground state.

Largest probability of the ground state

Explicit formulation

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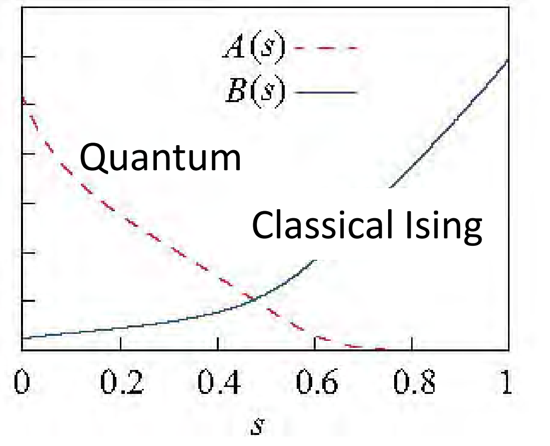
Quantum Classical

Transverse-field Ising model

$$H(s) = -\frac{A(s)}{2}\sum_i \sigma_i^x - \frac{B(s)}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$



Time evolution

$$H(0) = -\frac{A(0)}{2}\sum_i \sigma_i^x \rightarrow H(1) = -\frac{B(1)}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$

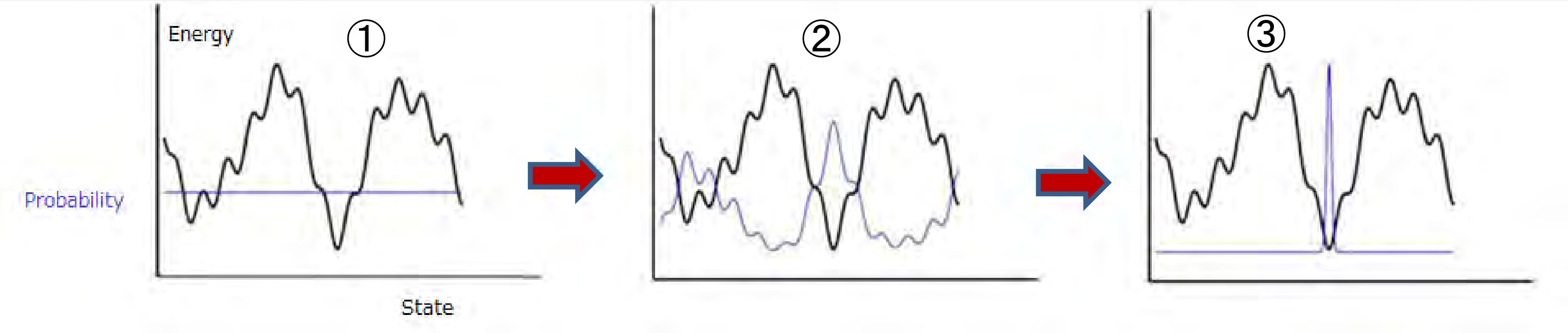
① ② ③

Superposition of all states with equal probability

|00000>+|00001>
+|00010>+|00011>
+….
+|11110>+|11111>

→ 10110 Solution

→ Time-dependent Schrödinger equation



All states with equal probability

Less quantum effects
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Largest probability of the ground state

