Constraining the phase diagram of strong-interaction matter

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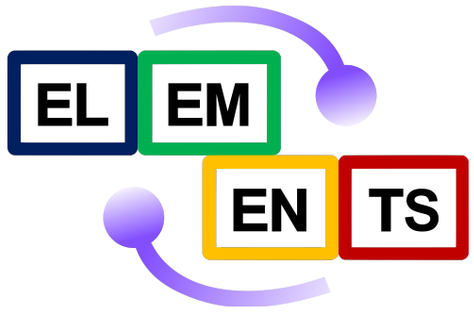
Lattice gauge theory for heavy ion and nuclear physics



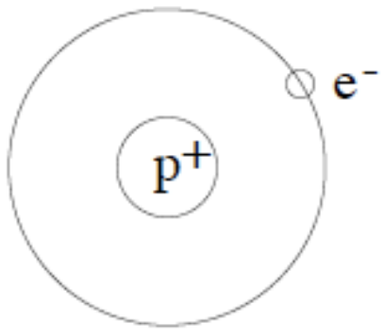
Quantum Chromodynamics, theory of strong interactions

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4g^2}\,\mathrm{Tr}\,F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{3}\bar{\psi}_i[\gamma_\mu D_\mu + m_i]\psi_i$$

$$m_u \sim 3\text{MeV}, \quad m_d \sim 6\text{MeV}, \quad m_s \sim 120\text{MeV} \quad \Rightarrow \quad N_f \approx 2+1$$

weak vs. strong coupling:

QED



$M_e = 0.5$ MeV

$M_p = 938$ MeV

$E_{bind} = 13.6$ eV

hydrogen (e.m. force)

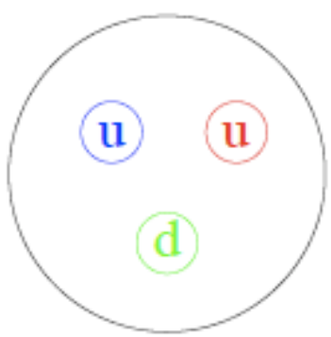
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

photons e,p gauge group U(1)

↓ ↓ ↓

gluons quarks gauge group SU(3)

QCD



$M_u \sim 3$ MeV

$M_d \sim 6$ MeV

$M_p = 938$ MeV

proton (strong force)

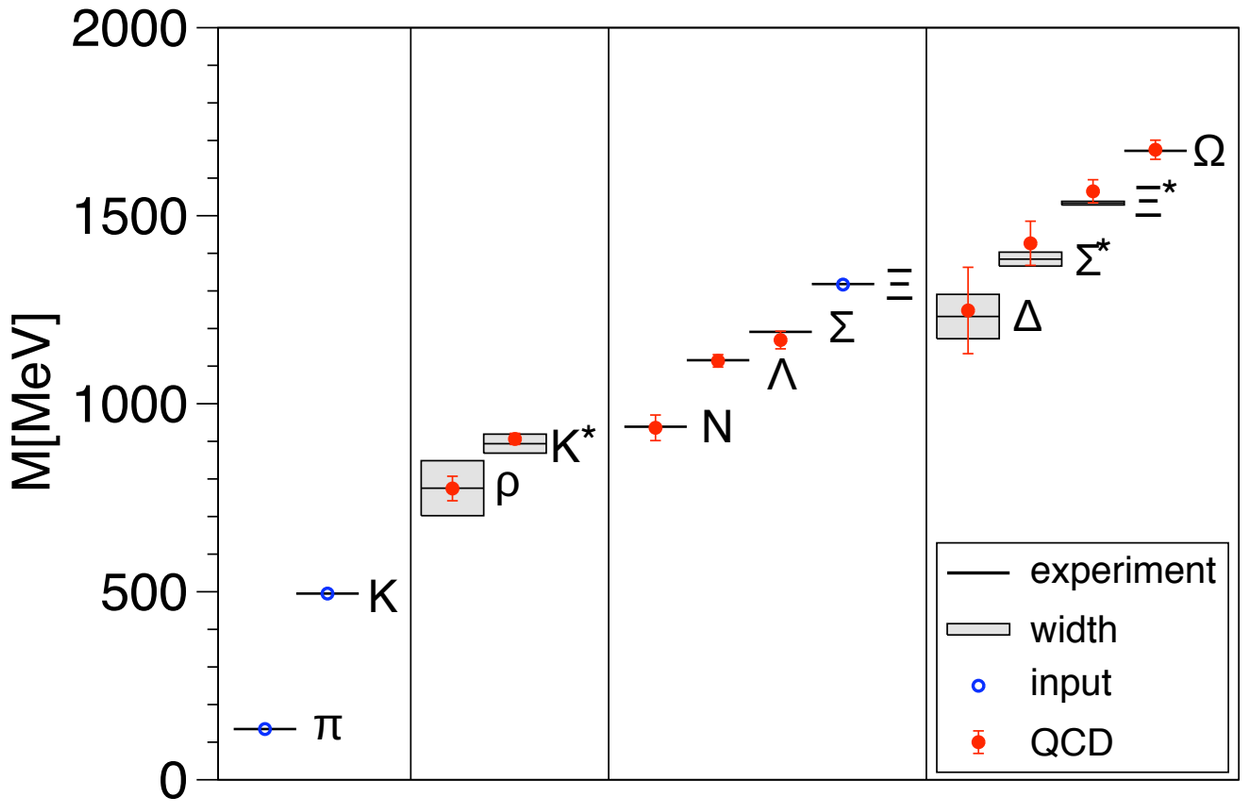
$$\alpha_s = \frac{g^2}{4\pi} \approx 1$$