History

1998 Kadowaki and Nishimori formulated quantum annealing as it is known now.

1999 D-Wave Systems was founded.

2001 Farhi et al formulated a similar idea of quantum adiabatic algorithm.

2007 Orion prototype was announced. 16 qubits

2011 D-Wave One commercialized with 128 qubits. Installed at USC with Lockheed-Martin.

2013 D-Wave Two with 516 qubits. Installed at NASA with Google.

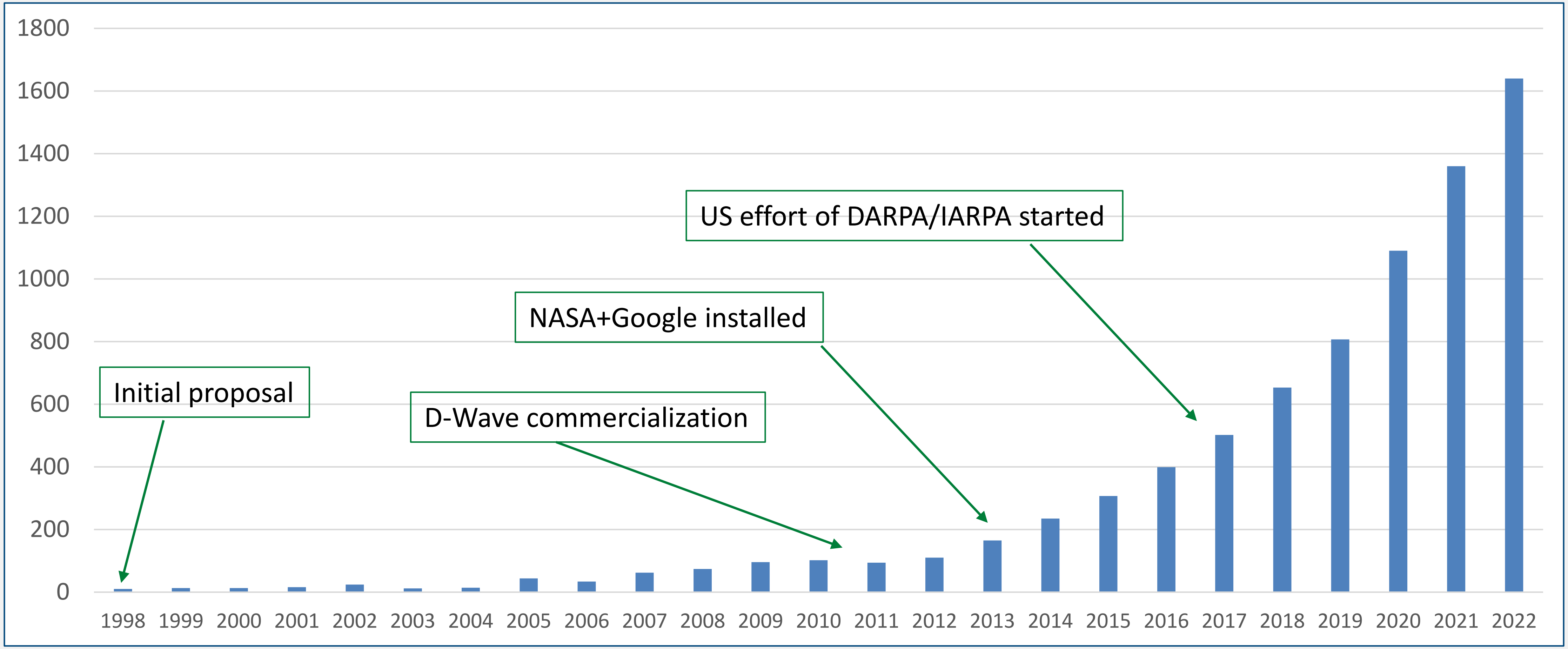
2015 D-Wave 2X with 1000+ qubits. Installed at Los Alamos National Lab.

2017 U.S. effort of IARPA/DARPA started for building its own annealer. D-Wave 2000Q.

2019 D-Wave Advantage with 5000+ qubits.

2022 D-Wave Advantage at FZJ.

Rapidly expanding field of quantum annealing



Number of papers with the keyword “quantum annealing”

Current efforts

Device improvements.

Theories for better control of parameters and better types of operators in the Hamiltonian.

Quantum simulations.

Applications to real-world problems.

Proof-of-concept experiments for applications

Table 1. Publications on real-world applications of QA. References discussed in the main text are in bold.

Application field	References
Mobility	Traffic flow optimization: [**38**, **85**, **133**, **140**, **166**, **195**]
Scheduling and logistics	Scheduling problems: [**84**, 139, 144, **157**, 165, **181**, **188**, **193**, **197**]
	VRP: [25, **62**, 194]
Quantum simulation	Chemistry: [82, **123**, **129**, 147, **168**, **192**]
	Physics: [**78**, **96**]
	Biology: [**109**, **187**]
Machine learning	Classification: [**127**, **136**, **137**]
	Reinforcement learning: [40, **135**]
	Cluster analysis: [**104**, **134**]
	Matrix factorization: [**67**, **142**, **145**]
Finance	Portfolio optimization: [**56**, **69**, **128**, 148, 160, **179**]
Miscellaneous	Finite-element design: [178]
	Material design: [99, 189]