⇒ **Confinement, non-perturbative**

gluon self-interaction!

Monte Carlo evaluation

[Budapest,Marseille,Wuppertal, Nature 2010]



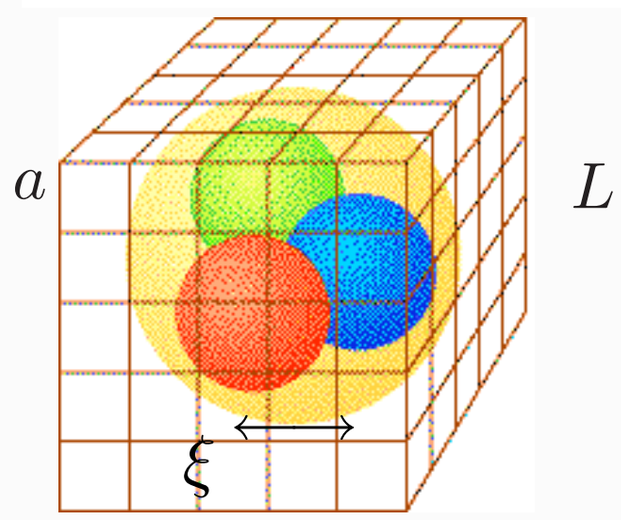
Euclidean partition function:

$$Z = \int D\bar{\psi} D\psi DU \, \mathrm{e}^{-S_g[U] - S_f[U,\bar{\psi},\psi]}$$

Systematics: finite V,a effects

for hadron with m_H, $\xi \sim m_H^{-1}$

$$a \ll \xi \ll aL \text{ !}$$



$\Rightarrow$e.g. $30^4 \sim 10^6$ lattice points

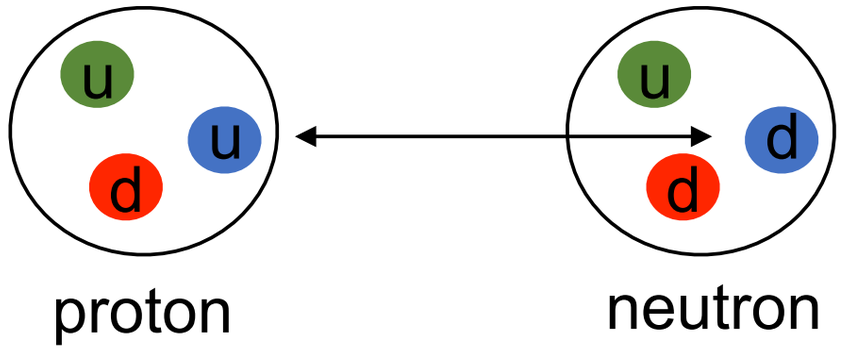
every point $\Rightarrow$4 U's, every $U \in$ SU(3) $\Rightarrow$8 independent components $\Rightarrow 10^8$-dimensional integral!

Directly calculable: particle masses, decay constants, equilibrium thermodynamics

Chiral symmetry breaking and restoration

For $m_q = 0$: QCD symmetric under rotations in flavor space

$$SU(2)_L \times SU(2)_R \times U(1)_A$$



Confinement + spontaneous symmetry breaking: $\sigma = \langle \psi\psi \rangle \neq 0$

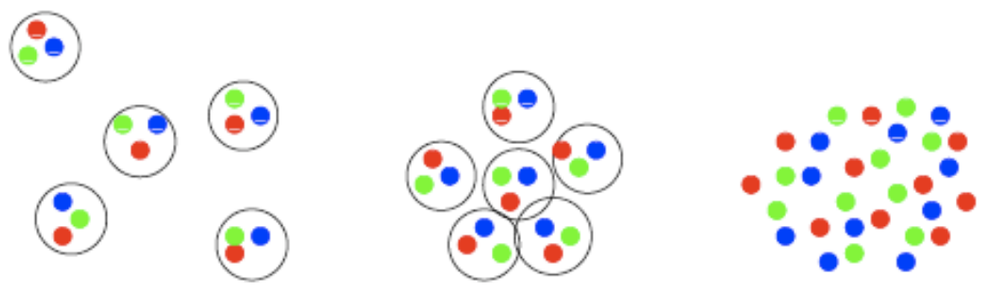
Responsible for visible mass in the Universe

Extreme conditions:

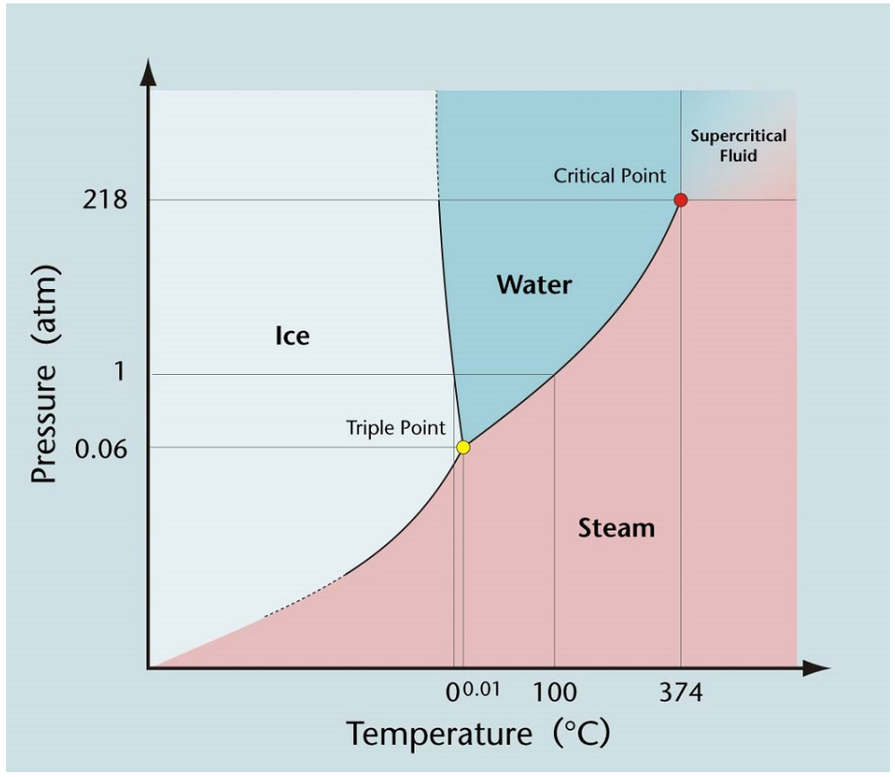
$$\xrightarrow{T,\,\mu_B} \quad \alpha_s(p \to \infty) \to 0$$

Phase transitions?