Yarkoni et al,
Reports on Progress in Physics (2022)

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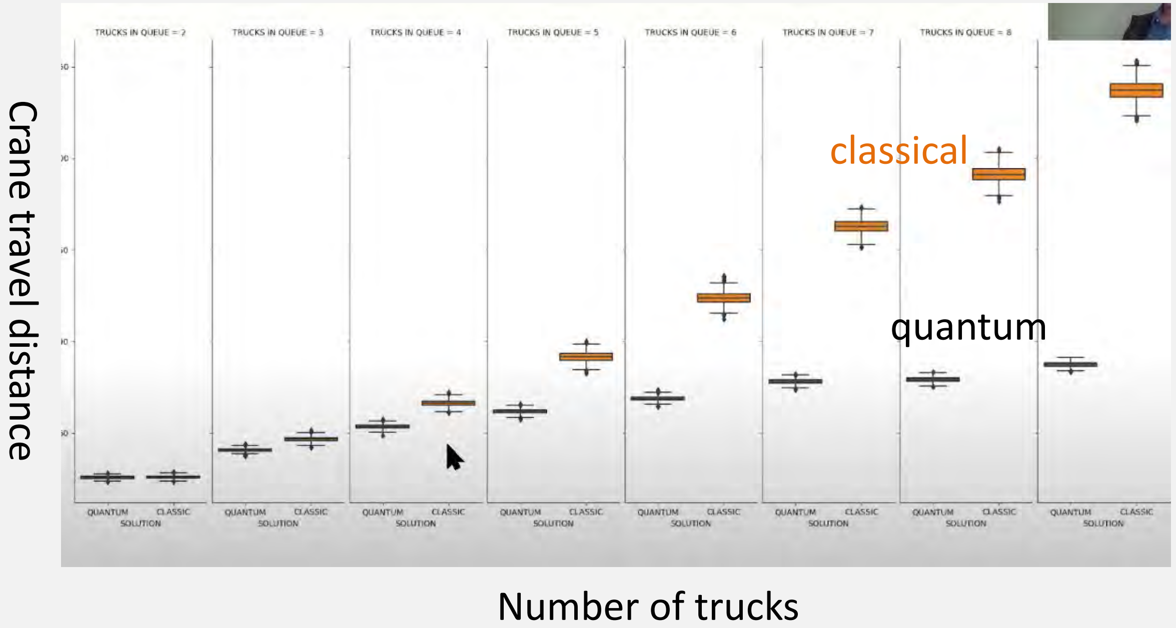
Quantum annealing in practical operation

SAVANT X's HONE engine (AI+QA)

To minimize waiting time of trucks by optimization of the motion of cranes.

Pier 300 of the port of Los Angeles

Reduction in comparison with a classical approach.



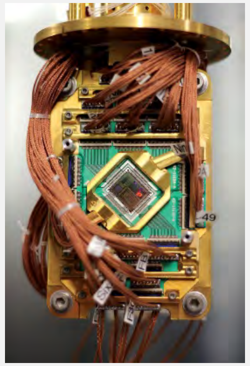
- Instructions to the cranes should be sent out about every 10 seconds.
- D-Wave sends back its solution within a few seconds.
- Classical method takes longer than 10 seconds, thus unpractical.
- Constant speedup is sufficient for this type of real-time processing.

Quantum simulation by quantum annealing

Quantum simulation

Richard Feynman (1981)

"I want to talk about the possibility that there is to be an ***exact*** simulation, that the computer will do ***exactly*** the same as nature."



- **(my) Definition of quantum simulation**
  To reproduce the behavior of a quantum-mechanical system on an artificial quantum device.
- **Advantage over real experiments**
  Precise control microscopic parameters, e.g., interactions between spins.
- **Disadvantage**
  Difficult to build large systems with high fidelity.
- **Our contribution**
  Successfully mitigated the above-mentioned difficulty in a large system with several thousand qubits.