-Heavy-ion collisions
-Early Universe
-Compact stars



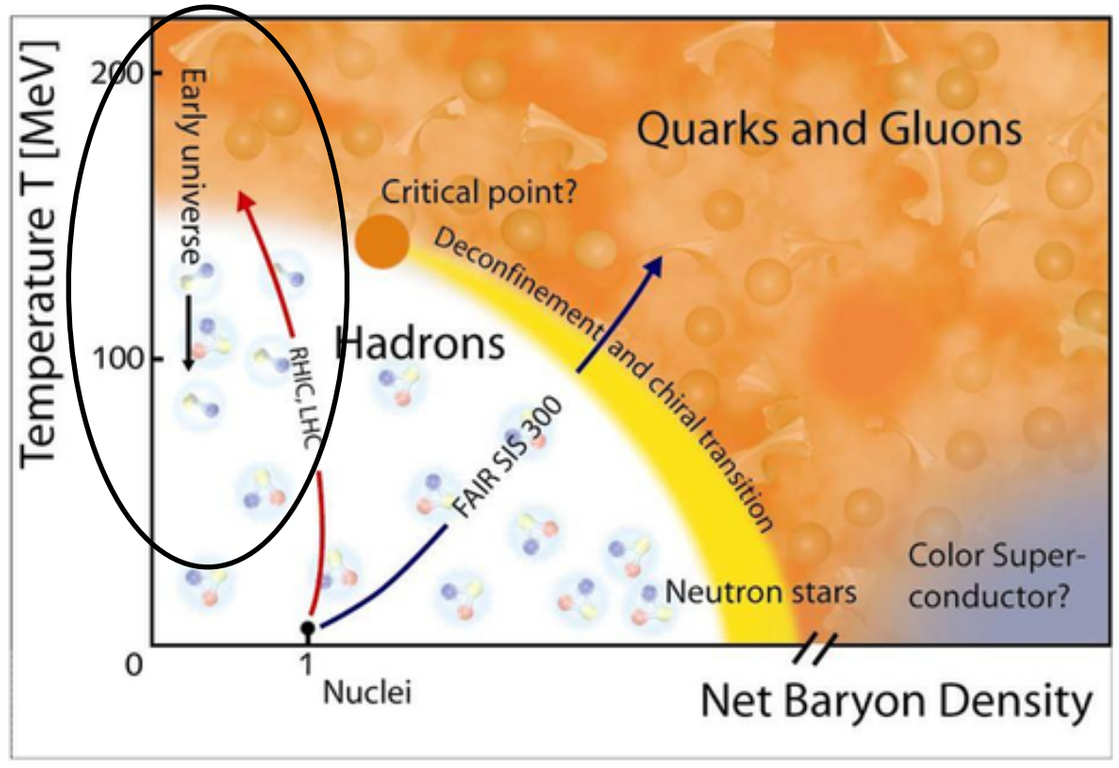
Phase diagram of water



https://www.101diagrams.com/phase-diagrams-of-water-printable/

Phase diagram of QCD?

Controlled region, smooth crossover observed



Source: GSI Darmstadt

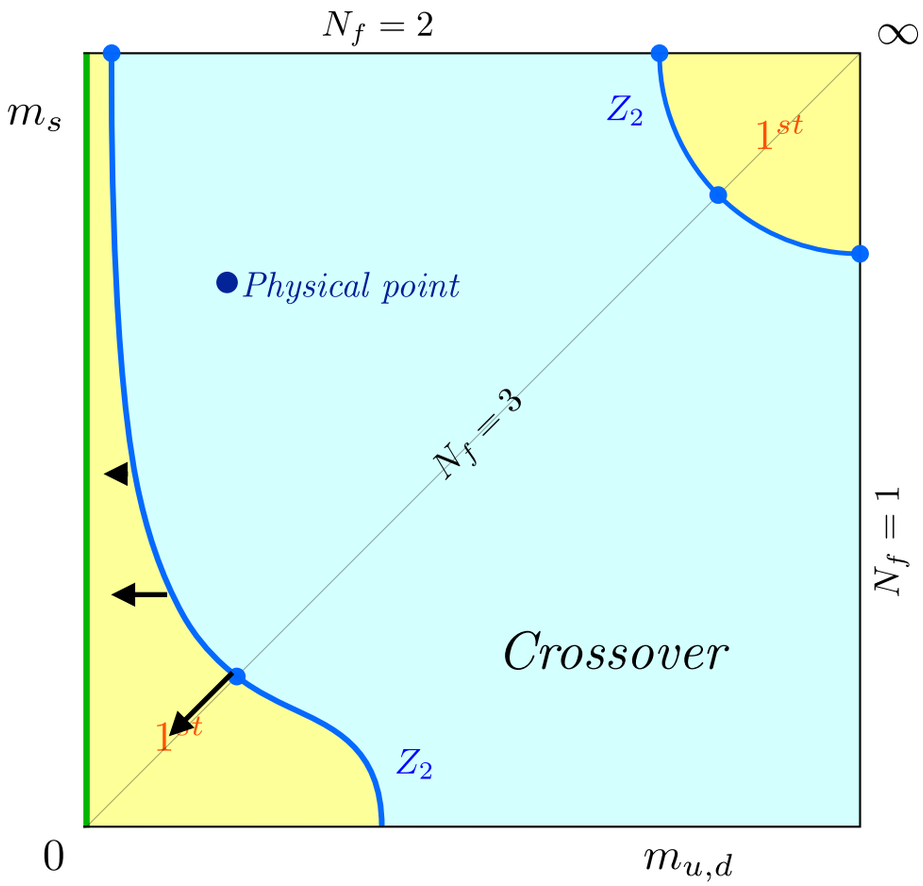
Situation with finite density (nuclear matter)

not simulable by Monte Carlo (sign problem)

Indirect methods!