Examples of quantum simulations by quantum annealing

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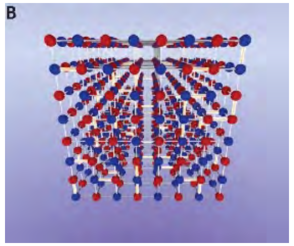
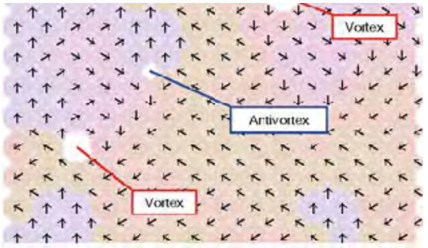
Static (equilibrium) properties

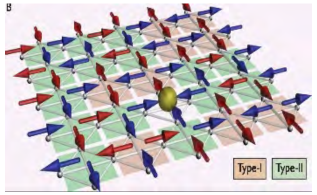
- Spin glass — *Harris et al., Science (2018)*
- Kosterlitz-Thouless transition — *King et al., Nature (2018)*
- Spin ice — *King et al., Science (2021)*
- Shastry-Sutherland model — *Kairys et al., PRX Quantum (2020)*
- Scalar field theory — *Abel et al., PRX Quantum (2021)*
- Griffiths-McCoy singularity — *Nishimura et al., Phys. Rev. A (2020)*
- Quasi crystal — *Lopez-Bezanilla and Nisoli (2923)*

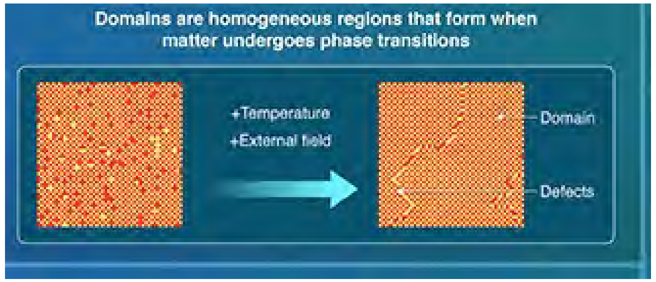
Dynamical (non-equilibrium) properties

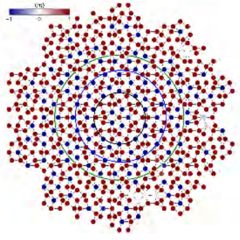
- Kibble-Zurek mechanism — *Gardas et al., Sci. Rep. (2018)*; *Weinberg et al., Phys. Rev. Lett. (2020)*
- Generalized Kibble-Zurek — *Bando et al., Phys. Rev. Res. (2020)*

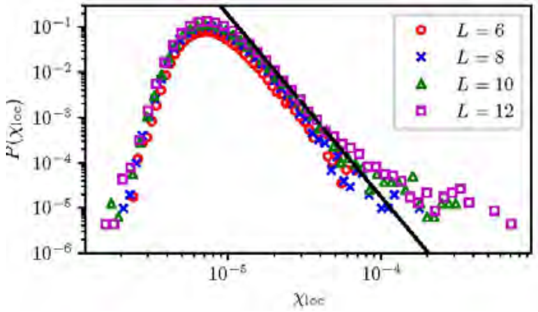


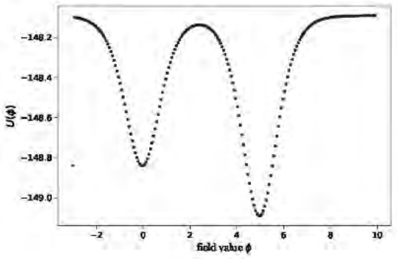


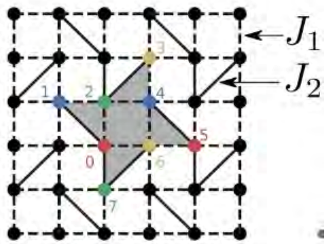












Coherent quantum simulation of the Kibble-Zurek mechanism

LETTERS
https://doi.org/10.1038/s41567-022-01741-6
nature physics

Check for updates

Coherent quantum annealing in a programmable 2,000 qubit Ising chain

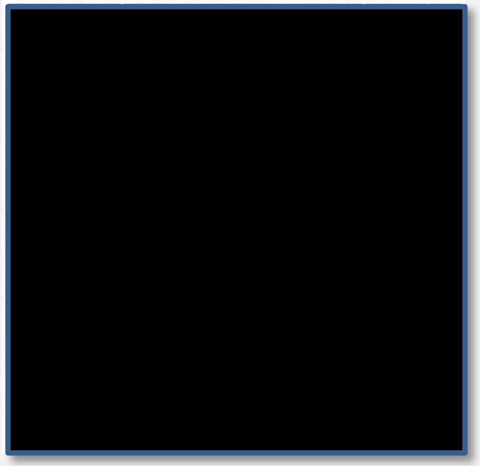
Andrew D. King[1], Sei Suzuki[2], Jack Raymond[1], Alex Zucca[1], Trevor Lanting[1], Fabio Altomare[1], Andrew J. Berkley[1], Sara Ejtemaee[1], Emile Hoskinson[1], Shuiyuan Huang[1], Eric Ladizinsky[1], Allison J. R. MacDonald[1], Gaelen Marsden[1], Travis Oh[1], Gabriel Poulin-Lamarre[1], Mauricio Reis[1], Chris Rich[1], Yuki Sato[1], Jed D. Whittaker[1], Jason Yao[1], Richard Harris[1], Daniel A. Lidar[3,4], Hidetoshi Nishimori[5,6,7] and Mohammad H. Amin[1,8]

Nature Physics **18**, 1324 (2022)

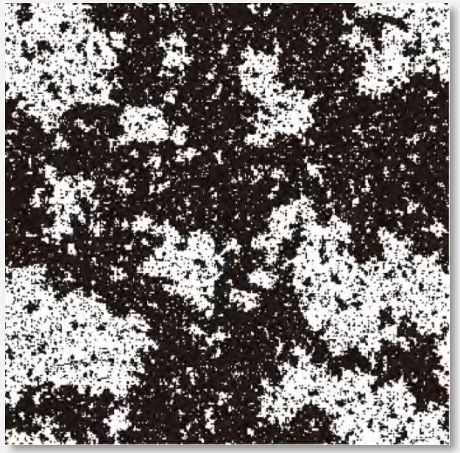
Phase transition (classical)

2d Ising model

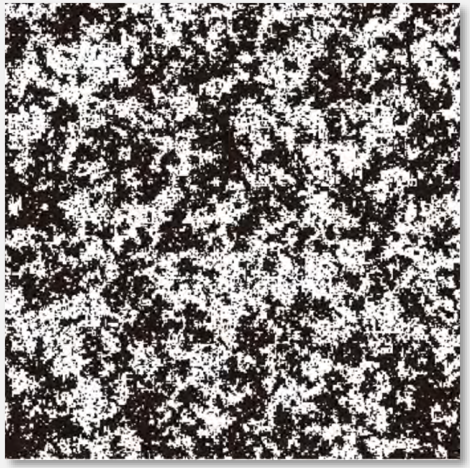
Low temperature (ordered)



Critical temperature



High temperature (disordered)



Temperature decrease

Slow decrease of temperature: System traces equilibrium states.

Fast decrease of temperature : Falls out of equilibrium. Defects are created even at *T*=0.

Problem: Relation of the density of defects ρ and the time scale t_a of temperature decrease $\rho = f(t_a)$

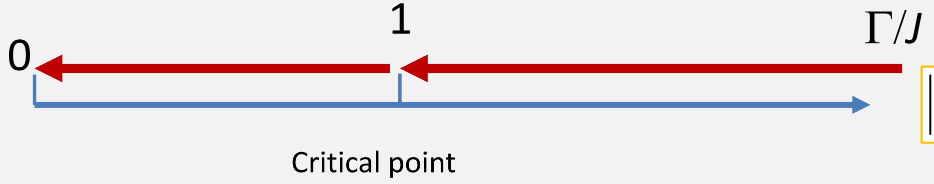
1 dimensional transverse-field Ising model

Classical Quantum

$$H = -J \sum_{i=1}^{L} \sigma_i^z \sigma_{i+1}^z - \Gamma \sum_{i=1}^{L} \sigma_i^x \quad (J > 0)$$

$$|\uparrow_1\uparrow_2\uparrow_3 \cdots \uparrow_{L-1}\uparrow_L\rangle$$
$$+ |\downarrow_1\downarrow_2\downarrow_3 \cdots \downarrow_{L-1}\downarrow_L\rangle$$