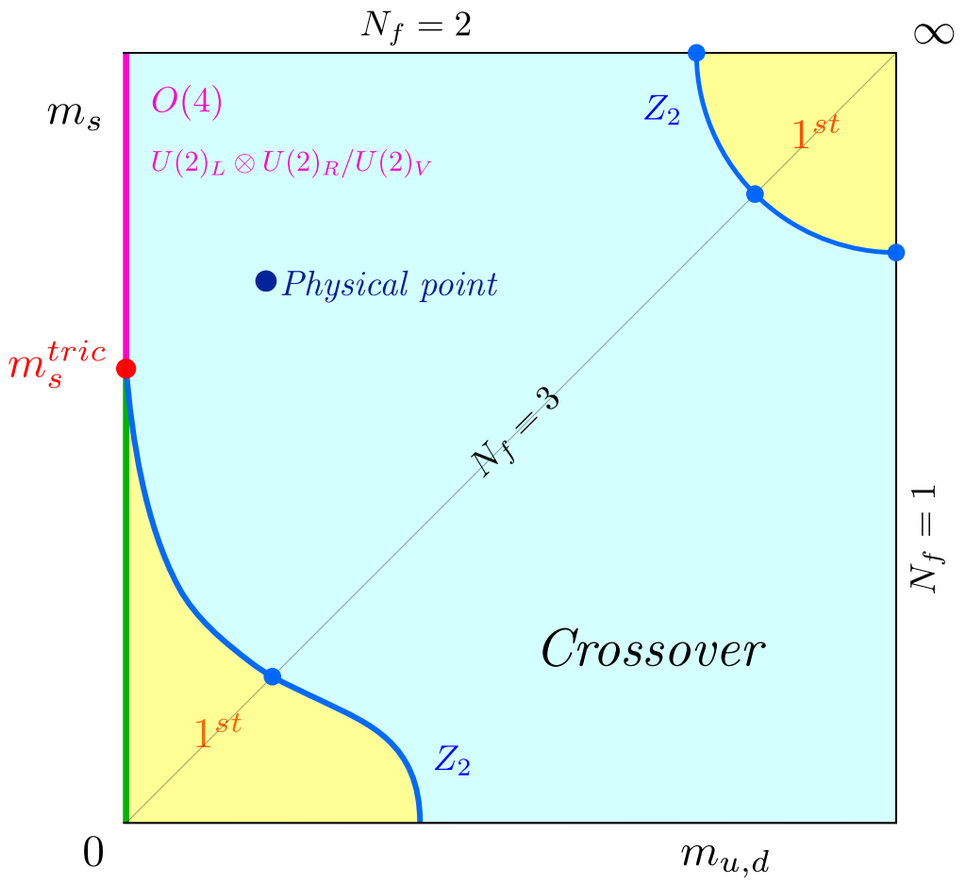
The nature of the QCD chiral transition

…is elusive, massless limit not simulable!



?

$a \to 0$



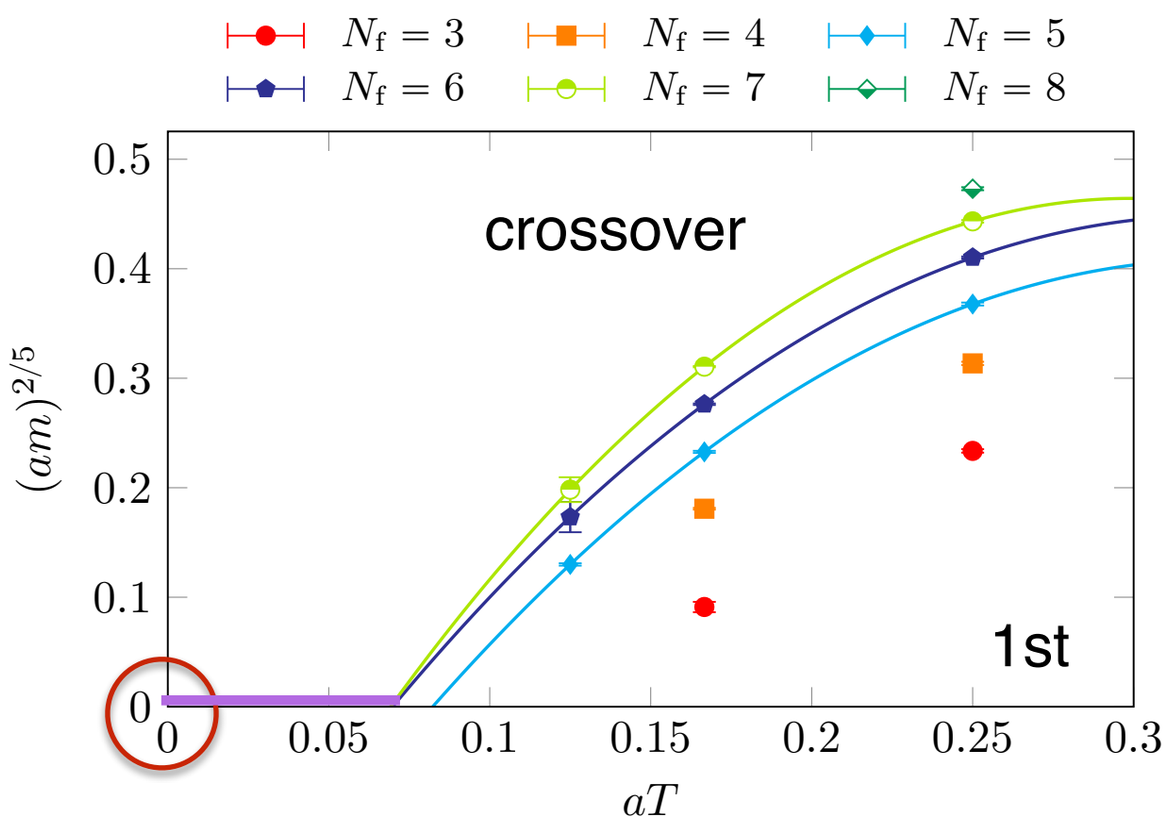
analytic predictions: [Pisarski, Wilczek, Phys. Rev. D, 84]

- Coarse lattices or unimproved actions: 1st order for $N_f = 2, 3$
- 1st order region shrinks rapidly as $a \to 0$
- Improved staggered actions: no 1st order region so far, even for $N_f = 3$

Details and reference list: [O.P., Symmetry 13, 2021]

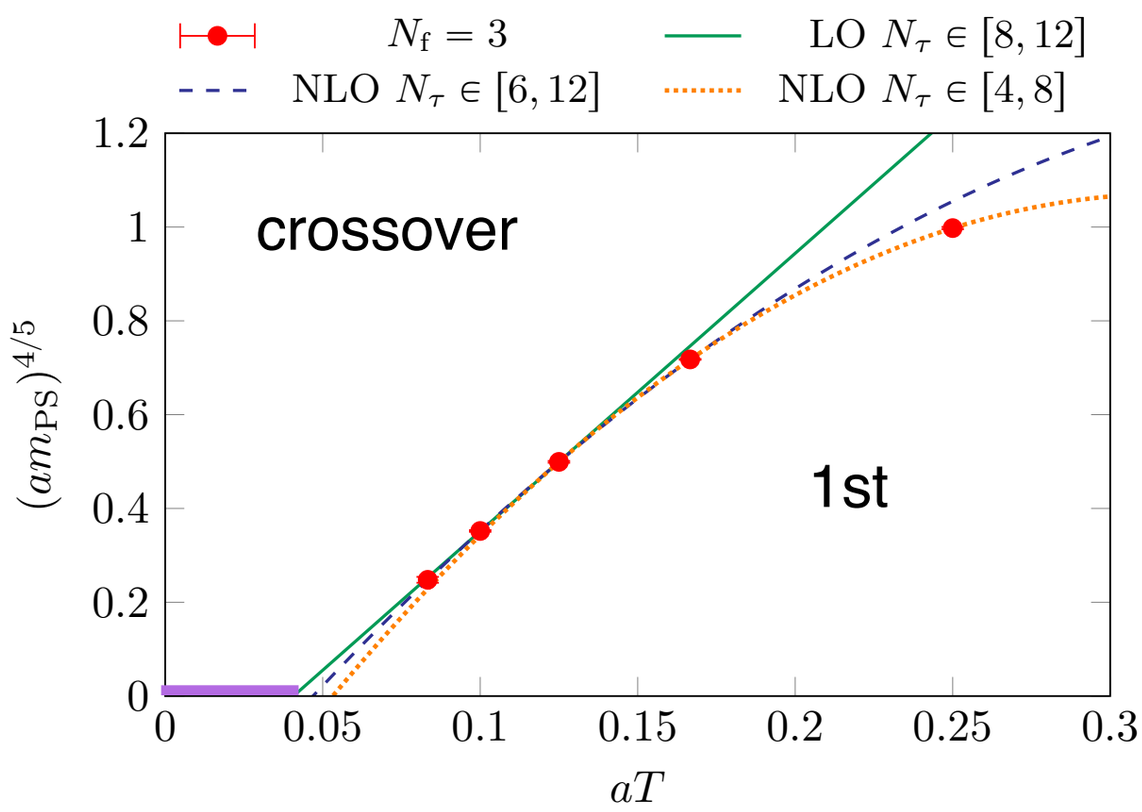
Resolution: scaling in lattice parameter space