Classical



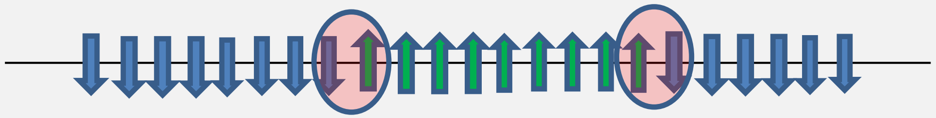
Slow change

Superposition of all states

$$|\uparrow_1\uparrow_2\uparrow_3 \cdots \uparrow_L\rangle + |\uparrow_1\uparrow_2\uparrow_3 \cdots \downarrow_L\rangle + \cdots + |\downarrow_1\downarrow_2\downarrow_3 \cdots \downarrow_L\rangle$$

Quantum

Defect (Kink)



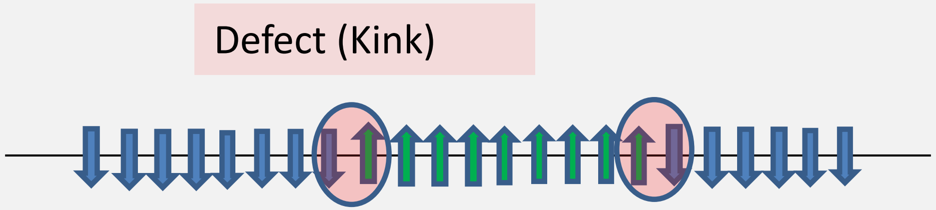
By decreasing Γ/J at a finite rate, defects are created at $\Gamma/J=0$.

Problem : What is the number (density) of defects ρ as a function of annealing time t_a? $\rho = f(t_a)$

Kibble-Zurek mechanism

For the 1d (chain) problem, the exact formula for the number (density) of defects is known.

$$\rho = \frac{1}{2\pi}\sqrt{\frac{\hbar}{2J}}\; t_a^{-1/2}$$



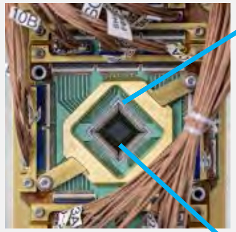
Quantum-mechanical formula without thermal noise (T=0).

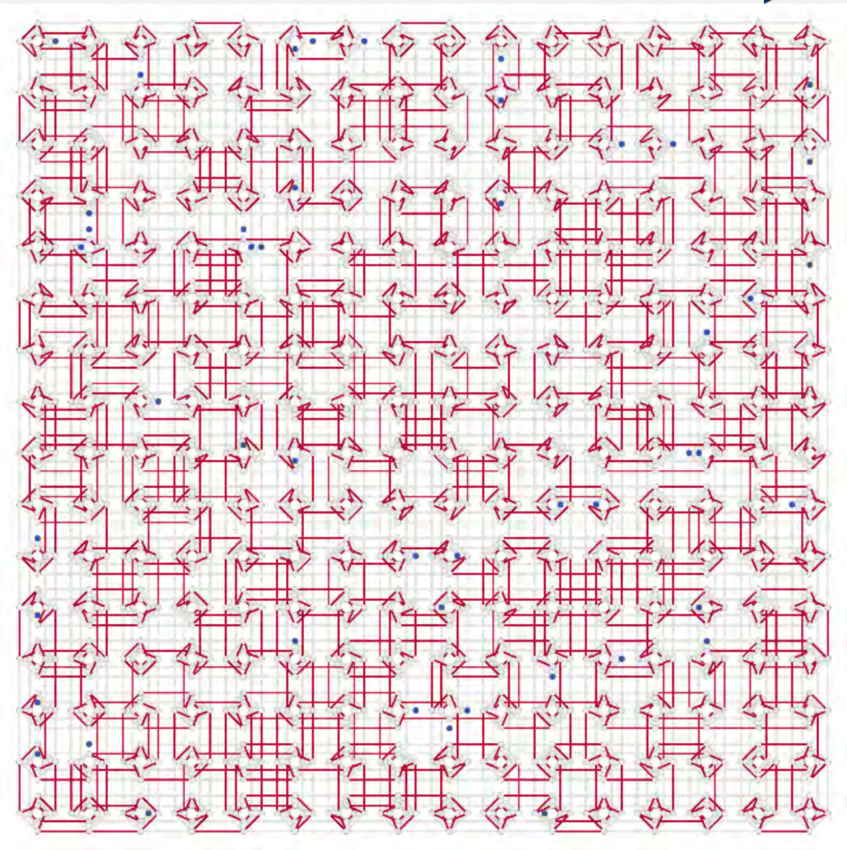
Goal

To verify this theoretical formula through quantum simulation by quantum annealing.