[Cuteri, O.P., Sciarra JHEP 21] Staggered



Continuum limit

O(a) improved Wilson rescaled

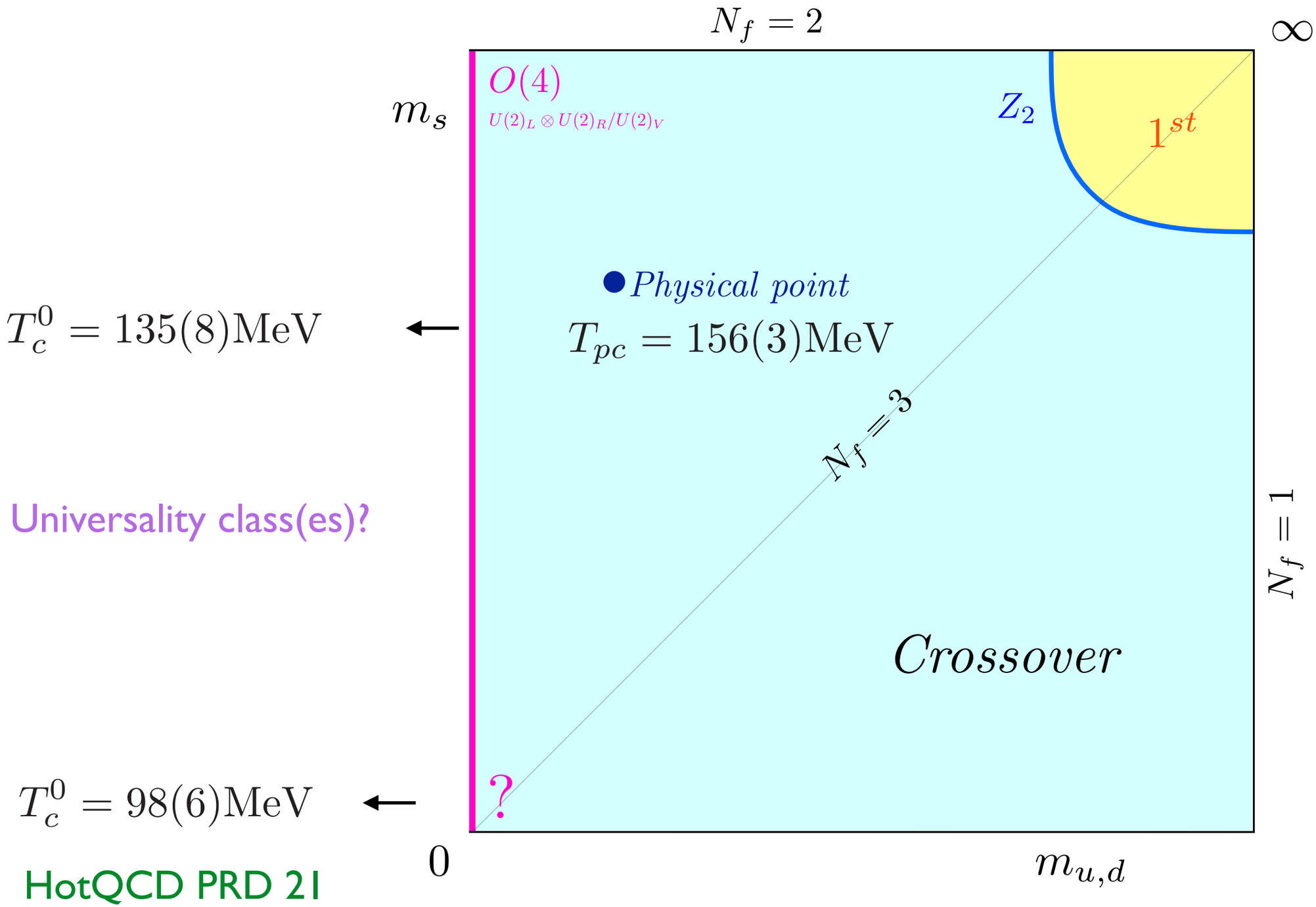


Data from: [Kuramashi et al. Phys.Rev. D, 20]