Representation of the 1d Ising model on the device

QPU



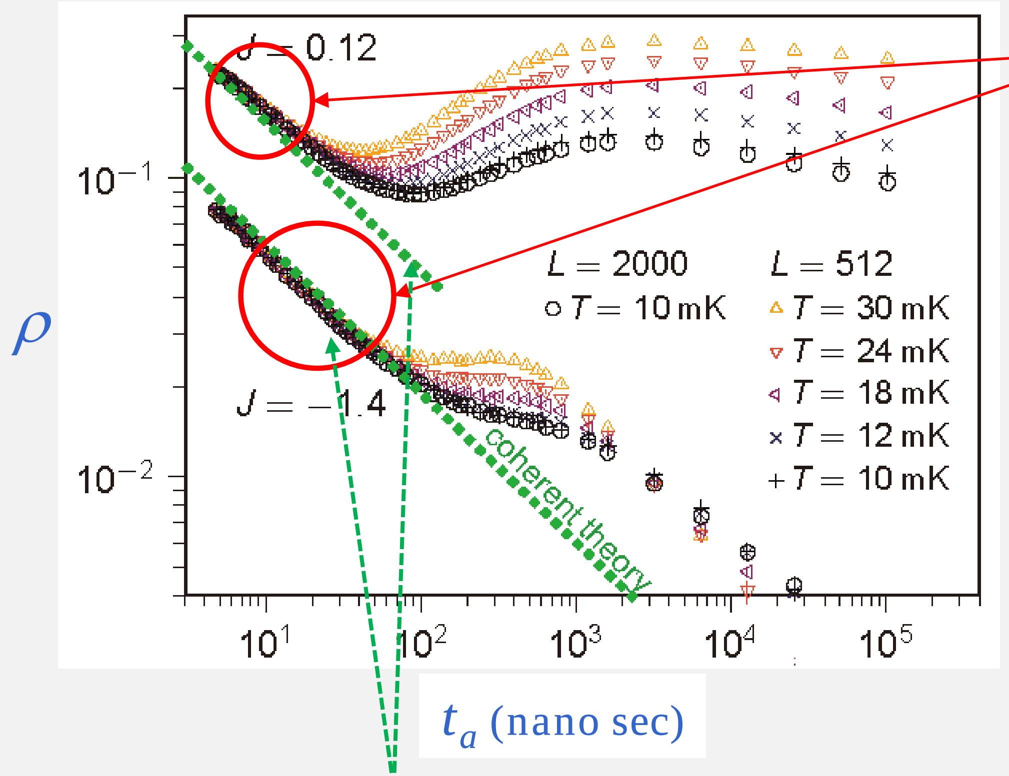


2,000 qubits are connected with periodic boundary.

$$H(s) = -\frac{A(s)}{2}\sum_i \sigma_i^x - \frac{B(s)}{2}\sum_{i,j} J_{ij}\sigma_i^z\sigma_j^z$$

The Ising model representing the problem can be implemented on the chip.