- Tricritical scaling observed in lattice bare parameter space
- Allows extrapolation to lattice chiral limit, tricritical points $N_\tau^{\rm tric}(N_f)$
- If tricritical point exists: region of 1st-order transitions not connected to continuum
- QCD chiral transition is second order for $N_f = 2 - 7$

5-6 years of simulations on Goethe-HLR (Frankfurt) , LQCD and VIRGO (GSI), JUQUEEN (NIC)

The QCD thermal transition in the continuum



Fully non-perturbative calculations necessary!

From the chiral limit to the physical point

The "standard scenario": [Halasz et al., PRD 98; Hatta, Ikeda, PRD 03…]

Importance of the chiral limit!

2nd order verified!

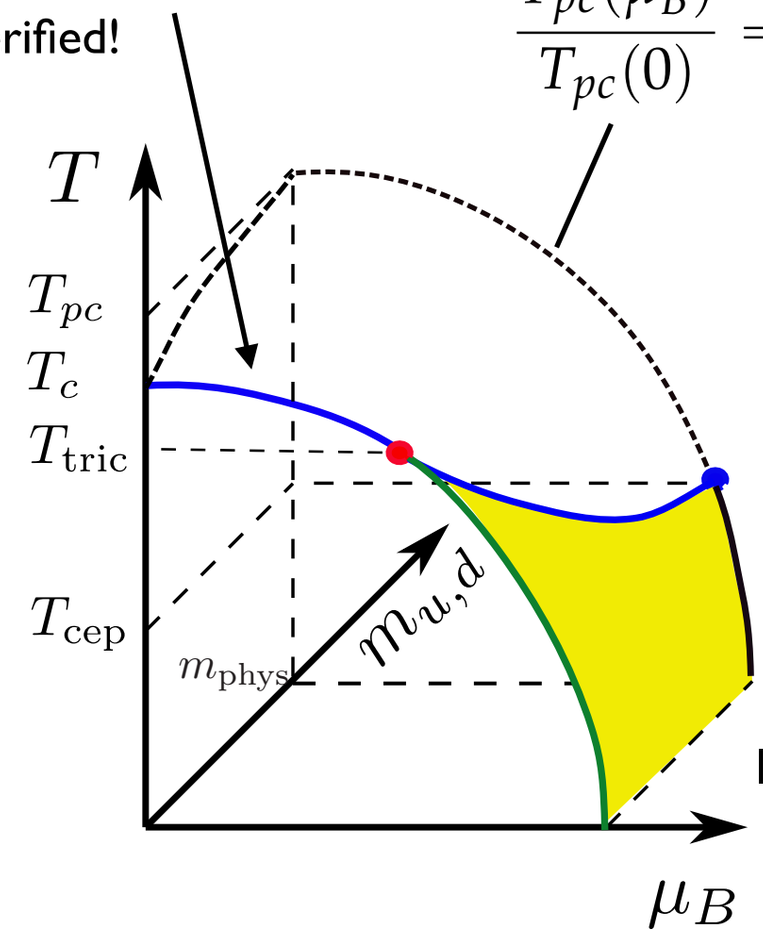
$$\frac{T_{pc}(\mu_B)}{T_{pc}(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_{pc}(0)} \right)^2 + \dots$$

[Bellwied et al, PLB 15]

[Bonati et al, PRD 18]

[HotQCD, PLB 19]

132 MeV [HotQCD, PRD 22]



1st order is model prediction, to be verified

$$T_{pc} > T_c > T_{\rm tric} > T_{\rm cep} \quad \Rightarrow \quad \mu_B^{\rm cep} > 3.1\, T_{pc}(0) \approx 485 \text{ MeV}$$

Direct simulations with refined reweighting $\mu_B^{\rm cep} > 2.5T$ [Wuppertal-Budpest collaboration, PRD 21]

Conclusions

- Chiral phase transition is at zero density is second order for Nf=2-7
- Phenomenologically relevant constraints on phase diagram emerging
- Complete phase diagram in ~5 years?