Result



$$\rho = \frac{1}{2\pi}\sqrt{\frac{\hbar}{2J}}\, t_a^{-1/2}$$

slope

offset

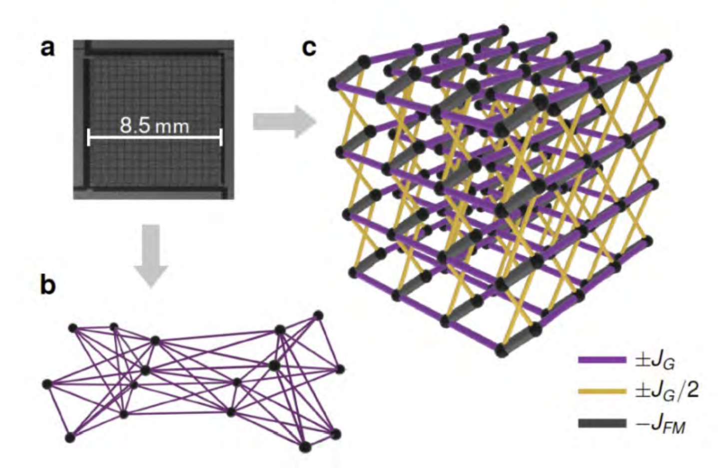
- Data agrees with the theory including the quantum coefficient at short time scale up to about 50 ns.
- No fitting parameters including the offset.
- The device runs coherently without thermal noise at short time scale.
- Environment (thermal effect) affects after about 50 ns.
- Kibble-Zurek theory has been confirmed including the coefficient.

Significance

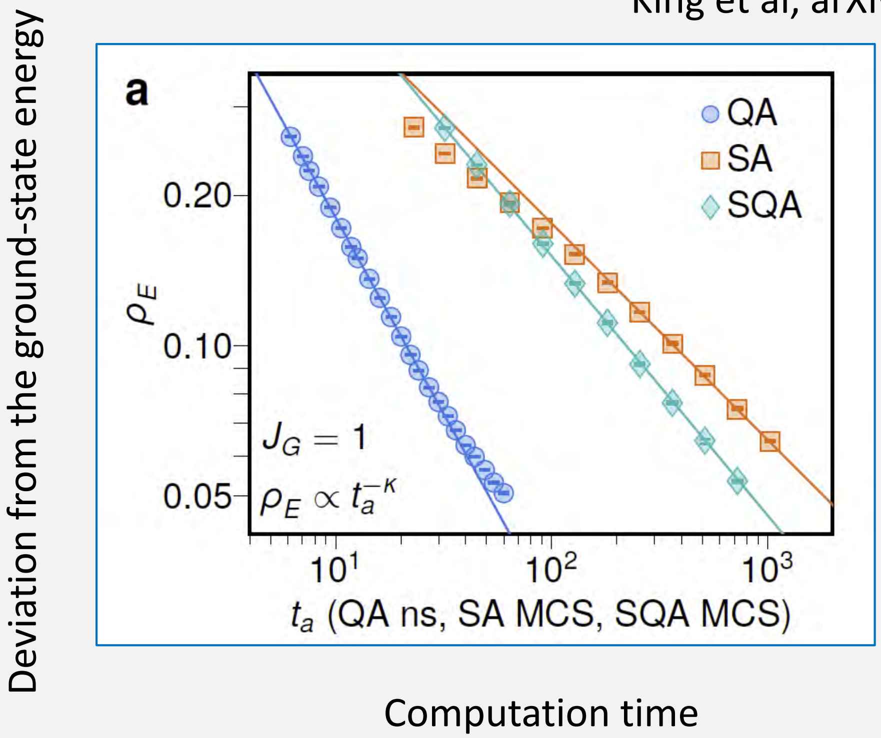
- The first case to run a large-scale artificial quantum device with L=**2000 qubits noise-free**.
  (cf. The largest quantum simulation on other platforms is about 200 qubits.)
- 1d problem can be classically simulated by a tensor-network method. But it does not work for other problems.
- Opened a path toward large-scale quantum simulations beyond the capacity of classical computers.

Quantum simulation of the 3D spin glass

King et al, arXiv:2207.13800



3d spin glass of size 15 × 15 × 12



- Quantum simulation of **3d spin glass dynamics with 5300 qubits**.
- **Faster dynamics** toward optimal **by quantum (QA)** than classical (SA, SQA).
- Still within reach by a classical method (parallel tempering) as an optimization problem.
- But the dynamics is far beyond classical capacities.

Conclusion

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- Quantum annealing is originally designed for optimization problems.
- Proof-of-concept experiments are conducted extensively including those for industrial applications.
- A few cases already exist for practical applications.
- A new field of quantum simulations by quantum annealing has been developing quickly.
- Large-scale quantum simulations have been conducted with thousands of qubits.
- The device operates coherently (without noise) at short time scales.
- We have laid a foundation for better solutions to optimization problems:
  The quality of the data will be greatly improved by running the device with improved coherence.